

Applying Method of Reference 2 to Effectively Calculating Performance of Microwave Radiation Thruster

Yang Juan, Yang Le, Zhu Yu, Ma Nan

(College of Astronautics, Northwestern Polytechnical University, Xi'an 710072, China)

Microwave thrusters without propellant are a new concept. In these thrusters, microwaves are radiated into a sealed conical cavity through a waveguide, which act on the surface of the cavity and follow the contour of the thrusters to produce thrust. The advantages of these thrusters are: (1) Producing thrust without propellant; without erosion, wear and thermal stress from the hot exhaust gas; and at the same time increasing quality. (2) If the microwave output power is stable, the performance of thrusters is not affected by its working environment. (3) Existing experiments and theories show that using different structure materials, with microwave output power of 1kW, the range of thrust is 0.1 – 31500N. From above mentioned, these thrusters can be used in satellites, deep space explorers and spacecraft, bringing the advantages of increasing their loading capability and operating life.

Microwave thrusters without propellant is invented by Roger Shawyer in a British company that research on satellite thrusters (SPR Ltd), he has already started the theories analysis and prototyping. Roger Shawyer has designed a conical cavity resonator, similar to the transverse wave TE mode, with a quality factor of 50,000, rectangular waveguide radiates the microwave into the resonator, using a non-resistance test turntable, the practical results show thrust produce by the resonator with the microwave output power of 300W is 86.2mN, this measurement match the calculation.

From the limited papers and materials, the research to this type of thrusters is limited. Research has only been done on the calculation and practical measurement of thrust on conical resonators in TE modes. The theoretical calculation first quantize the microwave field, the net thrust is then calculated from the change in momentum from the interaction of quantum to the cavity wall, and only the consider the contribution to the thrust from the interaction of the two End-Sides of the cavity with quantum, this method is rather crude.

Reference 2 studies the interaction of the electromagnetic wave and plasma, using the electromagnetic tension to process the force produced by microwave under a surface of a volume, producing an effective and practical research methods for this paper. This paper first shows the thought behind the invention

of the thruster, then uses the classical theory of electrodynamics to analyse the theory of propulsion, to derive the calculation of electromagnetic tensor for the calculation of thrust. With different microwave modes at no load, and from the equation of Maxwell, using finite element method to perform numerical calculations for different idealised conical cavity structures in different modes, from this to obtain the cavity distribution of electromagnetic field at resonant and quality factor at no load. Under 1000W of microwave radiation, using finite element method to numerically calculate the distribution of electromagnetic field of the non-idealised conical resonator that has microwave coupling window, under different structures and different microwave modes, from these to obtain the distribution of the electric and magnetic fields, then using surface tensor to calculate thrust along axes. In comparison to the calculation from Roger Shawyer, the calculation of electromagnetic tensor in this paper is not constrained by the cavity structure, so can accurately analyse the performance of the thrusters.

1 Original idea of the microwave thruster without propellant

Roger Shawyer's idea [1] of the microwave thrusters without propellant is originated from Planck hypothesis, Einstein's quantum theory of light, and theory of microwave.

The vibration of charge particles (where object is made from), near their equilibrium positions can be seen as charged oscillators, this oscillators can emit or absorb radiation energy of $E = nhf$, n is the quantum number, h is the Planck's constant and f is the oscillators frequency. Light and electromagnetic waves also have properties of particles, are photons that travel at the speed C , each photon has energy of $E = hf$, f in here is the frequency of light or electromagnetic waves. According to relativity, $E = mc^2$, the mass of photon is $m = E/C^2 = hf/c^2 = h/c\lambda$. For light wave, energy of quantum is transferred at the speed of light, so the photon momentum is $P = mc = hf/c = h/\lambda$. For electromagnetic wave, energy of quantum is transferred at the group velocity v_g , its momentum is $p = mv_g = hf v_g / c^2$. When the electromagnetic wave in the waveguide is transmitting in travelling-wave state, $v_g = c\lambda_0 / \lambda_g \sqrt{\mu_r \epsilon_r}$, where μ_r and ϵ_r are the transmission medium relative permeability and relative permittivity respectively, the wavelength of the electromagnetic wave within the waveguide is $\lambda_g = \lambda_0 / \sqrt{\mu_r \epsilon_r - (\lambda_0 / \lambda_c)^2}$, λ_c is the waveguide cut-off wavelength. Assuming the electromagnetic radiates into the waveguide metal sealed end, and the energy of the injected wave per second is P_0 . According to Einstein's quantum theory, each quantum has energy of hf , so the amount of injected quantum is $N = P_0 / hf$. When all quantum elastic collision with the metal, the reaction to the metal surface is

$$F_g = 2Nhf v_g / c^2 = 2P_0(v_g/c)/c = 2P_0\lambda_0 / c\lambda_g \quad (1)$$

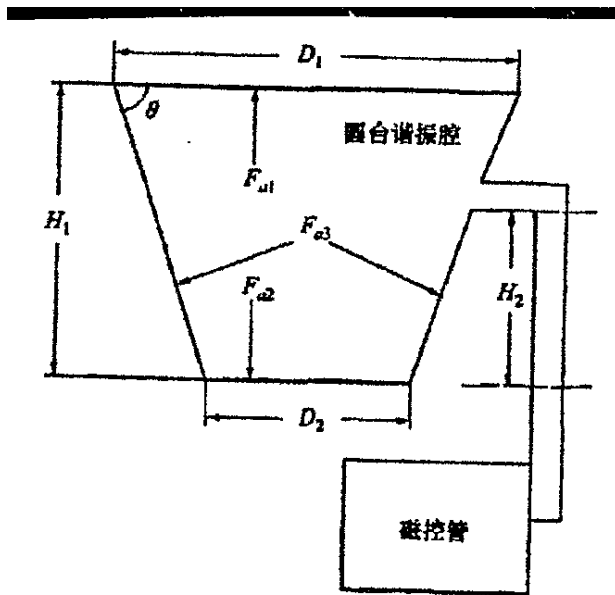


Figure 1

In the conical microwave thrusters in Figure 1, the radiated microwave produce three forces normal to the axes of the front, back end and its side, F_{a1} , F_{a2} and F_{a3} , the net thrust obtained by the thrusters along the axes are

$F_a = F_{a1} - F_{a2} - F_{a3} \cos \theta$. In order to obtain the largest thrust, the design of the cavity requires F_{a1}/F_{a2} to be the largest, F_{a3}/F_{a1} to be the smallest, so $F_a \approx F_{a1} - F_{a2}$.

With fixed microwave power, in order to obtain the maximum thrust along the axes, the cavity of the thrusters should be designed according to the resonator, so the reflected microwave energy of the cavity to minimum. According to microwave theory, the quality factor of the resonator is:

$$Q = 2\pi(\text{stored energy}/\text{periodic loss}) \quad (2)$$

Under the best resonant state, the injected microwave energy to the resonator is lost continually through absorption. Based on the injected energy, energy stored in the cavity is amplified by Q times, so the reaction on front and back ends of the cavity is also amplified by Q times, or $F \approx Q(F_{a1} - F_{a2})$.

2 Electrodynamics theory of the microwave thrust without propellant

In addition, the microwave thrusters can be explained using the classical electrodynamic theory.

First, using the Slater perturbation theorem as a starting point, in a tuning cavity, when the cavity wall deform, the resonance frequency of the cavity drifts. The drift in resonant frequency can be expressed by the energy of the electromagnetic field energy density and in electromagnetic field strength, or

$$Df/f = -(\epsilon_0 E^2 - u_0 H^2) dV / \int (\epsilon_0 E^2 + u_0 H^2) dV$$

When the disturbance within the cavity occurs in an area dominated by electric field, the resonant frequency decreases, then the electromagnetic force lead to the reduction of store energy within the cavity. When the disturbance within the cavity occurs in an area dominated by magnetic field, the resonant frequency increases, then the external force lead to the increase in the store of energy within the cavity. It can then inferred, designing a symmetric (along the axis) microwave resonator, using the electromagnetic field components of the cavity wall, thrust can be produced along the axes of the cavity, this resonator become a thruster that produce thrust using microwave radiated electromagnetic field.

If the microwave electromagnetic field consists of charge particles, due to the electromagnetic force, the charge particles can travel within the electromagnetic field, so the charge particles can acquire energy and momentum from the electromagnetic field. This indicates that electromagnetic field have energy and momentum. Charge particle energy and momentum fulfil the following relationship:

$$Dgp/dt=pE+JB \quad dwp/dt=J.E \quad (3)$$

Where J is current density of the moving particles, from the equation of Maxwell, the following is obtained:

$$V.(ExH)=-J.E-d/dt(1/2E.D+1/2H.B) \quad (4)$$

where $S=ExH$ represents the flux density vector of electromagnetic field or Poynting vector,

$wf=1/2E.D+1/2H.B$ represents the density of electromagnetic.

$$D/dt(wp+wf)+V.S=0$$

$$\int S.nds=-d/dt\int(wp+wf)dv=0 \quad (5)$$

so

$$\int wpdv+\int wfdv=const \quad (6)$$

Differentiate the Poynting vector and consider the Maxwell equation, the following equation can be derived:

$$D/dt(uoeo+gp)=-V \\ ((1/2eoE^2 +1/2 uoH^2)I-eoEE-uoHH) \quad (7)$$

Because gp is the density of the charge particles, compare the term, $uoeoS=uoeoExH$, in the equation above, it represents the density of momentum of the electromagnetic field gf . The right hand side of the above equation can be define as the momentum flux density tensor of electromagnetic field

$$\Phi=1/2(eoE^2+uoH^2)I=eoEE+uoHH \quad (8)$$

Introducing a new symbol $T = -\Phi$, used for the tension tensor of electromagnetic field per unit area, this is first proposed by Maxwell, so it is also called Maxwell tension tensor. Integrating Equation 6 to:

$$D/dt \int_V (g_f + g_p) dV = \int_S n \cdot T dS \quad (9)$$

compare with the classical conservation of momentum $dG/dt = F$, the right hand side of Equation 9 represents the electromagnetic force produced by the electromagnetic tensor acting on the surface V , regardless whether charge particles are presented within the volume, the surface electromagnetic force can change the momentum within the volume V .

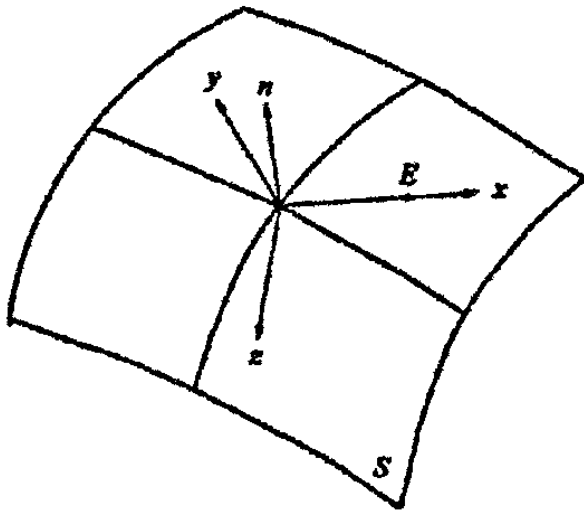


Figure 2.

From Equation 8, it can be seen that Maxwell tension tensor is made from tensors of electric and magnetic fields. As shown in Figure 2, define a surface of a volume and its coordinate system (u_x, u_y, u_z) and unit normal vector $n = n_x u_x + n_y u_y + n_z u_z$ and the coordinate system x axis coincides with the electric field vector, the tensor of electric field is:

$$T_e = \epsilon_0 E_x E_x - 1/2 \epsilon_0 E^2 (u_x u_x + u_y u_y + u_z u_z) = 1/2 \epsilon_0 E^2 (u_x u_x - u_y u_y - u_z u_z) \quad (10)$$

So the effect of electric field force acting per unit area on the surface S is

$$F_e = T_e \cdot n = 1/2 \epsilon_0 E^2 (n_x u_x - n_y u_y - n_z u_z) \quad (11)$$

The amount of force is $|F_e| = 1/2 \epsilon_0 E^2$

Similarly, the effect of magnetic field force acting per unit area on the surface S can be obtained:

$$F_m = \epsilon_0 n^2 \frac{1}{2} \mu_0 H^2 (n_x u_x - n_y u_y - n_z u_z) \quad (12)$$

The amount of force is $|F_m| = \frac{1}{2} \mu_0 H^2$

The schematic of microwave thrust is in Figure 1, the electric field of the cavity wall is in the same direction as the normal of the wall, and magnetic field is vertical to the wall. Also consider Equation 2, under the best resonant mode, the microwave energy injected to the resonator also amplified by Q times, cavity wall E^2, H^2 also amplified by Q times, so the thrust along the axes are:

$$F_a = Q \left[\int_{A_1} (F_{e1} + F_{m1}) dA - \int_{A_2} (f_{e2} + f_{m2}) dA - \int_{A_3} (F_{e3} + F_{m3}) \cos \theta dA \right] \quad (13)$$

where A_1, A_2 and A_3 in the equation are the areas of the Large-End, the Small-End and side walls.

3 Electromagnetic characteristic and thrust calculation of the thruster

3.1 Modes calculation and analysis

The purpose of analysing the modes of the microwave thrusters is to calculate, under a fixed frequency and fixed idealised cavity structure, the distribution of the electromagnetic field and the quality factor under no load, to find the maximum quality factor and a suitable method for microwave coupling, to find the best cavity structure under a fixed resonant frequency. Currently have two ways to find the electromagnetic field of the rectangular and circular waveguides, the eigen-value equation which is an analytical method and numerical solution, when finding solution for the resonator, Maxwell equation in is need to be created in a spherical coordinate system, because the complexity of the spherical coordinate fielder equation, has not found anyone using eigen-value method to calculated the distribution of the resonant field. Only find in Paper [4] using asymptotic method for conical waveguide. That method assume a equivalent radius a_e , believes field of wavefront sphere of cone waveguide $E_o, E\Phi, H_o, H\Phi$ can use its wavefront position radius a_e equivalent circular waveguide field $E_r, E\Phi, H_r, H\Phi$, this method of finding the field distribution within the conical resonator can be used as reference, but the accuracy reduced as the cone half opening angle increases. Using finite element to numerically simulate the Maxwell electromagnetic equation for the idealised conical resonator, the distribution of electromagnetic can be obtained directly, this method is not limited by the cavity structure and microwave mode.

With the three main modes of cylinder waveguide, TE011, TE012, TE111, using finite element to calculated the distribution of electromagnetic field of the conical resonators under the four modes TE011, TE012, TE111 and TM011 with frequency near 2.45GHz. To select the conical resonator diameters, first follow the cylinder resonator operation mode in Figure 3 to select cylinder diameter and

height in single mode, that diameter is the conical cavity average diameter, select the diameter of the Small-End as the waveguide cut-off diameter, the diameter of the large end can be obtained according to average diameter. Perform the finite element numerical simulate the distribution of microwave electromagnetic field in resonant for the cavity. Repeat adjusting the Small and Large ends diameters D_1 , D_2 and height H_1 of cavity within a limit, to obtain the distribution of the electromagnetic field. The quality factor of this resonator under no load can be calculated by the following equation:

$$Q_u = \frac{\int |H|^2 dv}{h/2 \int |n \times H|^2 ds + \text{tg} \delta \int |H|^2 dv} = (14)$$

Where tg is the electric loss within the cavity, n is the normal vector of the wall, s is the cavity surface area, v is the volume of the cavity.

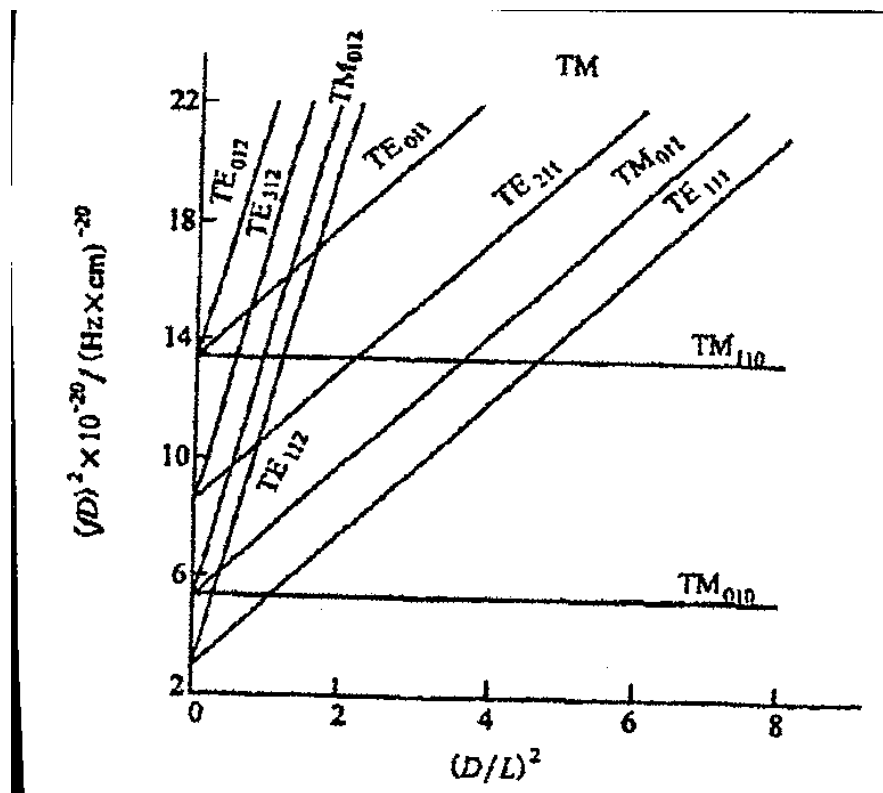


Figure 3.

In order to obtain the change in quality factor under different structures in the same mode, select different diameter of the Large End of the cavity, using electromagnetic field in finite element to numerically simulate the microwave electromagnetic field at resonance within the cavity, and change the height of the cavity to obtain almost the same resonant frequency. Using brass as the

material, under the same resonant mode, assumed the Large End diameter from small to large order with symbols of sn1 and sn2, where n=1 represent mode TE011, n=2 for mode TE012, n=3 for TE111, n=4 for TM011, using three dimensional adaptive mesh analysis, divide the TE011 to 5247 elements, divide TE012 to 3770 units, divide TE111 to 6126 units and TM011 to 3954 units.

Using finite element method to solve Maxwell equation under the mesh to quantize, to calculate the results of the idealised conical cavity thrusters in different modes as in Figure 1, results show: (1) within the cavity, in modes TE011, TE012, TE111 and TM011, modes TE021 has the higher quality factor, followed by mode TE011. The reasons can be explain from the the distribution of electromagnetic field in modes TE012 and TE011: the symmetric distribution of electromagnetic field in modes TE012 and TE011, electric field only has horizontal component of $E\Phi$, and $E\Phi$ is zero along the axis and near the wall, the magnetic field near the side wall follow the circular direction, and magnetic field strength in the front and back is almost zero. These electromagnetic characteristics determine the cavity in modes TE012 and TE011 to have relatively small wall current loss and high no load quality factor. (2) By keep the diameter of the Small End constant, increase the large end of the cavity, in order to have the same resonant frequency, cavity height must be reduced, quality factor also reduce.

Conical resonator must connect the microwave transmitting path, microwave energy is transfer through the coupling device to the cavity and excite the electromagnetic vibration for correct operation. The requirement of the coupling device is to create a vibration mode within the cavity, at the same time, preventing interference mode.

Table 1.

模态	TE ₀₁₁		TE ₀₁₂		TE ₁₁₁		TM ₀₁₁	
频率/GHz	2.50	2.45	2.45	2.41	2.45	2.45	2.45	2.45
腔体	S ₁₁	S ₁₂	S ₂₁	S ₂₂	S ₃₁	S ₃₂	S ₄₁	S ₄₂
高度/mm	118	89	240	175	96	87	150	122
Q 值	62705	46622	79523	77105	25883	25630	25914	25025

Analyse the distribution of electromagnetic field for the different structure of the cavity given in table 1, distribution of field in mode TE011 is symmetrical and it has an area dominated by magnetic field in the centre of the side of cavity, there exist an area of strongest electric field within the cavity; distribution of field in mode TE012 is symmetrical and it also has an area dominated by magnetic field in the side of cavity, there exist an area of strongest electric field within the cavity; distribution of electromagnetic field in mode TE111 is non-symmetrical

and it has an area dominated by electric field in the centre axis, there exist an area of strongest magnetic field within the cavity; electromagnetic field in mode TE011 is symmetrical and it has an area dominated by electric field in the axis between the Large and Small Ends. These results determined the following microwave coupling for the conical resonator: TE011 and TE012 use the rectangular waveguide in mode TE10 in the resonator side wall where magnetic field is relatively large to couple as in Figure 1, Mode TE111 use coaxial cable under the resonator axis using coaxial cable in resonator Small End where electric field is relatively large to probe coupling.

3.2 Thrust calculation

The calculation of thrust of the loaded thrusters is done through the calculation of electromagnetic field. Assume the loaded condition is, microwave input power is 1000W, frequency around 2.45GHz, brass as the material of the cavity, following the different dimensions of the cavity in Table 1, with air as filling medium and electric wall boundary condition, using the same three dimensional adaptive meshing method, dividing into 69549 points, 50088 units, using finite element method in mesh to quantize the Maxwell equation [5], calculate the distribution of electromagnetic field with an input microwave input power of 1000W for modes TE012, TE111 and TM011, then use equations 13 and 14 to calculate, under different resonant modes and different structure, the time average of the net thrust in one cycle and the loaded quality factor as in Figure 2. Compare the calculated quality factor in Table 1 and Table 2, the following are found: (1) Under input microwave power of 1000W, quality factor of cavity reduce differently, this is because when compare the practical input power of 1000W to the input of 1mW, the loss to the cavity wall increase proportional, so reducing the quality factor. (2) The calculation of the different modes and different cavity structure, the mode TM012 which has smallest cavity Large-End has the largest thrust, so has the highest quality factor and thrust. Mode TM011 thrusters has the worst performance. (3) As the Large-End of the cavity increases, the height of cavity reduce, cavity volume and wall surface area also reduced, leading to low quality factor and producing less thrust.

Table 2

模态	TE ₀₁₁		TE ₀₁₂		TE ₁₁₁		TM ₀₁₁	
频率/GHz	2.50	2.45	2.45	2.41	2.45	2.45	2.45	2.45
腔体	S ₁₁	S ₁₂	S ₂₁	S ₂₂	S ₃₁	S ₃₂	S ₄₁	S ₄₂
耦合高度 H ₂ /mm	15	1.3	40	40				
计算值 Q ₀	43 414	37 212	53 036	14 256	17 500	16 743	20 262	6 080
计算推力 F ₀ /mN	411	186	456	270	264	227	190	88
实验值 Q ₀	32 000		50 000					
实验推力 F ₀ /mN	214		315					
Q 值修正系数 c ₁	0.737	0.737	0.943	0.943				
推力修正系数 c ₂	0.521	0.521	0.691	0.691				
修正值 c ₁ Q ₀	32 000	27 429	50 000	13 439				
修正推力 c ₂ F ₀ /mN	214	97	315	187				

4 Conclusion

Microwave thrusters without propellant does not require propellant, so without erosion from the high temperature gas stream, erosion and heat transfer problem, the performance of the thrusters is not affected by the working environment, at the same time, increasing the spacecraft quality, using different material for the structure can increase the range of thrust, suitable for use in space and near space spacecraft.

There are two ways to explain this new thrusters, (1) from the Plank's hypothesis and Einstein's quantum theory of light, also the theory of microwave to explain the thrust from the thrusters, that is to quantise the injected microwave to the sealed cavity into photons, its travelling speed is the group speed, photons and the thrust cavity wall elastic collision produce the net thrust, (2) From the classic theory of electrodynamics to explain how the thrust is produce by the thrusters, according to the kinetic energy and conservation of momentum of the electromagnetic system within its volume, Maxwell equation and electromagnetic flux density vector can found out the source of the thrust is coming from the integration of the electromagnetic tensor along the surface of the volume.

Using finite element numerical method to numerical analyse the classical Maxwell equation of electric field of the idealised conical resonator, to obtain the model and practical of the distribution of the electric field of the cavity under 1000W. By analyse the properties under different modes and the different properties. Calculation show that under the four modes, TE011, TE012, TE111 and TM011, the quality factor of TE012 is highest and with highest thrust, followed by TE011. With the Small End of the cavity unchanged, the quality factor and thrust decrease with the increase in the Large End.

The TE10 mode rectangular waveguide is suitable for use in modes TE011 and TE012, and coupling to the side wall of the resonator in where the magnetic field is relatively strong, coupling using coaxial cable is suitable in modes TE011 and TM011, in axis of the resonator where electric field is relatively large.

With 1000W microwave input, using brass as the material of the cavity, using the classical theory of electrodynamics, the maximum theoretical thrust produced in modes TE011 and TE012 is 411mN and 456mN respectively, and the practical measurements are 214mN and 315mN.

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