

to which the components ( $P, Q, R$ ) belong — corresponding to a homogeneous elastic solid, or a mass of incompressible liquid held in a rigid vessel, with the density so distributed as to be in equilibrium. The second equation of (67) is satisfied by this condition, and we have,

$$dX/dx + dY/dy + dZ/dz = \nabla^2 W. \quad (71)$$

Accordingly by (42) we have the original equation of an elastic solid:

$$(m+n) \nabla^2 \delta + \nabla^2 W = 0 \quad (72)$$

which is satisfied by the assumption  $\delta = -W/(m+n)$ .

#### The Aether as an Infinite Elastic Solid.

Hence if this analysis applies to the aether, as an infinite elastic solid, the density of the medium must be arranged so as to give a potential augmenting about each mass of matter embedded in it, as shown in my Dynamical Theory of Globular Clusters, 1912. This latter condition of the potential is described analytically as follows:

$$\omega = \iiint \frac{\rho \, dx \, dy \, dz}{V[(x-x')^2 + (y-y')^2 + (z-z')^2]} = \int \frac{dm}{r} \quad (73)$$

and the inference, from Dynamical Theory, that the potential is greater towards the centres of matter, finds obvious physical illustration in the accumulated arrangement of globular clusters, with the starlight increasing in brightness till it attains a perfect blaze near the centre, in such splendid globular clusters as 47 Tucani and  $\omega$  Centauri.

This increase in potential towards the centres occupied by matter can only be attributed to centripetal stresses in the aether: the medium is thus filled with waves receding from these masses, and the density in the agitated medium is inversely as the wave amplitude or directly as the radius (cf. *Electrod. Wave-Theory of Phys. Forc.* I. 134, 157-8, 1917).

Since the dilatation

$$\delta = \partial\alpha/\partial x + \partial\beta/\partial y + \partial\gamma/\partial z \quad (74)$$

is required to fulfil the equation

$$\nabla^2 (\delta - V) = 0 \quad (75)$$

where  $V$  is the potential, we see at once that the dilatation throughout the aether is similar to the potential. The potential is merely an expression for the total accumulated stress based on average amplitude of the waves,  $A = k/r$ , and the density  $\sigma = \nu r$ , and the attractive force  $f = k^2/r^2 = \partial V/\partial r = -M/r^2$ . This proves the *Electrod. Wave-Theory of Phys. Forc.* to represent the true order of nature.

Accordingly, we have the following table for the displacement or wave amplitude, density, potential and force:

Displacement or amplitude $A = k/r$	$f = k^2/r^2 = A^2 = \delta^2$ $= \partial V/\partial r = -M/r^2$
Density of the aether $\sigma = \nu r$ , $\nu = 1/k$	
Potential $= V = M/r$	
$V = \delta = A$	

Since the direction of the force always is central, and the waves react towards the origin at the centre of gravity, we conclude from this whole investigation:

1. That the aether behaves as an infinite aeolotropic elastic solid, with displacements everywhere identical with the electrodynamic wave amplitude  $\delta$  and also identical with

the potential  $V$ . This gives a geometrical and physical significance to the potential, which hitherto has been entirely lacking, and long proved bewildering to the geometer and the natural philosopher.

2. If this were not true, the general equations for an infinite elastic solid could not have been integrated by Lord *Kelvin* as outlined above (cf. *Cambridge and Dublin Mathematical Journal*, 1848). But as this celebrated geometer effected such an integration for the general equations of an infinite isotropic elastic solid, without giving a physical interpretation to the solution found, we see that Lord *Kelvin's* mathematical genius builded better than he knew, and natural philosophers are now enabled for the first time to interpret physically one of the sublimest results in the whole range of mathematical science.

*Newton* surmised that if the density of the aether varied directly as the distance from the centre, it would press towards the centre so as to develop the force of gravitation. *Maxwell* holds that *Newton* conceived this action as analogous to hydrostatic pressure, but we have shown that the reaction of the waves with amplitudes  $A = k/r$  produces this arrangement of density and would generate an effect similar to mere hydrostatic pressure (cf. *Electrod. Wave-Theory of Phys. Forc.*, I. 134, 1917).

Why the Forces between the Sun and Planets Operate in Right Lines: Weierstrassian Theory of the Resulting Least Action.

a) Imagine waves propagated from the sun and earth as shown in the accompanying diagram: and let the velocities of the mutually interpenetrating waves from the centres  $S$  and  $E$  be  $V_1$  and  $V_2$ .

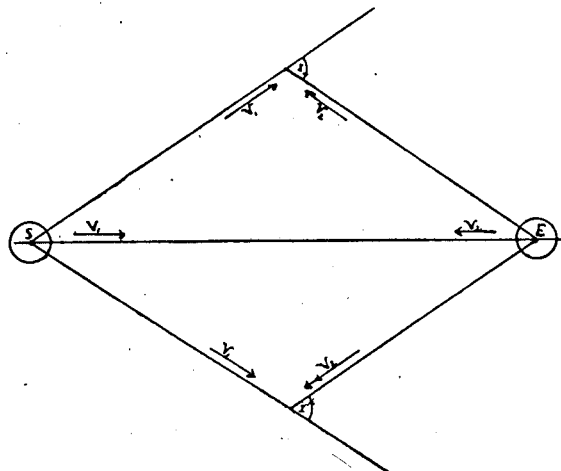


Fig. 4. Illustration of the interpenetration of waves between the sun and earth, which gives maximum tension along the line  $SE$ , where the interpenetration is with double the velocity of light.

The problem arises as to the effect of the relative interpenetration of the waves, the velocities  $V_1$  and  $V_2$  being equal, but the amplitude and direction of propagation different at every point of space.

\*) By referring to fig. 1, section 2, we see the physical meaning of this equation: the aether has dilatation,  $\delta = V$ , near the sun, owing to the increasing amplitude of the waves. This dilatation and decreased density of the medium exists about every star and planet.