INFLATIONARY UNIVERSE GENERATED BY THE COMBINED ACTION OF A SCALAR FIELD AND GRAVITATIONAL VACUUM POLARIZATION

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Received 25 February 1985

The chaotic inflationary universe scenario based on a joint account of vacuum polarization and scalar field effects is considered. The probability for the universe to go through the inflationary stage increases compared to that in the separate scenarios. The total expansion during the inflationary stage is equal to the product of the expansions in the separate scenarios.

1. In the development of cosmology during the last few years, ever increasing hopes are related to models in which the universe was exponentially (or quasi-exponentially) expanding at very early stages of its evolution. There are two main types of models. The models of the first type are connected with the account of the one-loop corrections to the Einstein equations and higher-derivative terms in the gravitational action [1, 2]. The second-type models are based on the investigation of cosmological consequences of a slow evolution of a scalar field φ in the exponentially expanding (inflationary) universe [3–5]. Below, these models will be called for brevity type I and type II models respectively. For an extensive review of the history of the development of these models, their advantages and difficulties and also of the objectives which were pursued by their creators see ref. [6].

For a long time the type I and type II models seemed to be competing and probably even mutually incompatible. The exponential expansion of the universe was thought of as a rather exotic phenomenon and it seemed unlikely that both abovementioned mechanisms could work simultaneously. Moreover, different initial conditions seemed to be necessary for the realization of the scenarios [1,3,4]. Since quantum gravity is far from being completely elaborated and since it was not possible to solve the primordial monopole problem [7] in the first models of the type I [1], most scientists originally preferred to study the type II models [3,4], and the favourable one was the new inflationary universe scenario [4]. Meanwhile, since 1982 the status of the models of both types has significantly changed.

2. First of all, the type II models were essentially modified. It was shown that in order to obtain small density perturbations after inflation $\delta\rho/\rho \sim 10^{-4}$, which are necessary for galaxy formation, one should consider a theory of an extremely weakly interacting field φ [8–11]. Such a field usually does not undergo any high-temperature phase transitions in the early universe since the time necessary for a considerable modification of its initial value due to high-temperature effects usually exceeds the age of the universe [6, 12]. Therefore it is extremely difficult (if possible at all) to obtain a completely satisfactory realization of the new inflationary universe scenario [4], which is based on the theory of high-temperature phase transitions. Despite many efforts, no consistent realization of the new inflationary universe scenario has been suggested so far.

Fortunately, this difficulty disappears in the chaotic inflation scenario [5], according to which the universe was initially filled with some chaotic initial distribution of a scalar field φ. It turns out that, in a wide class of theories, the domains of the universe with $|\varphi| \gtrsim CM_P$ [$C = O(1), M_P = G^{-1/2}$ is the Planck mass] expanded more than e^{70} times during the slow rolling of the field to the minimum of its potential energy $V(\varphi)$. The specific property of the chaotic inflation scenario is that the expansion of the universe at the inflationary stage is not precisely exponential. The Hubble parameter $H = \dot{a}/a$ [$a(t)$ is the scale factor of the universe] in this scenario slowly changes during inflation. However, the variation of H at the stage of inflation is small compared to the rate of the expansion itself, i.e. the condition

$$|\dot{H}| \ll H^2 \qquad (1)$$

is satisfied. Then $R_i^k \approx \frac{1}{4}\delta_i^k R \approx -3H^2\delta_i^k$. The stages when eq. (1) holds will be called the quasi-de Sitter ones.

According to the chaotic inflation scenario, the more the initial value of φ is, the greater is the inflation of the universe. The initial potential energy density of the scalar field $V(\varphi)$ in this scenario can be arbitrarily large. This circumstance has two important consequences. Firstly, in contrast to other versions of the inflationary universe scenario, the flatness problem can be solved here even if the universe was spatially closed initially [6]. [Let us remind [13] that if the universe is spatially closed and its energy density at the Planck time $t_P \sim M_P^{-1}$ is far from the critical one, then it recollapses during a typical time $t \sim t_P$, i.e. before the beginning of inflation, unless inflation starts with $V(\varphi) \gtrsim M_P^4$]. Secondly, the possibility for inflation to begin with $V(\varphi) \gtrsim M_P^4$ makes possible a realization of the inflationary scenario for the type II models based on the idea of quantum creation of the universe [6,14]. (Analogous realization for the type I models was suggested in ref. [15].)

It is especially important that in the infinite (e.g. open) universe, there should be infinitely many domains of a size $l \gtrsim 2H^{-1}$ [5] where $|\varphi| \gtrsim CM_P$ just before the beginning of the inflationary stage. The size of these domains at the present moment is larger than the size of the observable part of the universe. In this scenario inflation of the universe, from an exotic phenomenon which could occur in a number of rather specific theories only, becomes a natural consequence of chaotic initial conditions in the expanding universe.

3. Simultaneously, a similar metamorphosis has taken place with the type I models. The first model of this type [1] was constructed on the basis of the conformal anomaly of the total vacuum energy–momentum tensor of massless (or light enough) quantum fields,

$$\langle T^l{}_l \rangle = -(1/2880\pi^2)[k_1 C_{iklm}C^{iklm} + k_2(R_{ik}R^{ik} - \tfrac{1}{3}R^2) + k_3 \Box R] \qquad (2)$$

where $R = R^l{}_l$ and the constants k_1, k_2, k_3 depend on the number and types of the quantum fields contributing to (2). However, it was realized soon that to provide a sufficient duration of the inflationary stage one should assume that $k_2 > 0, k_3 < 0, |k_3| \gg k_2$, which is not satisfied in typical cases (in particular, there exist supergravity theories where $k_2 = 0$). Therefore another version of the type II model was considered [2] in which the local term $M_P^2 R^2/96\pi M^2$ with $M \ll M_P$ was added to the lagrangian of the gravitational field. This corresponds to the effective renormalization of k_3. After that, the effective constant k_3 need not be proportional to the number of light quantum fields. More generally, a local term $f(R)$ can be added to the gravitational lagrangian, where f is an arbitrary function restricted by the condition $\lim_{R\to 0}[f(R)/R] = 0$ only (see ref. [16] in this connection).

It should be mentioned that the procedure of addition of quadratic terms to the Einstein gravitational action in general is not completely harmless since it may lead to the appearance of ghost particles with negative energy [17]. Fortunately, the introduction of the R^2 or $f(R)$ term (in contrast to the $C_{iklm}C^{iklm}$ term) does not lead to the appearance of ghosts [17]. If the sign of the R^2 term is chosen correctly then we obtain only one new scalar particle with positive energy and positive mass squared. It is this particle that was called scalaron in ref. [1]. Therefore, the hypothesis that the scalaron rest mass M is significantly smaller than the Planck mass $[M \sim (10^{-4}-10^{-5}) M_{\mathrm{P}}]$ does not lead to any difficulties or inconsistencies. If $M \ll M_{\mathrm{P}}$ and $R_{iklm}R^{iklm} \ll M_{\mathrm{P}}^4$ then the k_1 and k_2 terms in eq. (2) can be neglected. Thus, the type I models need not be based on the non-local part of conformal anomaly [we call the k_1 and k_2 terms in eq. (2) non-local because they cannot be obtained by variation of any local action with respect to the metric]. This conclusion completely eliminates the criticism of the type I models expressed in ref. [18].

It was shown in ref. [2] that the modified type I models possess a prolonged quasi-de Sitter stage. Inflation in these models can also be chaotic, i.e. the quantity H can be different in different points of space, $H = H(t, r)$. Therefore, the inflationary universe scenario based on a type I model with the R^2 term (we shall continue to call this term vacuum polarization for brevity) can be realized over a wide range of initial conditions for the scalar curvature R.

If $M \lesssim 10^{15}$ GeV, then the monopole problem can be also solved in this scenario. This is the case because the phase transition with SU(5) breaking takes place before the end of the inflationary stage [even if the unbroken SU(5) phase was present at the very beginning of the expansion] and the temperature never grows to 10^{15} GeV after the end of inflation (the scalaron lifetime $\tau \sim M_{\mathrm{P}}^2/M^3$ [1] for $M \lesssim 10^{15}$ GeV is large enough, and therefore reheating occurs comparatively late). The same reason is responsible for the absence of the monopole problem in the type II models based on supergravity [12].

4. Thus, it appears now that scenarios of both types do not need significantly different initial conditions and, therefore, they can be realized simultaneously. So, we consider the generalized model where the expansion of the universe is determined by the combined influence of a scalar field φ with a potential $V(\varphi)$ and by the vacuum polarization described by the $M_{\mathrm{P}}^2 R^2/96\pi M^2$ term in the total lagrangian (the numerical coefficient is chosen here in such a way that the scalaron mass is equal to M). Then the evolution of the flat Friedmann–Robertson–Walker cosmological model and of the scalar field follows from the equations

$$H^2 = (8\pi/3M_{\mathrm{P}}^2)[\tfrac{1}{2}\dot{\varphi}^2 + V(\varphi)] - M^{-2}(2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2),$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0, \quad H = \dot{a}/a, \quad M \ll M_{\mathrm{P}}, \tag{3}$$

where the dot denotes differentiation with respect to the usual time t and the prime, differentiation with respect to φ [the term in eq. (2) has been neglected].

If the scalar field changes slowly at the quasi-de Sitter stage (1), $|\dot{\varphi}/\varphi| \ll H$, we can neglect $\ddot{\varphi}, \dot{\varphi}^2$, $H\ddot{H}$ and $\dot{H}^2$ in eqs. (3), and they take the following form

$$6H^2\dot{H}/M^2 + H^2 = (8\pi/3M_{\mathrm{P}}^2)V(\varphi),$$

$$3H\dot{\varphi} + V'(\varphi) = 0. \tag{4}$$

Eq. (4) can be integrated in the general case by using φ as an independent variable:

$$t = -3\int \frac{\mathrm{d}\varphi\, H}{V'(\varphi)},$$

$$\mathrm{d}H^2/\mathrm{d}\varphi = [M^2/V'(\varphi)]\,[H^2 - 8\pi V(\varphi)/3M_{\mathrm{P}}^2],$$

$$H^2 = -\tfrac{8}{3}\pi(M/M_{\mathrm{P}})^2 \exp\left(M^2 \int^{\varphi} \frac{\mathrm{d}\varphi_2}{V'(\varphi_2)}\right) \times \int^{\varphi} \frac{V(\varphi_1)}{V'(\varphi_1)} \exp\left(-M^2 \int^{\varphi_1} \frac{\mathrm{d}\varphi_2}{V'(\varphi_2)}\right) \mathrm{d}\varphi_1. \tag{5}$$

If the condition $|\dot{\varphi}/\varphi| \ll H$ is not satisfied at the quasi-de Sitter stage (1), then the scalar field rapidly vanishes and can be neglected at all, and we return to the inflationary regime generated by the vacuum polarization only [2]:

$$H = \tfrac{1}{6}M^2(t_1 - t), \quad M \ll H \ll M_{\mathrm{P}},$$

$$a(t) = a_1 \exp[-\tfrac{1}{12}M^2(t_1 - t)^2], \tag{6}$$

where t_1 = const. is the time when the quasi-de Sitter stage ends and a_1 is the size of the universe at the end of inflation.

An important result can be obtained from eqs. (5):

$$\ln\frac{a(t)}{a_0} = \int_{t_0}^{t} H\,\mathrm{d}t = -\frac{8\pi}{M_{\mathrm{P}}^2} \frac{\mathrm{d}\varphi_1 V(\varphi_1)}{V'(\varphi_1)} + (3/M^2)(H_0^2 - H^2). \tag{7}$$

Here t_0 is the moment of the beginning of the quasi-de Sitter stage and a_0, H_0, φ_0 are the corresponding initial values of a, H, φ. Comparing eq. (7) with the expressions obtained in refs. [2,5] for the separate scenarios where only one of the two species (either a scalar field or vacuum polarization) was taken into account but not both, we arrive at the following simple rule: the total expansion of the universe during the inflationary stage (i.e., a_1/a_0) in the combined scenario, considered as a function of φ and H, is equal to the product of the total expansion in the separate scenario. This rule is rather general and can be applied also to the inflationary scenario with an arbitrary number of scalar fields interacting only gravitationally.

Let us further consider, following ref. [5], the theory with the potential $V(\varphi) = \frac{1}{4}\lambda\varphi^4$, $\lambda \ll 1$. Then eq. (7) and the last of eqs. (5) take the form

$$H^2 = -\tfrac{1}{3}\pi(M/M_P)^2 \exp(-M^2/2\lambda\varphi^2) \times \int^{\varphi} \exp(M^2/2\lambda\varphi_1^2)\, d(\varphi_1^2),$$

$$\ln(a/a_0) = (\pi/M_P^2)(\varphi_0^2 - \varphi^2) + (3/M^2)(H_0^2 - H^2). \quad (8)$$

Denote $m = M_P\sqrt{\lambda/6\pi}$, $m \ll M_P$. The ratio m/M is the important parameter in our combined scenario.

Let us consider the question about the origin of the quasi-de Sitter stage. In the case when the Friedmann model has a flat three-space (the total energy density is equal to the critical one), all solutions, except those having zero measure, reach singularity. The most general solution of eqs. (3) near singularity ($t \to 0$) has the following form:

$$a(t) \sim \sqrt{t},$$

$$\varphi = C_1/\sqrt{t} + C_2 - \sqrt{t}\, C_1^3(2\lambda + \pi M^2/M_P^2) + \ldots,$$

$$H = 1/2t - (\pi M^2/3M_P^2)C_1^2 + C_3\sqrt{t} + \ldots,$$

$$R = -6(\dot{H} + 2H^2) = t^{-1}(4\pi M^2/M_P^2)C_1^2 - 15C_3/\sqrt{t} + \ldots,$$

$$R^2 \ll R_{ik}R^{ik} \sim t^{-4}, \quad (9)$$

where C_1, C_2, C_3 are the arbitrary constants (independent initial conditions). It is worthwhile to note here that, in the absence of vacuum polarization, the general solution would behave as $a(t) \sim t^{1/3}$ near singularity which corresponds to the effective equation of state $p = \epsilon$ (i.e., the Zeldovich limiting equation of state) [‡1]. So, the effect of vacuum polarization near singularity is the change of the effective equation of state to that of massless radiation (the behaviour of the general solution near singularity in the pure vacuum polarization case was considered earlier in ref. [19]). However, one should not think that the R^2 term is completely analogous to radiation. The crucial difference between them becomes clear when one considers the more general anisotropic singularity. For example, in the case of the homogeneous Bianchi type models, radiation becomes unimportant near singularity, but the vacuum polarization transforms the classical one-parametric vacuum Kasner solution to the new two-parametric one where the three exponents q_α ($\alpha = 1, 2, 3$) satisfy only one equality,

$$\sum_\alpha q_\alpha^2 = \Big(2 - \sum_\alpha q_\alpha\Big)\sum_\alpha q_\alpha, \quad \text{with } 1 < \sum_\alpha q_\alpha \leq \tfrac{3}{2}$$

(this condition guarantees that $R^2 \ll R_{iklm}R^{iklm}$ as $t \to 0$).

The evolution of our combined model during expansion (in the direction of growing t) depends on C_1, C_2, C_3. Without going into details, we present here the results of our investigation. It appears that the general solution reaches the quasi-de Sitter stage (6) or (8) or (12) in the course of time if one of the following three conditions is satisfied:

(i) $|C_3| \gg M^{3/2}$,

(ii) $|C_3| \lesssim M^{3/2}$, $C_1^2 \gg (M_P^2/M)\min(1, M/m)$,

(iii) $|C_3| \lesssim M^{3/2}$, $C_1^2 \lesssim (M_P^2/M)\min(1, M/m)$,

$|C_2| \gg M_P$. (10)

These inequalities cover the most part of possible initial conditions. This means that it is highly probable for our combined model to go through the inflationary stage during its evolution.

If $m > M$, the quasi-de Sitter stage begins either with the solution (8) where both the scalar field and vacuum polarization work or, if the scalar field was sufficiently small initially, with the solution (6). H

‡1 This regime for the theory $m^2\varphi^2/2$ without vacuum polarization terms was recently investigated by Belinski, Grishchuk, Khalatnikov and Zeldovich (to be published).

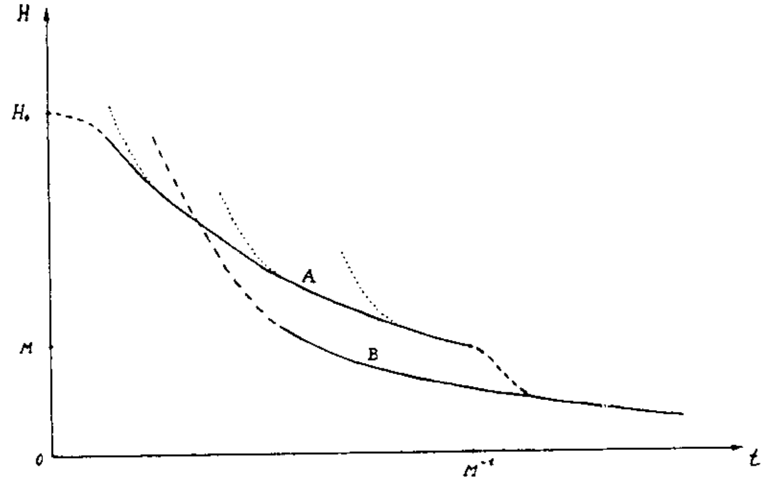

Fig. 1. The dependence of the Hubble parameter H on t for the quasi-de Sitter stages.

$> M$ throughout the whole stage. At the end of it, only vacuum polarization works, so the evolution is described by the solution (6) also. This situation corresponds to the curve A in fig. 1. It is interesting that if the scalar field was ever in the "slow rolling" regime $|\dot{\varphi}/\varphi| \ll H$ during the quasi-de Sitter stage in this case, it remains in this regime up to the end of inflation even when $|\varphi| \ll M_P$. Its evolution during the stage (6) has the form

$$\varphi^2 = \varphi_2^2 \{1 + (4\lambda\varphi_2^2/M^2) \ln[t_1/(t_1 - t)]\}^{-1},$$

$$\varphi_2 = \text{const}, \quad M(t_1 - t) \gg 1. \qquad (11)$$

In the opposite case $m < M$, the model can pass through two successive quasi-de Sitter stages: the first with $H > M$, described by eqs. (5), (8) and the second with $H < M$, where only the scalar field works [5],

$$\varphi = \varphi_0 e^{-mt}, \quad H = \sqrt{\tfrac{2}{3}\pi\lambda}\,(\varphi_0^2/M_P)\, e^{-2mt},$$

$$|\varphi_0| \gg M_P. \qquad (12)$$

The stage (12) corresponds to the curve B in fig. 1. Depending on initial conditions (9), there may or may not be a rather short period of a power-law expansion with $a(t) \sim t^{2/3}$ dominated by massive dust-like scalarons [1] between the two quasi-de Sitter stages. The solution can also pass through the only one quasi-de Sitter stage (12) in this case.

Let us finally mention the possibilities that the quasi-de Sitter regimes (8), (6) or (12) were preceeded by a hot radiation-dominated isotropic stage where $T_i^k \sim a^{-4}$ or by some anisotropic stage supported by vacuum polarization [16,20]. It can be shown that the curves A and B in fig. 1 play the role of envelopes for various evolution curves (depicted by dots in fig. 1) corresponding to different initial conditions.

It is not difficult to generalize our results to the non-homogeneous case by assuming C_1, C_2, C_3 and H to be functions of spatial coordinates. This corresponds to the case of chaotic inflation. In particular, after the proper generalization of the results obtained in ref. [21], it appears that the non-homogeneous quasi-de Sitter stage has the following form in the synchronous system of reference:

$$ds^2 = dt^2 - \gamma_{\alpha\beta} dx^\alpha dx^\beta, \quad \alpha, \beta = 1, 2, 3,$$

$$\gamma_{\alpha\beta} = a_{\alpha\beta}(r) \exp\left(2 \int H(t, r)\, dt\right)$$

$$\times [1 + O(t^2 e^{-\int H\, dt})], \qquad (13)$$

where $a_{\alpha\beta}(r)$ are arbitrary functions of three spatial coordinates restricted by the only condition that the spatial gradients should be smaller than H^{-1} at the beginning of the stage (the same is assumed for the spatial dependence of H also) and the dependence of H on t is given by eqs. (5), (6), (8) in each point of space.

5. Thus, we have shown that the two mechanisms

of inflation, which earlier were considered separately, can happily coexist with each other and their joint account results in the substantial enlargement of the number of domains in an initially chaotic universe which pass through the inflationary stage. The initial state of such a universe either could be dominated by hot matter or it could be vacuum-like. We can describe the ensemble of initial conditions for the curvature R, the scalar field φ and the temperature T of matter in different domains by the initial probability amplitude $\psi(R, \varphi, T)$.

Different classical initial conditions in an isotropic universe correspond either to some pure state of ψ in the case of a quantum creation of the universe [14,15, 22] and in the case in which the isotropic state appears after the decay of a more complex anisotropic initial state [20], or to a mixed state in the case of the ordinary hot beginning.

Another important conclusion is that the inflationary stage produced by one of the mechanisms can typically create conditions necessary for the other mechanism to come into play. In particular, if $m > M$ and only scalar fields work initially (so that $H_0^2 = 2\pi\lambda\varphi_0^4/3M_P^2$, $|\varphi_0| \gg M_P$ at the beginning of the quasi-de Sitter stage), the vacuum polarization terms always become significant dynamically [and the regime (6) takes place finally] because $H_0 \gg M$. In the opposite case $m < M$, if the scalar field was less than M_P initially, it can become greater than M_P due to the growth of its long-wave vacuum fluctuations [9,23,24] during the quasi-de Sitter stage (6) produced by the vacuum polarization provided $H_0 > \sqrt{MM_P}$ initially, and then the second quasi-de Sitter stage (12) will take place (a similar mechanism was used in ref. [25] in order to solve the problem of symmetry breaking in supersymmetric grand unified theories).

6. Let us finally say a few words about density perturbations which are generated in our model during the quasi-de Sitter stage. By using the expressions obtained in refs. [2,8–11] (see also ref. [26]), it can be shown (the details will be given elsewhere) that the amplitude of adiabatic perturbations generated in the combined scenario is determined by the mechanism that dominates at the end of the quasi-de Sitter stage and is given by the smallest of the amplitudes which would be generated in each of the two scenarios separately. At the present dust-like stage $a(t) \sim t^{2/3} \sim \eta^2$ [where $\eta = \int dt/a(t)$] we obtain:

$$\delta\rho/\rho = (2\pi)^{-3/2} \int d^3k\, e^{ikr} (\delta\rho/\rho)_k ,$$

$$[\langle(\delta\rho/\rho)_k^2\rangle]^{1/2} = \tfrac{1}{20} A(k)\, k^{1/2} \eta^2 ,$$

$$A(k) = (M/M_P)\sqrt{\tfrac{8}{3}\pi}\, \ln(k/k_1), \quad M < m,$$

$$= \sqrt{\tfrac{16}{3}\lambda}\, \ln^{3/2}(k/k_1), \quad M > m, \qquad (14)$$

where $a/2\pi k_1 = (1-10^5)$ sm at the present time.

These results are in an agreement with the estimates made in refs. [6,27] and with the results obtained in ref. [28], but somewhat differ from the results obtained in ref. [29]. The perturbations (14) correspond to the following large-scale anisotropy of the microwave background radiation [2,30]:

$$\frac{\Delta T}{T}(\theta, \varphi) = \sum_{lm} (\Delta T/T)_{lm} Y_{lm}(\theta, \varphi),$$

$$\langle(\Delta T/T)_{lm}^2\rangle = A^2/100\pi l(l+1), \qquad (15)$$

where $A \equiv A(k_{hor})$, $k_{hor} = 2\pi/a\eta$ at the present time. It follows from direct observations (using the correlation function [2]) that $A < 1.5 \times 10^{-3}$. This, together with eq. (14), implies that either $M/M_P \lesssim 10^{-5}$ or $\lambda \lesssim 10^{-12}$.

We would like to note also that in our scenario not only adiabatic but also the isothermal density perturbations are generated, which may become very important at the late stages of the universe evolution [31].

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