ELECTRODYNAMIC WAVE-THEORY OF PHYSICAL FORCES

11445

VOLUME I

BULLETINS 1 TO 6 INCLUSIVE

ANNOUNCING THE DISCOVERY OF THE PHYSICAL CAUSE OF MAGNETISM, OF ELECTRODYNAMIC ACTION, AND OF UNIVERSAL GRAVITATION. WITH PROOF THAT THESE FUNDAMENTAL FORCES OF NATURE ARE DUE TO THE INTERPENETRATION OF WAVES PROPAGATED WITH THE VELOCITY OF LIGHT THROUGH THE FREE AETHER, BUT MORE SLOWLY THROUGH THE SOLID MASSES, WHENCE ARISES ALSO THE REFRACTION, DISPERSION AND PERHAPS ABSORPTION OF PART OF THE WAVE ENERGY, AND THUS THE HITHERTO UNEXPLAINED FLUCTUATIONS OF THE MOON'S MEAN MOTION ESTABLISHED BY NEWCOMB IN 1909, AND JUSTLY PRONOUNCED THE MOST ENIGMATICAL PHENOMENON PRESENTED BY THE CELESTIAL MOTIONS.

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Ο Θεὸς ἀεὶ γεωμέτρει — *Plato.*

1917

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DEDICATED

TO THE MEMORY OF MY REVERED MASTER

DR. GEORGE WILLIAM HILL

MOST ILLUSTRIOUS OF ASTRONOMICAL GEOMETERS

WHO DID NOT HESITATE TO DEPART FROM BEATEN PATHS,

AND IN ORDER TO PRESERVE HIS INDEPENDENCE EVEN PUBLISHED

AT HIS OWN EXPENSE THE IMMORTAL

RESEARCHES IN THE LUNAR THEORY, 1877.

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INTRODUCTION

The Science of Astronomy is essentially one of Geometry on a great scale. It consists in the tracing out of regular orbits from exact observations, the deduction of the masses under whose central forces the orbits are described, and the calculation of the perturbations to which these principal motions may be subjected. Cosmogony goes further yet, and attempts to discover the foundations of the permanent Geometry of the Heavens. It explains the gradual origin of the system of the world through excessively slow Processes of Capture known to be going on in Nature, and made possible by the primordial diffusion of Cosmical Dust under the action of repulsive forces for the formation of nebulæ in the vacant regions of starless space.

Both Astronomy and Cosmogony thus require an insight into the physical cause which underlies the observed central forces. This primordial agency, combined with the Process of Capture, in the course of millions of ages, has established the recognized order of the heavens. The astronomer therefore labors to throw light on the Cause of Universal Gravitation, so as to find out whether the received Law of NEWTON is rigorously exact, and will hold true for all time, or only a close approximation which may give rise to no large errors of calculation inside of several centuries.

Sir ISAAC NEWTON himself was greatly occupied with these questions, while composing the *Principia*, 1687, and not very much additional light has been shed on them since, because the Cause of Universal Gravitation has remained as completely hidden from our view as in the days of the formulation of the Law of Gravitation. Notwithstanding this long Stationary Period, the time has now come for a great and significant advance, if we can correctly make out the nature of the physical forces governing the Universe by connecting them with Magnetism, which is simpler and more easily understood.

Forty years have now elapsed since the writer's attention was attracted to the Physical Cause of Universal Gravitation as a problem awaiting investigation. The central force which governs the motion of the Moon came under special consideration in 1881, and in 1882 the Transit of *Venus* and the Great Comet which passed so near the Sun, and for months remained visible in the morning sky till about sunrise, gave these early studies a more definite direction. Yet the solution of the problem still appeared remote, and for a third of a century, indeed, it was not forthcoming, so that heretofore nothing has been published bearing on the subject.

But in April, 1914, a mode of attacking this baffling problem, through the related phenomenon of Magnetism, finally presented itself; and there seemed to be a reasonable hope that if the nature of Magnetism could be clearly made out, the physical cause of Universal Gravitation also could be established. During the past three years much attention has been given to the subject, with the result that the problem of Magnetism now seems to be solved in so clear and simple a manner that little remains to be desired in the way of improvement.

The discovery of a Physical Mechanism, conformable to the well established laws of Dynamics, by which stresses in the medium can be exerted across space, so as to give physical forces acting in right lines, when the transmission is through the free Æther, has been a most urgent desideratum of Science for many centuries. Up to very recent times, however, such a mechanism had not been disclosed. For in an Address on the "Æther of Space," at Bedford College, England, in February, 1914, Sir OLIVER LODGE, declared that there is a *mysterious force, which he did not understand in the least, the force of gravity. It puts the force of cohesion absolutely to shame. This force of gravity binds the Cosmos together. It must be transmitted by the Æther, but as there is no mechanical connection between Æther and Matter, the nature of the attractive process has not been made out, though* SIR OLIVER *believed it to be electrical.*

Now whatever shades of opinion may develop among natural philosophers as to the *details* of the processes disclosed in these *Bulletins*, it cannot, I think, be denied that Electrodynamic Waves will explain Magnetic, Electrodynamic and Gravitational Attraction, in accordance with the definitely established Laws of Nature. The historical difficulty of finding a physical mechanism between bodies under which they will mutually attract, owing to the interpenetration of waves changing the separate stresses and thus generating *forces pulling in straight lines,* seems therefore to be overcome.

The Electrodynamic Wave-Theory of Universal Gravitation is a necessary consequence of the discovery of the Physical Cause of Magnetism. For some time, it has seemed to me that there could be no mode of attacking the problem of Gravitation except through the more familiar attraction of Magnetism; and the line of reasoning here adopted has therefore been based on the laws of wave action shown to be operative in Electrodynamics.

The explanation of Magnetic and of Gravitational Attraction now put forth has the indispensable merit of simplicity and of conformity to FARADAY's great experimental discovery that all bodies are magnetic. The magnetism of all matter means, in other words, that it was shown by FARADAY's careful Laboratory

Experiments that *all bodies emit waves which may be more or less coerced into parallelism by the action of suitable electric currents.*

This comprehensive Electrodynamic Wave-Theory obviously is *sufficient* to account for the phenomena of Universal Gravitation, and it has the additional advantage of illustrating clearly the unity and correlation of all the forces of Nature, with which FARADAY had become so deeply impressed. Whether the Electrodynamic Wave-Theory also fulfills the *necessary condition of being the only possible explanation of gravitation* is a question which may be fairly left to the judgment of the reader.

The philosophic methods of ARCHIMEDES and of NEWTON naturally explain Gravitational Attraction by the same mechanism which accounts for Magnetic Attraction; and as this Electrodynamic Wave-Theory seems definitely established by the rigorous geometrical and physical conditions fulfilled by the forces producing the *Lunar Fluctuations* natural philosophers doubtless will hold that it *is also the only possible Theory of Gravitation.*

It seems certain that no other theory will explain the *Attraction* operating between bodies, which we ascribe to *forces pulling in right lines. Such tension in the medium becomes a maximum directly between the masses, because along these connecting right lines the oppositely directed waves always mutually interpenetrate with double the velocity of Light, thus putting the medium under extraordinary tension, like that of a stretched mass of India rubber.*

Heretofore, no such *natural mechanism* as is now outlined has been known to the investigator. And from the author's studies of the rigorous geometrical and physical conditions fulfilled by the forces producing the *Lunar Fluctuations* he believes that no explanation other than that now offered is mathematically possible; so that the *Lunar Fluctuations* appear to constitute an *observational experimentum crucis* of the physical cause underlying Universal Gravitation.

The discovery of the physical cause of the hitherto *Unexplained Fluctuations of the Moon's Mean Motion* has therefore confirmed the Electrodynamic Wave-Theory of Gravitation, and given this theory both a mathematical and physical basis, which will, I think, deserve the earnest consideration of natural philosophers.

Accordingly there results the Electrodynamic Wave-Theory of Magnetism, of Electrodynamic Action, and of Universal Gravitation, with proof that these Fundamental Forces of Nature are due to waves propagated with the velocity of Light through the free Æther, but more slowly through solid masses — whence arises also Refraction, Dispersion, and perhaps Absorption of part of the wave-energy, and thus the hitherto *Unexplained Fluctuations* of the Moon's Mean Motion.

The *Lunar Fluctuations* discovered by NEWCOMB were justly regarded as the most enigmatical phenomenon presented by the Celestial Motions. It is remarkable that they now become an *experimentum crucis* for establishing the undulatory nature of Gravitation, owing to the perturbative effects actually observed to be exerted upon the Moon when near the shadow of the Earth.

The Greek astronomer HIPPARCHUS initiated the investigation of the Lunar Inequalities through the study of the *Evection,* about 140 B.C., but left it to be more fully worked out by PTOLEMY some 300 years later. After steady progress extending over 2000 years and culminating in the recent exhaustive researches of Professor E. W. BROWN, the Lunar Theory was almost perfected, — when at last the cause of the *Lunar Fluctuations* unexpectedly comes to light, and every sensible Inequality in the Motion of the Moon thus finally disappears!

These results are of no small interest, and I gladly would have offered them to one of the numerous Learned Societies which exist for the promotion of Science. Several of the foremost mathematicians of Europe and America have generously urged this course, and laid me under lasting obligations for their kindness. But there are too many well known instances of discoveries being misunderstood to justify the belief that the reception accorded to such new work would be sufficiently favorable to ensure its prompt publication by conservative committees having no part in its development. It is a somewhat melancholy reflection, but one quite justified by History, that it is chiefly individual discoverers who have the vision to perceive and the moral courage to support new Truth in advance of its triumph.

For example, it is well known that the Royal Society delayed unduly and thereby evaded the publication of NEWTON'S *Principia,* 1686; and to prevent the loss of the work HALLEY had to print at his private expense the most immortal production of the human intellect.

Some things even worse than the smothering of priceless discoveries occasionally occur; for in his *History of Astronomy,* LAPLACE gives an account of the persecution of ANAXAGORAS, at Athens, and justly observes: "To establish itself on earth, Truth has often had to combat accredited errors, which more than once has proved fatal to those who have discovered it."

In the well known case of GALILEO the discoveries which he made and promulgated did not quite prove fatal, indeed, yet this first philosopher of Europe, in a blind old age, after a laborious life devoted to the founding of Modern Science, was thrown into prison, because he persisted in publishing the discoveries which established the Copernican System of the world.

Laplace comments on the difficulties of Kepler as follows: "With so many claims to admiration this great man lived in misery, while judicial Astrology, everywhere honored, was magnificently recompensed. Fortunately, the enjoyment which a man of genius receives from the truths which he discovers, and the prospect of a just and grateful posterity, console him for the ingratitude of his contemporaries."

Thus the labor of the discoverer always is unduly increased, yet the investigator who succeeds, — simply because he is a *labor-loving and truth-loving man*,

ἀνὴρ φιλόπονος καὶ φιλαλήθης ,

such as Hipparchus was, according to the description given by Ptolemy in the *Almagest*, Lib. III, Ch. I, — is not without the noblest of rewards. And such a true discoverer naturally is *unwilling to condescend* to secure publication of his priceless treasures.

As is well known, in past struggles for *new ideas* I have often stood entirely alone; I can again do so, cheerful and hopeful of the ultimate triumph of reason and justice. It took the Capture Theory of Cosmogony eight years to triumph in some unprogressive countries, yet that period is short in the long History of Astronomy.

If there be those who hesitate to welcome the present results on the nature and mode of propagation of Physical Forces, let us hope that they will not lay themselves open to the grave charge that there are none so blind as those who do not wish to see. Galileo has related how certain learned professors in Padua refused to look through the newly-invented Telescope lest they see *Jupiter's* satellites, and thus be compelled to admit the truth of his discovery. Unjust criticism is not only useless, but philosophically beneath contempt. He who is deserving of the name of natural philosopher shows a genuine interest in Truth, and labors to make it more accessible to *those worthy to take part in such studies*. That moral integrity is a prerequisite to the successful cultivation of Science is impressively emphasized by Plato (*Republic*, Book VII, §535 of Jowett's Translation), and specifically stated by Apollonius of Perga in his second letter to Eudemus.

It is conceded that very little progress in regard to the Cause of Universal Gravitation has been made during the past 250 years. As the laws of Electrodynamic Action were quite unknown in the time of Newton, that great philosopher naturally was unable to solve the problem; yet this is no reason why we should idly fold our hands today and ignore all the progress made since the time of Ampère. We should rather assume the responsibility of leadership in the new advance which is demanded by the state of Science in our time.

Probably the present work could have been somewhat improved by still further delay in publication; but I deemed this course unjust to contemporary investigators, and will cheerfully take the blame for any incompleteness in the present development. It is only by the united efforts of many minds that a new subject may be rounded out in all its aspects. In the course of this thorough test the discoverer himself must make many sacrifices.

In the first instance, he has the choice, on the one hand, between following the beaten path, which is the course of least resistance, — safe and conservative, yet involves no step forward; and of opening a new path, on the other, which may lead through thorns and underbrush, and call for the energy and force of character appropriate to the pioneer. Enormous are the difficulties of the explorer, but great also may be the reward of his daring courage!

In the contemplation of the world the philosopher finds that wonders are many, and thus he never ceases to marvel at the mysteries of Nature disclosed to his searching vision. Yet sublime as are the wonders opened to us in the past, it seems that they are as nothing compared to the unseen but stupendous power operative in the process of Universal Gravitation. This power results from the interpenetration of waves traveling from all bodies with the Velocity of Light and by decreasing the stress in the Æther between them thus generating the tremendous physical forces required for holding the planets in their orbits!

Lest we forget the stupendous power of Gravitation, in terms of the *Strength of Steel*, we may recall two instances of the mighty forces now traced to the interpenetration of Electrodynamic Waves traveling with the Velocity of Light:

1. In the *Æther of Space*, 1909, pp. 112–126, Sir Oliver Lodge shows from the recognized laws of Dynamics that the pull of the Earth on the Moon is equivalent to the breaking strength of a steel column 400 miles in diameter, or of a forest of five million million weightless pillars each a square foot in cross section — the tenacity of the steel being thirty tons to the square inch.

2. In the same way Sir Oliver Lodge shows (p. 130) that the pull of the Sun on the Earth is equivalent to the tenacity of a forest of weightless steel pillars each eleven inches in diameter, acting on every square foot of a hemispherical section of the globe — the steel again having the breaking strength of thirty tons to the square inch.

To inquire into the unseen mechanism for producing these wonderful forces would have appealed to the genius of Archimedes or of Newton! No physical problem could be more worthy of the meditation of the natural philosopher!

With uncovered heads we stand in silent awe at the amazing mysteries unfolded to our contemplation! The curtain of the ages never before was thus parted to open such a vision to the human mind! The revelation of the previously unseen mechanism of the physical world must be pronounced, in Sophoclean phrase, beyond mortals wonderful!

It may not be inappropriate to point out a certain Analogy between the *Infinite Series* of Higher Analysis, so well known to the geometer, and indispensable to the numerical evaluation of the curves traced by the heavenly bodies in their motions and mutual perturbations; and the *Electrodynamic Waves* which by mutual interpenetration generate the forces for holding the planets in their orbits and thus cause the phenomenon of Universal Gravitation.

The Series of Analysis represent *numerical oscillations* by which finite and definite values of transcendental functions may be approximated; the Electrodynamic Waves are the *physical oscillations* for generating the corresponding continuous forces under which the heavenly bodies describe the higher curves recognized in the Geometry of the Heavens. If the nature of Series has been mysterious to the geometer, so also are the Waves to the natural philosopher; yet with this Analogy before us the problem of the philosopher is somewhat illuminated by the close correspondence and unbroken continuity shown to pervade the order of Nature.

The author is much indebted to his former Secretary, Mr. D. R. Adams, for efficient service in preparing the first outline of the Electrodynamic Wave-Theory, 1914; and to Mr. Joseph Pedroni, his present Secretary, for a like service in completing this Volume, 1916–17.

But of all those who have contributed to the entire development I owe most to Mrs. See, for steadfast support and patient interest in a labor presenting many difficulties. It is not yet quite finished, it is true, but with the appearance of the present Volume the ultimate triumph of these discoveries seems fully assured; and when the significance of the advance made comes to be appreciated, it will be only right that those who aided in overcoming almost insurmountable obstacles, in a period of universal distress and world darkness, should be gratefully remembered as bearers of the Light.

T. J. J. SEE.

Starlight on Loutre,
Montgomery City, Missouri,
September 19, 1917.

ELECTRODYNAMIC WAVE-THEORY OF PHYSICAL FORCES

BULLETINS

I, II, III, IV, V, VI

RESULTS OF RESEARCHES

ON THE

ELECTRODYNAMIC WAVE-THEORY OF PHYSICAL FORCES

BULLETIN NO. 1

ANNOUNCEMENT OF TWO IMPORTANT ASTRONOMICAL DISCOVERIES:

I. The Physical Cause of the Hitherto Unexplained Fluctuations in the Moon's Mean Motion, Dec. 10, 1916.

II. The Physical Cause of Universal Gravitation, 1914–17.

By T. J. J. SEE

After most thorough investigation, — including the independent confirmation of the results from both the mathematical and physical points of view, as well as by comparison of the new Physical Theory with the observed *Fluctuations*, which are the outstanding differences between pure Gravitational Theory and actual observation of the Moon's Mean Motion, as set forth by Newcomb, 1909 — the author of this Bulletin announces that on December 10, 1916, he was able to discover the Physical Cause of the hitherto *Unexplained Fluctuations* in the Moon's Mean Motion.

This discovery will prove to be remarkable as revealing the existence of a New Type of Long Inequality in the Theories of the Celestial Motions. These Inequalities depend on the Electrodynamic Nature of Gravitation, and may occasionally become sensible to observation, but heretofore their existence has scarcely been suspected and never considered probable.

The late Professor Simon Newcomb regarded the *Unexplained Fluctuations* in the Moon's Mean Motion as the most enigmatical phenomenon presented by the celestial motions, and thus the discovery of the cause of these perplexing inequalities will no doubt occasion surprise among investigators. There naturally will be curiosity to know in what way the discovery was made.

Unfortunately, owing to the serious problem of publication for *new work*, it is not practicable at present to give details of the *processes* of investigation.

The *results* here set forth, however, will leave no doubt of the rigor of the indicated laws of the Moon's Mean Motion.

In explanation of this statement it remains to add that the discovery of the Cause of the Moon's *Fluctuations* was made in the extension of researches on the *Cause of Universal Gravitation*, with which the author has never ceased to be occupied since 1881, and first had his attention directed to in 1878. It is well known that in April, 1892, he had the honor of discussing the Cause of Gravitation in some detail with Professor Sir GEORGE DARWIN on the occasion of a visit to this eminent mathematician at Cambridge.

In spite of all the attention previously given to the subject no conclusion could be reached at that time; and although the problem was kept steadily in view it was twenty-two years later, April, 1914, before any definite promise of a solution opened up. Even then the greater part of the spare moments of the past three years had to be given to perfecting the development of the new Electrodynamic Wave-Theory.

Early in 1915 a *first outline* of the work on the Cause of Gravitation was sent to one of the oldest and most celebrated of the Scientific Societies of Europe. Although not published, it seems to have been given much consideration, and in spite of the evidence that it was not fully understood, was found worth filing away in the Archives for future reference. New conceptions seldom are welcomed at first, and besides the conditions in Europe during the past three years have been little suited to philosophic contemplation.

The researches on the Cause of Gravitation necessarily have extended over all the related investigations of the past two hundred fifty years, and thus covered a very wide field. Accordingly they cannot be adequately described in a Bulletin, and it is deemed best to defer details of the new Electrodynamic Wave-Theory, in the hope that before many years the work may be brought out as a whole. The intervening time may be utilized for additional researches, and for the preparation and publication of the results. But as the *Cause of Gravitation* seems clearly and definitely made out, and confirmed by the discovery of the cause of the hitherto *Unexplained Fluctuations* in the Moon's Mean Motion, it appears advisable to announce also this discovery in the present Bulletin.

The first outline of the Electrodynamic Theory, ascribing the stresses for transmitting forces across space to wave action, in an unpublished Manuscript, 570 pages, December 10, 1914, bears the title: "Electrodynamic Theory of Magnetism and of Universal Gravitation: Discovery of the Cause of Gravitation, with proof that this Fundamental Force of Nature is propagated with the velocity of Light."

WEBER'S Fundamental Electrodynamic Law of 1846 is shown to be the Law of Nature. Thus the formula of WEBER

$$F = \frac{mm'}{r^2}\left\{1 - \frac{1}{c^2}\left(\frac{dr}{dt}\right)^2 + \frac{2r}{c^2}\left(\frac{d^2r}{dt^2}\right)\right\}.$$

where m and m' are the masses, r the distance, and c the velocity of transmission for the wave action, reduces to NEWTON'S Law of 1686

$$F = \frac{mm'}{r^2},$$

when the orbit is circular, so that $\frac{dr}{dt} = 0$, $\frac{d^2r}{dt^2} = 0$, and there is no relative motion between the bodies of the system.

All the periodic phenomena of Terrestrial Magnetism are explained, including Earth Currents, Auroras, "Magnetic Storms," and the dependence of "Magnetic Storms" on the sun spot cycle, and a connection is shown to exist between the Magnetism of the Earth and the Electrodynamic Forces which control the motions of the Planets in their orbits.

Within the past few years Professor EINSTEIN, who had previously failed to connect Gravitation with Electrodynamics, has published a Relativity Theory for explaining the *true law* of Gravitation, including the motion of *Mercury's* Perihelion, etc.; but so far as I know he makes no attempt to assign the *Cause* of Gravitation. Indeed, under EINSTEIN'S vague and chimerical theory, Gravity is not a "force," but "a property of space!" This Relativity Theory of EINSTEIN has appealed to some English mathematicians, but they evidently are misled by deceptive analysis, and reasoning on false premises, as so frequently happens when *appropriate physical conditions* are overlooked.

The present author has now shown that at least half a dozen different explanations of the motion of *Mercury's* perihelion are possible. Any one of them *a priori* is admissible, yet not one of them is established by the necessary and sufficient conditions required to make a conclusive argument. Moreover each of these independent explanations may be varied and combined with the others, so as to multiply indefinitely the possible solutions. The motion of *Mercury's* perihelion is therefore in no way decisive for or against any theory of Gravitation.

The *Fluctuations* of the Moon's Mean Motion, on the other hand, have been generally regarded as essentially incapable of theoretical explanation. In 1909 Professor NEWCOMB wrote: "I regard these *fluctuations* as the most enigmatical phenomenon presented by the celestial motions, being so difficult to account for by the action of any known causes, that we cannot but suspect them to arise from some action in nature hitherto unknown" (*Monthly Notices*, January, 1909).

This prediction was verified through the discovery of a cause not hitherto suspected, connected with the Electrodynamic Wave-Theory of Gravitation, by which alone the present author was able to solve the mystery of the Moon's *Fluctuations*.

The accompanying table and plate illustrates the completeness of the new physical theory.

It seems quite evident that no outstanding *fluctuation* now remains in the Moon's motion in excess of $1''.0$; for the average residual shown for the past eighty years is about $0''.70$, and on that basis we know from the theory of probability that a periodic inequality of $1''.0$ cannot remain undetected in the Moon's Mean Motion.

This quantity is so small as to be near the limit of vision with a 6-inch telescope such as a transit circle. Thus it appears that on the basis of true physical theory, resulting from the extension of the work on Gravitation, I have been able to improve the accuracy of the Moon's predicted mean longitude at least a *dozen fold*.

There are in all *three hitherto undiscovered terms* of sensible magnitude in the Moon's Mean Motion as follows:

1. The *Short Period Fluctuation*, recurring in 18.0293 years, with a coefficient of $1''.0$:

$$\Delta L_1 = 1''.0 \sin \{19^\circ.9675\ (t - 1800.0)\ + 239^\circ.42\} \ . \ . \ . \qquad (1)$$

2. The *Large Fluctuation* of 61.7006 years' period, with coefficient of $3''.0$, and having an exceedingly close resemblance to the Great Inequality in the mean motion of *Jupiter* and *Saturn*, of which the physical cause was discovered by LAPLACE in 1785:

$$\Delta L_2 = 3''.0 \sin \{5^\circ.83597\ (t - 1800.0)\ + 126^\circ.35\} \ . \ . \ . \qquad (2)$$

3. The *Great Fluctuation*, estimated by NEWCOMB to have a period of 275 years, but now found theoretically to have a period of 277.590 years. This involves also the auxiliary period of 17.9971 years. This Great Fluctuation likewise has some analogy to the Great Inequality of *Jupiter* and *Saturn*. The coefficient is $13''.0$ and the formula:

$$\Delta L_3 = 13''.0 \sin \{1^\circ.29691\ (t - 1800.0) + 100^\circ.6\} \ . \ . \ . \qquad (3)$$

Any Theory of Gravitation which explains the cause of the Moon's *Fluctuations* must be regarded as meeting the conditions of an *experimentum crucis*. As the Electrodynamic Wave-Theory triumphs under this severe test it must be held to rest on a true Law of Nature.

On the other hand the light afforded by the Electrodynamic Wave-Theory for making this difficult discovery in the Lunar Theory, which successfully challenged the greatest mathematicians for more than a century, shows the exceeding usefulness of the unpublished researches on the *Cause of Gravitation*.

Having first drawn attention, in 1869, to the departure of the Moon's actual motion from the theoretical places indicated by pure Gravitational Theory, NEWCOMB worked out the chief *Fluctuations* in his *Researches on the Motion of the Moon*, 1878, and finished his final memoir on the subject just before his death in 1909.

HILL became deeply interested in the Lunar Theory in the early seventies, and in 1877 published his celebrated *Researches in the Lunar Theory*. To the date of his death in 1914, Dr. HILL never ceased to be interested in the Motion of the Moon. Like NEWCOMB he was occupied with the subject over forty years, yet he was unable to throw any light on these mysterious *Fluctuations*.

It was thus left to Professor E. W. BROWN to make the most exhaustive researches on the Lunar Theory yet attempted. He has now worked on the subject for more than a quarter of a century, but has not found any satisfactory explanation of the *Fluctuations* established by NEWCOMB.

Accordingly it would appear that the discovery of the Physical Cause of these mysterious *Fluctuations* may be regarded as no ordinary achievement. If there be those who hesitate to welcome the new results, they will still have the opportunity of improving on them in their own way, without embarrassment with the details of the author's processes of discovery.

Universal Gravitation is now shown to be due to Electrodynamic Waves traveling with the velocity of light, and reacting on matter through stresses in the medium due to such waves, which are shown to be decidedly different from those conceived by MAXWELL in his Electromagnetic Theory of Light. In correcting the researches of MAXWELL and removing a traditional defect in the Wave-Theory of Light, the author has been able to harmonize them essentially with some curious views of NEWTON, on the cause of Gravitation, dating from 1721.

AMPÈRE's explanation of Magnetic Forces, as due to elementary currents of electricity circulating around the atoms is also harmonized with the Electrodynamic Wave-Theory; and altogether the unity and harmony introduced into our Theories of the Physical Universe is remarkable.

Not only does the assigned *Cause of Gravitation* explain all known celestial phenomena, and triumph under the *experimentum crucis* of the *Lunar Fluctuations*, the explanation of which appears to be unique, but it seems thereby effectually to exclude every other conceivable explanation. For mere coincidence could not give this perfect physical theory of the intricate phenomena of the Lunar Fluctuations.

It is only for these weighty reasons that the author has finally decided to announce the discovery of the *cause* of the *Moon's Fluctuations* and of *Universal Gravitation*. These two discoveries necessarily are associated, one and inseparable.

Details of the more elaborate researches on the Cause of Gravitation must await publication, but the accompanying proof of the value of the discovery in the Lunar Theory is ample, and it should be of interest to investigators in more than one of the Physical Sciences.

T. J. J. SEE

Starlight on Loutre,
Montgomery City, Missouri, February 19, 1917.

Physical Cause of the Fluctuations of the Moon's Mean Motion

T. J. J. See, Dec. 10, 1916

t	A_1	$1''.0 \sin A_1 = \Delta L_1$	A_2	$3''.0 \sin A_2 = \Delta L_2$	A_3	$13''.0 \sin A_3 = \Delta L_3$	Obs. Fl.	Cal. Fl. $= \sum_{i=1}^{i=3} \Delta L_i$	Final Resid. O−C	Wt. Newcomb
	°	″	°	″	°	″	″	″	″	
1829.5	108.53	+0.95	298.51	−2.34	138.86	+ 8.55	+ 7.0	+ 7.16	−0.2	20.0
1833.5	188.42	−0.15	321.85	−1.85	144.05	+ 7.63	+ 6.1	+ 5.63	+0.5	10.0
1838.5	308.27	−0.95	351.03	−0.47	150.53	+ 6.40	+ 5.8	+ 4.98	+0.8	30.0
1843.0	18.14	+0.31	17.30	+0.89	156.37	+ 5.21	+ 6.3	+ 6.41	−0.1	20.0
1846.5	88.04	+1.00	37.72	+1.83	160.91	+ 4.25	+ 5.3	+ 7.08	−1.8	10.0
1848.5	127.84	+0.79	49.39	+2.28	163.50	+ 3.69	+ 5.6	+ 6.76	−1.1	8.0
1849.5	147.81	+0.53	55.23	+2.46	164.80	+ 3.41	+ 3.5	+ 6.40	−2.9	15.0
1850.5	167.78	+0.21	61.07	+2.62	166.10	+ 3.12	+ 4.2	+ 5.94	−1.7	18.0
1851.5	187.75	−0.13	66.91	+2.76	167.40	+ 2.84	+ 3.7	+ 5.47	−1.7	12.0
1852.5	207.72	−0.46	72.75	+2.86	168.70	+ 2.55	+ 3.5	+ 4.95	−1.4	8.0
1853.5	227.69	−0.74	78.59	+2.94	170.00	+ 2.26	+ 3.0	+ 4.46	−1.4	7.0
1854.5	247.66	−0.92	84.43	+2.98	171.30	+ 1.97	+ 3.4	+ 4.03	−0.6	14.0
1855.5	267.63	−1.00	90.27	+3.00	172.60	+ 1.77	+ 3.8	+ 3.77	±0.0	6.0
1856.5	287.60	−0.95	96.11	+2.98	173.90	+ 1.38	+ 3.3	+ 3.41	−0.1	6.0
1857.5	307.57	−0.79	101.95	+2.93	175.20	+ 1.09	+ 3.2	+ 3.23	±0.0	9.0
1858.5	327.54	−0.54	107.75	+2.86	176.47	+ 0.80	+ 4.3	+ 3.12	+1.2	8.0
1859.5	347.51	−0.22	113.59	+2.75	177.77	+ 0.51	+ 4.4	+ 3.04	+1.4	6.0
1860.5	7.48	+0.13	119.43	+2.61	179.07	+ 0.21	+ 4.1	+ 2.95	+1.1	12.0
1861.5	27.45	+0.46	125.27	+2.45	180.37	− 0.08	+ 3.2	+ 2.83	+0.4	5.0
1862.5	47.42	+0.74	131.11	+2.26	181.67	− 0.28	+ 3.5	+ 2.72	+0.8	7.0
1863.5	67.39	+0.92	136.95	+2.05	182.97	− 0.67	+ 2.3	+ 2.30	±0.0	6.0
1864.5	87.36	+1.00	142.79	+1.81	184.27	− 0.97	+ 1.9	+ 1.84	+0.1	10.0
1865.5	107.33	+0.95	148.63	+1.56	185.57	− 1.26	+ 1.3	+ 1.25	±0.0	5.0
1866.5	127.30	+0.79	154.47	+1.29	186.87	− 1.56	+ 1.2	+ 0.52	+0.7	6.0
1867.5	147.27	+0.54	160.31	+1.01	188.17	− 1.85	− 0.8	− 0.30	−0.5	5.0
1868.5	167.24	+0.22	166.11	+0.72	189.44	− 2.13	− 1.2	− 1.19	±0.0	10.0
1869.5	187.21	−0.12	171.95	+0.42	190.74	− 2.42	− 0.9	− 2.12	+1.2	9.0
1870.5	207.18	−0.46	177.79	+0.11	192.04	− 2.71	− 2.1	− 3.06	+1.0	6.0
1871.5	227.15	−0.73	183.63	−0.19	193.34	− 3.00	− 4.4	− 3.92	−0.5	10.0
1872.5	247.06	−0.92	189.47	−0.49	194.64	− 3.29	− 5.1	− 4.70	−0.4	16.0
1873.5	267.03	−1.00	195.31	−0.79	195.94	− 3.57	− 5.4	− 5.36	±0.0	12.0
1874.5	287.00	−0.95	201.15	−1.08	197.24	− 3.85	− 6.1	− 5.88	−0.2	8.0
1875.5	306.97	−0.80	206.99	−1.36	198.54	− 4.13	− 6.5	− 6.29	−0.2	8.0
1876.5	326.95	−0.54	212.83	−1.63	199.54	− 4.41	− 6.5	− 6.58	+0.1	30.0
1878.5	6.88	+0.12	224.47	−2.10	202.41	− 4.96	− 6.6	− 6.94	+0.3	18.0
1879.5	26.83	+0.45	230.31	−2.21	203.71	− 5.23	− 5.7	− 6.99	+1.3	14.0
1880.5	46.82	+0.72	236.15	−2.49	205.01	− 5.49	− 6.9	− 7.26	+0.3	20.0
1881.5	66.79	+0.92	241.99	−2.65	206.31	− 5.76	− 7.3	− 7.49	+0.2	12.0
1882.5	86.76	+1.00	247.83	−2.78	207.61	− 6.03	− 7.4	− 7.81	+0.4	8.0
1883.5	106.73	+0.96	253.67	−2.88	208.91	− 6.29	− 8.4	− 8.21	−0.2	7.0
1884.5	126.70	+0.80	259.51	−2.95	210.21	− 6.54	− 8.6	− 8.69	+0.1	30.0
1885.5	146.67	+0.54	265.35	−2.99	211.51	− 6.79	− 9.2	− 9.24	±0.0	50.0
1886.5	166.64	+0.23	271.19	−3.00	212.81	− 7.04	− 9.8	− 9.81	±0.0	18.0
1887.5	186.61	−0.11	277.03	−2.98	214.11	− 7.29	− 9.8	−10.38	+0.6	20.0

Physical Cause of the Fluctuations of the Moon's Mean Motion

(Continued)

t	A_1	$1''.0 \sin A_1 = \Delta L_1$	A_2	$3''.0 \sin A_2 = \Delta L_2$	A_3	$13''.0 \sin A_3 = \Delta L_3$	Obs. Fl.	Cal. Fl. $= \sum_{i=1}^{i=3} \Delta L_i$	Final Resid. O − C	Wt. Newcomb
	°	″	°	″	°	″	″	″	″	
1888.5	206.58	−0.45	282.83	−2.92	215.38	− 7.53	−11.0	−10.90	−0.1	8.0
1889.5	226.55	−0.72	288.67	−2.84	216.68	− 7.76	−11.2	−11.32	+0.1	7.0
1890.5	246.52	−0.92	294.51	−2.73	217.98	− 8.00	−11.4	−11.65	+0.2	10.0
1891.5	266.49	−1.00	300.35	−2.59	219.28	− 8.23	−11.3	−11.82	+0.2	16.0
1892.5	286.46	−0.96	306.19	−2.43	220.58	− 8.46	−11.0	−11.85	+0.8	17.0
1893.5	306.43	−0.80	312.03	−2.22	221.88	− 8.68	−11.3	−11.70	+0.4	15.0
1894.5	326.40	−0.55	317.87	−2.01	223.18	− 8.90	−11.8	−11.46	−0.3	30.0
1895.5	346.37	−0.23	323.71	−1.78	224.48	− 9.11	−11.2	−11.12	−0.1	60.0
1896.5	6.34	+0.11	329.35	−1.52	223.78	− 9.32	−10.4	−10.73	+0.3	60.0
1897.5	26.31	+0.44	335.39	−1.25	227.08	− 9.52	−11.5	−10.33	−1.2	20.0
1898.5	46.28	+0.72	341.19	−0.97	228.35	− 9.71	−11.2	− 9.96	−1.2	28.0
1899.5	66.25	+0.91	347.03	−0.67	229.65	− 9.91	−11.3	− 9.67	−1.6	12.0
1900.5	86.15	+1.00	352.87	−0.37	230.95	−10.10	−10.1	− 9.47	−0.6	15.0
1901.5	106.12	+0.90	358.71	−0.07	232.25	−10.28	−10.4	− 9.45	−0.9	16.0
1902.5	126.09	+0.81	4.55	+0.24	233.55	−10.46	−10.2	− 9.41	−0.8	18.0
1903.5	146.06	+0.56	10.39	+0.54	234.85	−10.63	−10.0	− 9.53	−0.5	12.0
1904.5	166.03	+0.24	16.23	+0.84	236.15	−10.80	− 9.7	− 9.72	±0.0	20.0
1905.5	186.00	+0.10	22.07	+1.13	237.45	−10.96	− 9.5	− 9.73	+0.2	20.0
1906.5	205.97	−0.44	27.91	+1.40	238.75	−11.12	− 9.8	−10.16	+0.3	16.0
1907.5	225.94	−0.72	33.75	+1.66	240.05	−11.26	− 9.3	−10.32	+1.0	15.0
1908.3	241.94	−0.88	38.38	+1.86	241.06	−11.38	− 9.3	−10.40	+1.1	9.0

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RESULTS OF RESEARCHES

ON THE

ELECTRODYNAMIC WAVE-THEORY OF PHYSICAL FORCES

BULLETIN NO. 2

DISCOVERY OF THE PHYSICAL CAUSE OF MAGNETISM

By T. J. J. SEE

I General Introductory Remarks

Having announced in the first of this series of Bulletins the discovery of the physical cause of the hitherto *Unexplained Fluctuations* in the Moon's Mean Motion, and of *Universal Gravitation*, and indicated the main features of the new theoretical explanation of the *Lunar Fluctuations*, which are shown to constitute an *experimentum crucis* for establishing the truth of the Electrodynamic Wave-Theory of *Universal Gravitation*, the author is able to follow these significant results with the announcement of the discovery of the *Cause of Magnetism*, — including the Magnetism of the Earth and the periodic changes therein depending on the Sun and Moon, — in the action of Electrodynamic Waves of a New Type. These waves proceed from the equators of the atoms, are flat in those planes and in all planes normal to the lines of force, and propagated with the Velocity of Light.

Notwithstanding the enormous development of the modern physical sciences which deal with *Electricity* and *Magnetism*, beginning with the publication of GILBERT's *Treatise De Magnete*, 1600, it is a recognized fact that up to the present time the nature of Magnetism has successfully challenged the ingenuity of the natural philosopher.

About half a century ago, however, MAXWELL put into mathematical language some penetrating observations of FARADAY (*Experimental Researches in Electricity*, §1297, etc., Nov., 1837) in regard to the properties of the lines of force: and was able to attribute the properties of the magnetic field to the effects of rotations around the lines of force. Indeed, at an earlier date Lord KELVIN had treated of FARADAY's discovery of 1845, that there is sensible rotation of a beam of polarized light when it is passed through heavy glass, carbon disulphide, etc., along the path of a magnetic line of force (*Proc. Roy. Soc.*, Vol. VIII, June, 1856, *Phil. Mag.*, March, 1857, *Baltimore Lectures*, 1904, Appendix F.)

Lord KELVIN inferred that "the magnetic influence on light discovered by FARADAY depends on the direction of motion of moving particles," and that "FARADAY's optical discovery affords a demonstration of the reality of AMPÈRE's explanation of the ultimate nature of magnetism."

After discussing RANKINE's hypothesis of *molecular vortices*, which he had himself developed at length, Lord KELVIN finally concludes: "I think we have good evidence for the opinion that some phenomenon of rotation is going on in the magnetic field, that this rotation is performed by a great number of very small portions of matter, each rotating on its own axis, this axis being parallel to the direction of the magnetic force, and that the rotations of these different vortices are made to depend on one another by means of some kind of mechanism connecting them."

MAXWELL followed up KELVIN's researches, and, having become associated with FARADAY in 1861, formulated a mathematical theory of the physical lines of force, 1861-2, and applied the theory of *molecular vortices* to Magnetic phenomena and to Statical Electricity. Notwithstanding the originality, elasticity and vigor of his powers at thirty, MAXWELL had great difficulty in understanding the mechanism of *molecular vortices*, because he had no suspicion that they represented *flat waves*, having common direction and velocity and thus would be devoid of friction and dissipation of the Energy of motion into heat.

This difficulty indeed has continued to the present day. For in his *Mathematical Theory of Electricity and Magnetism*, third edition, 1915, p. 486, Dr. JEANS dwells at length on "our utter ignorance" of the *mechanism* by which action is transmitted through the aether.

Accordingly, in his celebrated Memoir "On the Physical Lines of Force," 1861, (cf. *Scientific Papers*, Vol. I, pp. 468, 486) MAXWELL, by default of the hypothesis of *Waves*, had to imagine a very artificial constitution of the aether:

"I have found great difficulty in conceiving of the existence of vortices in a medium, side by side, revolving in the same direction about parallel axes. The contiguous portions of consecutive vortices must be moving in opposite directions; and it is difficult to understand how the motion of one part of the medium can coexist with, and even produce, an opposite motion of a part in contact with it."

"The only conception which has at all aided me in conceiving of this kind of motion is that of the vortices being separated by a layer of particles, revolving each on its own axis in the opposite direction to that of the vortices, so that the contiguous surfaces of the particles of the vortices have the same motion."

"In mechanism, when two wheels are intended to revolve in the same direction, a wheel is placed between them so as to be in gear with both, and this is called an 'idle wheel'. The hypothesis about the vortices which I have to suggest is that a layer of particles, acting as idle wheels, is interposed between each vortex and the next, so that each vortex has a tendency to make the neighboring vortices revolve in the same direction with itself."

It was from a new point of view, attained in 1914, that the present writer was able to refer Magnetic phenomena to waves imagined as flat in planes normal to the lines of force, and thus producing the observed rotation of the plane of polarized light discovered by FARADAY in 1845. As this view of magnetism overcomes entirely the difficulties just described by MAXWELL, and has since been fully confirmed, it is now deemed advisable to communicate some of the results to other investigators, in the hope that they may find the new lines of inquiry worthy of attention in their researches.

The present treatment is restricted to brief notice by Bulletin, yet this may suffice to elucidate the chief phenomena. Accordingly, anyone may join in these studies who has the power of independent thinking, if his mind is not too much biased by misleading traditions, which unfortunately still are very numerous.

II Analysis of Maxwell's Final Conclusions

As we have seen above it was in 1861 that MAXWELL confirmed KELVIN's conclusions from strict dynamical reasoning, 1856, that the transmission of Magnetic Force is associated with a rotatory motion of the small parts of the medium. (cf. MAXWELL's Article *Attraction, Encyc. Brit.*, 9th edition, 1875, reprinted in the *Scientific Papers*, Vol. II, p. 488, and *Treatise on Electricity and Magnetism*, sections 791, 813, 816, 821, 822, 830 and 831.)

From the first MAXWELL saw clearly that there is rotatory motion going on in the medium and that the axis of rotation is in the direction of the magnetic force. In his *Treatise*, §821, he concludes as follows: "The only resemblance which we can trace between a medium through which circularly polarized light is propagated and a medium through which lines of magnetic force pass, is that in both there is a motion of rotation about an axis. But here the resemblance stops . . ."

"There is nothing, therefore, in the magnetic phenomenon which corresponds to the wave length and the wave-propagation in the optical phenomenon. A medium in which a constant magnetic force is acting is not in consequence of that force, filled with waves traveling in one direction, as when light is propagated through it."

Accordingly, whilst MAXWELL's great mathematical and physical intuition enabled him to forecast and to outline the *Electromagnetic Theory of Light*, which was afterwards so brilliantly confirmed by HERTZ, he was never able to make out the undulatory nature of Magnetism.

It is evident that MAXWELL sought for waves *along the lines of force instead of at right angles to these lines;* and, not finding any trace of such waves along the lines of force, was led to the conclusion that they do not exist in Magnetic phenomena, as shown above in the quotation from §821 of the *Treatise*. Apparently it never occurred to him to consider magnetic waves in planes normal to the lines of force, although he knew the *observed rotations* were in these planes, and recognized that the magnetic force consists in *pressure* normal to and *tension* along these lines.

In §§641-46 of MAXWELL's *Treatise on Electricity and Magnetism*, it is shown how electric and magnetic stresses may be analyzed, and the phenomena referred to the state of the medium.

In the Article *Attraction*, 1875, MAXWELL describes the theory as follows: "It is there shown that, if we assume that the medium is in a state of stress, consisting of tension along the lines of force, and pressure in all directions at right angles to the lines of force, the tension and the pressure being equal in numerical value and proportional to the square of the intensity of the field at the given point, the observed electrostatic and electromagnetic forces will be completely accounted for."

"The next step is to account for this state of stress in the medium. In the case of electromagnetic force we avail ourselves of THOMSON's deduction from FARADAY's discovery stated above. We assume that the small parts of the medium are rotating about axes parallel to the lines of force. The centrifugal force due to this rotation produces the excess of pressure perpendicular to the lines of force. The explanation of electrostatic stress is less satisfactory, but there can be no doubt that a path is now open by which we may trace to the action of a medium all forces which, like the electric and magnetic forces, vary inversely as the square of the distance, and are attractive between bodies of different names, and repulsive between bodies of the same names."

MAXWELL then turns to the consideration of the attraction of gravitation, but as in all cases he imagines the pressure to be along the lines of the gravitative force, combined with a tension in all directions at right angles to these lines, he is unable to make progress. This view of the stresses is incomplete, and thus it is not remarkable that those who have followed him have also failed. (Cf. MINCHIN, *Treatise on Statics*, Vol. II, 1886, pp. 448-458).

MAXWELL adds in conclusion that stresses of the kind he imagined "would no doubt account for the observed effects of gravitation. We have not, however, been able hitherto to imagine any physical cause for such a state of stress."

It may be worth noting that Professor CHALLIS of Cambridge long gave considerable attention to the problems of *Attraction*, and had ascribed the observed forces to wave action. In reviewing CHALLIS' "Essay on the Mathematical Principles of Physics," 1873, MAXWELL was not convinced by the author's argument, yet adds that gravity is the most universal and mysterious of all actions. "Whatever theory of the constitution of bodies holds out a prospect of the ultimate explanation of the process by which gravitation is effected, men of science will be found ready to devote the whole remainder of their lives to the development of that theory."

If so much importance avowedly was attached to the solution of the problem of *Attraction* by the greatest mathematician and natural philosopher of the past age, the present effort to give an accurate account of the successive steps in our progress will not be considered superfluous.

III THE CAUSE OF THE ATTRACTION OF TWO MAGNETS

1. With this explanation of MAXWELL's researches we shall consider the cause of the mutual action* of two magnets, a chief magnet A, and a secondary magnet or needle B. Under the hypothesis of Electrodynamic Waves, we see that if the chief magnet acts alone, the waves proceeding from it give rise to increase of *pressure* along the plane of xy, which is here taken at right angles to the axis of the magnet.

2. At any distance $r = \sqrt{x^2 + y^2}$ from the axis of the magnet, the stress on a small rectangle $dxdy$ may be denoted by ϖ. And the *stress* over a larger area $S = \int\int dxdy$ of any figure in the xy-plane, will be given the double integral

$$\int\int \varpi dxdy \tag{1}$$

*We are here concerned with the *Physical Cause of the Stresses* in Magnetism. The full mathematical theory of these stresses, without explanation of the *mechanism* by which they are produced, is given by MAXWELL (*Treatise*, §§641-645) MINCHIN (*Treatise on Statics*, Vol. II, 1886, §396) JEANS (*Mathematical Theory of Electricity and Magnetism*, 1915, §§157-168, 193-203) and other authorities. The present discussion is restricted to the proof of the *mechanism* involved; for as Dr. JEANS says (p. 486) we have heretofore been "in utter ignorance of the ultimate laws which govern action in the ether."

This resultant stress always has a definite value, depending on the variation of the element of stress on the surface elements $dx\,dy$ and the limits of the double integral.

3. In general the *mean intensity of the stress* is given by the equation

$$P_0 = \frac{P}{S} = \frac{\int\int \varpi dxdy}{\int\int dxdy} \tag{2}$$

And this may be applied to a surface of any extent or figure. *Tension* is equivalent to negative *pressure*. And the two types of stress on equal elements of the surface of the medium are numerically of equal magnitude. In the equations (1) and (2) the stress on the xy-plane is really a *tension*, but its value is identical with the *pressure* in the direction of the xy-plane.

4. Let us now consider the cause of *Attraction* in the *First Case*, where the Magnets present *unlike poles*. We may take the medium already *stressed*, as MAXWELL showed it is, by the chief Magnet A, having increase of *pressure* normal to the lines of *force* and corresponding *tension* along these lines. At the centre of the Magnet the increase of *pressure* is in the direction of the xy-plane, and the *tension* therefore along the z-co-ordinate.

5. Now imagine a Needle or Secondary Magnet introduced into the field of the chief magnet A, with its center lying exactly in the plane of xy, as shown in the figure, which requires brief explanation.

The curve of *wave amplitude* is greatly magnified, so as to render the law of the variation with the distance visible to the eye. The curve is a rectangular hyperbola referred to its asymptotes, $y = \frac{k}{x}$, or $a = \frac{k}{r}$.

It is well known that in wave motion the *amplitude* varies inversely as the distance, but the *energy* of the wave action varies as the *square of the amplitude*, or $E = \frac{k^2}{r^2}$; which corresponds to the forces following the law of the inverse squares, established for the attraction of Magnets by GAUSS, *Intensitas Vis Magneticæ*, 1833, §21, and for the attraction of the Planets under Universal Gravitation by NEWTON, *Principia*, 1687.

Returning now to the stress in the medium between the magnets A and B, we perceive that the *rotatory elements* of the new waves arising from B will at every point of the plane tend to counteract those from the chief Magnet A. The *pressure* parallel to the plane of xy is thus *decreased*, as if by a *new tension superposed* in that plane, with the value $\tau = -\varpi$ on any element $dxdy$.

6. The resulting modified mean stress or mean new pressure in the direction of the xy-plane becomes

$$P' = P - p = \int\int \bar{\varpi} dxdy - \int\int \varpi dxdy \tag{3}$$

which necessarily is less than P. Accordingly, as the Medium is less stressed when the Needle acts than when it does not, there results between the two ponderable bodies decrease of *pressure* in the direction of the plane xy.

7. *The Pressure normal to the lines of force, as* MAXWELL *shows, gives rise to a tendency in the Medium to expand, just as the parts of compressed India rubber do.* Hence FARADAY's observation that the lines of force tend to repel one another and to separate. Thus the tendency to expansion of the compressed medium inevitably leads to the result observed by FARADAY. As the *pressure* is decreased by the action of the Needle, it is equivalent to a *new tension* in the direction of the xy-plane. Hence we inevitably have *attraction* between the chief magnet and the needle. *Like a stretched India rubber layer contracting in the xy-plane, the Medium contracts in that direction, thus drawing the magnets A and B together.*

8. It is readily understood that although the introduction of the needle relaxes the *pressure at every point of xy and decreases correspondingly the tension along the z-co-ordinate, the new stress, thereby superposed, is numerically greatest at the least distances from both bodies, and thus directly between the two magnets.* Accordingly in the *contraction of the Medium,* like the relaxing of a stretched sheet of India rubber, the magnets with unlike poles are drawn together. This explanation is simple, and in view of MAXWELL's analysis leaves nothing to be desired either mathematically or physically.

IV THE CAUSE OF THE REPULSION OF TWO MAGNETS

9. To reach the case of repulsion we now reverse the ends of the secondary magnet, so that the poles presented are like. In this case the *mean pressure* parallel to the plane of xy will become everywhere a *maximum:*

$$P' = P + p = \int\int \bar{\varpi} dxdy + \int\int \varpi dxdy \tag{4}$$

So far from developing tension between the two ponderable bodies, this case offers *increase of stress, increase of mean pressure parallel to the xy-plane, and increase of mean tension parallel to the z-axis.*

10. The Medium is thus given an *augmented stress* and the two ponderable bodies are *forced apart*, as actually observed. The stress appropriate to each body separately is augmented by the *pressure and tension superposed* by the other. Accordingly, either body, by the wave action of the other, can exert a given stress at a greater distance than without it, so that the double Magnetic system reacts as if by mutual repulsion of the masses.

11. In this way we explain very simply the celebrated phenomenon of the *attraction* and *repulsion* of two Magnets. The explanation involves no hypotheses except the single one of the *stressing of the medium by plane waves for producing the rotations* of the elements around the lines of force, in accordance with FARADAY's experiment of 1845, on the rotation of a beam of polarized light by magnetism.

12. It follows from the Electrodynamic Wave-Theory of Magnetism, that such waves proceed from every atom in each magnet. At any point of space the resulting rotations of the medium generally are not in the same plane, but in planes mutually inclined at the angle ε. The intensities of the wave energies are $I = \frac{\kappa^2 a^2}{r^2}$, $I' = \frac{\kappa'^2 a'^2}{r'^2}$, and therefore in a limited region the effect upon the medium from pairs of masses of atoms

$$dm_2 = \sigma_2\, dx_2\, dy_2\, dz_2\,, \quad dm_1 = \sigma_1\, dx_1\, dy_1\, dz_1\,,$$

would lead to the stress element due to the resultant forces

$$dP = \frac{\sigma_2 \kappa_2^2 a_2^2}{r_2^2} \cos\epsilon \frac{\sigma_1 \kappa_1^2 a_1^2}{r_1^2}\, dx_2\, dy_2\, dz_2\, dx_1\, dy_1\, dz_1 \tag{5}$$

And for the whole of the two magnets* exerting stresses in the medium throughout all space the sextuple integration would give for the resulting *attraction*

$$A = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left(\frac{\sigma_2 \kappa_2^2 a_2^2}{r_2^2} \cos\epsilon \frac{\sigma_1 \kappa_1^2 a_1^2}{r_1^2}\, dx_2\, dy_2\, dz_2\, dx_1\, dy_1\, dz_1\right) dx'\, dy'\, dz'\, dx\, dy\, dz \tag{6}$$

In this equation we have put $r_1^2 = (x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2$ for the squared distance of the point (x, y, z) from the atoms of the primary mass, and equally

*In many works on *Magnetism*, such as that of Dr. JEANS, §§421-2, p. 379, it is shown that the force between two magnets varies as the inverse fourth power of the distance. If the waves acting from each magnet be imagined concentrated at the mean distance of the other — which is essentially the view taken in an integral formula, where each set of waves is referred to its originating centre, — then in this expression $r' = r$, and we get the law of the inverse fourth power characteristic of magnetic mass action.

$r_2^2 = (x' - x_2)^2 + (y' - y_2)^2 + (z' - z_2)^2$, for the squared distance from the atoms of the secondary magnet. The co-ordinates (x_1, y_1, z_1), (x_2, y_2, z_2) relate to the spaces occupied by the atoms within the surfaces of the two bodies, while (x, y, z), (x', y', z') represent all space referred to the centres of gravity of these bodies as wave centres.

The sextuple integral (6) determines the action due to two magnetic systems, with their waves interpenetrating throughout all space, and by the stressing of the medium, thus causing the attraction observed.

If the matter of two Magnets be imagined expanded in to infinite cosmical clouds or nebulæ filling unlimited space, and interpenetrating each other in any manner, the mutual potential energy of their matter under Universal Gravitation would be

$$E = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{\sigma\sigma' dx'dy'dz'dxdydz}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} \tag{7}$$

Let I and I′ be the intensities of the magnetization due to the two magnets, and λ, μ, ν, λ′, μ′, ν′ the direction cosines of their axes; then the mutual potential energy of the two magnets, imagined as nebulæ of unlimited extent and interpenetrating each other in any manner, will be

$$\Omega = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left(\lambda'\frac{d}{dx'} + \mu'\frac{d}{dy'} + \nu'\frac{d}{dz'}\right) I'dx'dy'dz' \left(\lambda\frac{d}{dx} + \mu\frac{d}{dy}\ \nu\frac{d}{dz}\right) I\,dxdydz \tag{8}$$

When the two magnets are distinct and confined within finite surfaces, the above equations (6) and (8) may be reduced to the form of a triple integral appropriate for the mutual action of two finite magnetic systems each external to the other, but with their waves interpenetrating throughout space, and causing attraction by the stresses developed in the medium.

Let us denote by R and R' the *resultant forces* at any point of space from all the atoms in each of the two magnets. Then this equation becomes the triple integral:

$$A = \frac{1}{4\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} RR' \cos\theta\, dxdydz\ , \tag{9}$$

where θ is the angle between the direction of the resultant forces. By its infinite limits this integral includes all space, and thus expresses the action resulting from the stressing of the aethereal medium by the independent systems of electrodynamic waves emitted by the atoms of the two magnets and propagated throughout infinite space with the velocity of light.

In concluding this analysis of the attraction and repulsion of magnets, it only remains to point out, that the poles are *positive* or *negative*, according as the molecular rotations are *positive* or *negative*. The action thus depends on the side from which we view the *vortices* or the *plane waves* emitted by the atoms of the magnets. And as electro-magnets acquire their properties under the action of currents which line up their atoms in parallel planes, held by AMPÈRE, 1822, the electrodynamic wave-theory implies that *mutual interactions of waves alone* are concerned in magnetic attraction.

Accordingly the old conception of two fluids finally disappears from the theory of magnetism. Whether this will lead also to the disappearance of the terms *positive* and *negative* as applied to *electricity* remains to be decided; but it is not easy to see how our views on this hitherto obscure subject can escape material modification.

V NEW THEORY OF OERSTED'S EXPERIMENT OF 1819

In view of the foregoing theory of Magnetic Attraction by *stresses* in the Medium, due to waves, we are now able to give a simple dynamical explanation of OERSTED's celebrated experiment of 1819, as follows.

1. Imagine a wire bearing a current from South to North, as shown in the accompanying Figure 2, and let a magnetic needle be suspended below it.

2. Owing to the rotations in the earth's field it will be seen that looking north *right handed rotations* make up the elements of the electrodynamic waves from the wire, while viewed from the south pole *left handed rotations* proceed from the needle below the wire, as shown in the figure. This gives a general view of the nature of the mutual wave action.

3. The result is that at every point of space the *stress* in the medium, due to the current, is *decreased* by the *waves due to the magnet;* and when the *pressure* and *tension* are thus *decreased*, the medium contracts like a stretched mass of India rubber. The contraction is greatest between the needle and the wire, and thus the needle is drawn to the wire. This will apply to a copper wire, or wire of any conducting material, and hence the explanation is general, applying directly to ARAGO's experiment of 1820.

4. It has been held from the days of OERSTED that the current *only directs the needle, but neither attracts nor repels it* (cf. MAXWELL's address on *Action at a Distance*, *Scientific Papers*, Vol. II, p. 317). MAXWELL apparently was unable to detect the error in this reasoning, and, with his endorsement of the view of OER-

STED, it seems to have continued to the present time.* But the above reasoning shows the error in the traditional theory of this celebrated experiment, which was the very beginning of Electrodynamics.

VI Reconciliation of Ampère's Theory of Electric Currents Circulating about the Atoms with the Electrodynamic Wave-Theory of Magnetism

From experiments made in 1822, AMPÈRE found that when the wire conducting a positive current in a solenoid is *wound right handed* about the steel bar within, that end of it becomes the south pole. Instead of attributing magnetic phenomena to two fluids AMPÈRE assumed that each individual molecule of a magnetic substance is transversed by a closed electric current, and further that these molecular currents are free to move about their centers, and thus forced to line up in parallel planes by the action of the electric current. They are surrounded by an infinite system of plane waves traveling outward through space in all directions, and thus called a *polarized or magnetic field.*

AMPÈRE's Theory of Magnetism was proposed in 1822 (*Journal de Physique*, Tome XCIII, or *Recueil d'Observations*, Paris, 1822, pp. 164-174), and ascribes the power of magnetizing to the turnable character of the molecular currents about the molecules. He cites various considerations in support of this view, and in paragraph 7 says they "confirm the opinion, founded moreover on a comparison of all the facts, that the properties of magnets are really due to the continued movement of two electric fluids around their particles." (*Théorie Mathématique des Phénomèones Électro-dynamiques*, A. M. AMPÈRE, 2d edition, Paris, 1883, p. 151).

In the accompanying diagram, Fig. 3, we give the mathematical conditions which prove that the *Theory of* AMPÈRE is identical with the *Wave-Theory:* there is thus shown to exist an exact agreement of the two theories, as implied in equation (10), which underlies the whole theory of wave motion.

1. Imagine a series of plane waves propagated along a horizontal line, as shown in Fig. 3. All the waves

$$y = A \sin \left\{ \frac{2\pi}{\lambda} (Vt - x) + a_i \right\} \tag{10}$$

*In recognizing that by the action of its *poles* the needle tends to bend itself around a charged wire MAXWELL implies the attraction of the needle to the wire, but this view of the action of *poles* is so unsatisfactory that we proceed in a very different way.

are taken to have the *same length* (λ) though of *different phase* (α_i) as if coming from *successive atoms of a magnet at different distances*, but otherwise acting in concert. The successive waves are numbered α_1, α_2, α_3, α_4, α_5, α_6, α_7, α_8.

2. The circular figure represents an AMPÈRE atom, and for simplicity we have made the points of the circumference at 0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°, 360°, correspond to the waves of phase α_1, α_2, α_3, α_4, α_5, α_6, α_7, α_8. The intervals here adopted are for graphical illustration only, and it is evident that for a magnet with billions of billions of atoms in any diameter of the cross section, the successive waves would be infinite in number, 1 to ∞; and, owing to the atoms crowded in above and below the line, their spaces apart also are infinitely small.

3. *Acccordingly, it follows that this infinite order of successive waves, also from above and below, would give continuous motion at every point of the circuit of the* AMPÈRE *atom here represented. Thus the aether in continuous motion, oscillating as the waves pass, would give the effect of closed electric currents circulating about the atoms. These currents would attract or repel according to their direction of flow, as long held in Electrodynamics by* AMPÈRE, WEBER, *and their successors.*

4. *Therefore it follows that a succession of waves from the atoms would explain the closed electric currents imagined by* AMPÈRE *to circulate around the atoms.* And *vice versa,* the existence of the *closed currents* implies the existence of the *waves.* The two views are easily harmonized by such an explanation, which is both necessary and sufficient to account for all the phenomena. The Wave-Theory is therefore geometrically established as representing the true phenomena of Nature; for it is only by *waves depending on oscillations* within the atomic system that we can explain the *closed currents* imagined by AMPÈRE to circulate about the atoms of a Magnet. As the Aether is the vehicle of energy, it is a perfectly elastic medium; and *electrodynamic waves,* with half the energy kinetic and half potential, for stressing the medium, is Nature's way of exerting action across space, through forces traveling with the velocity of light.

VII NEW THEORY OF ARAGO'S EXPERIMENT OF 1820

In view of the above theory of OERSTED's experiment it is easy to explain in a very simple manner the *attraction* observed in ARAGO's celebrated experiment of 1820.

1. Imagine a copper wire bearing a current, with waves proceeding from it, as shown in the foregoing figure illustrating OERSTED's experiment. The

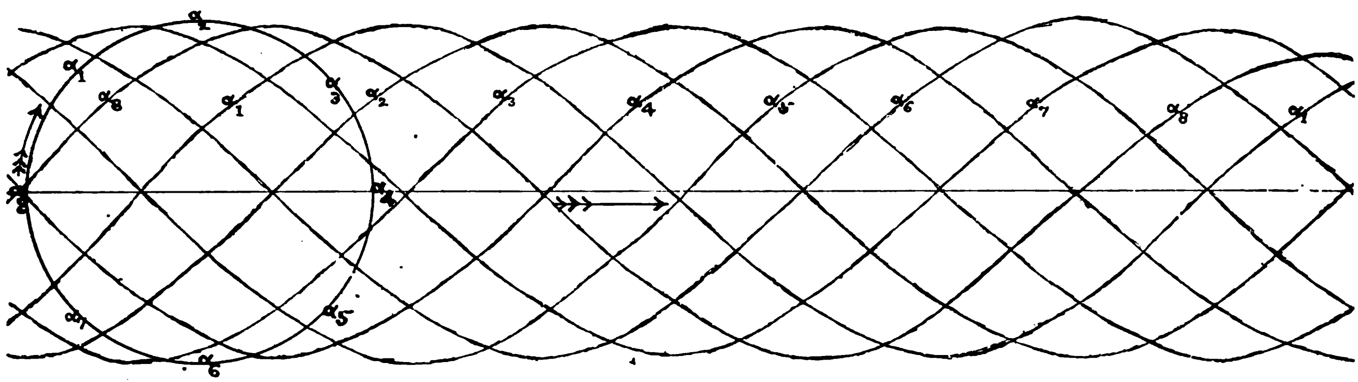

Fig. 3. Ampère's Theory of Elementary Electric Currents about Atoms reconciled with the Wave-Theory.

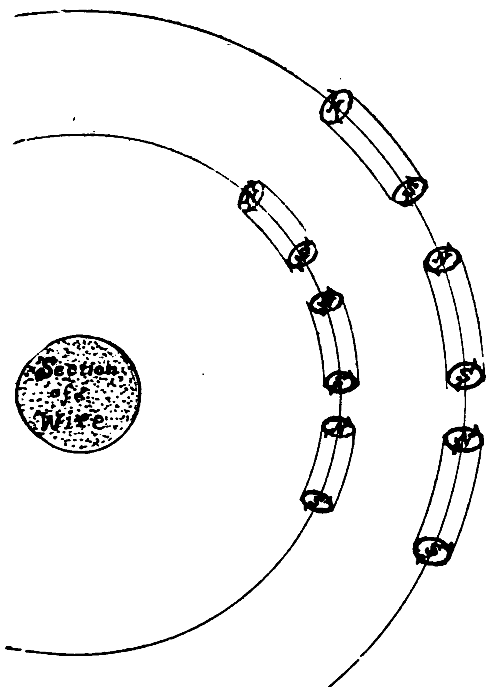

Fig. 4. New Theory of the Whirl of Iron Filings about a conducting wire.

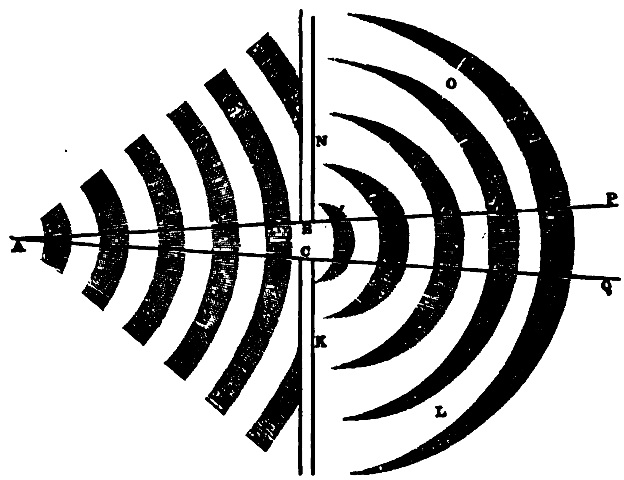

Fig. 5. Newton's Theory of the lateral spreading of waves propagated through an Orifice., *Principia*, 1687.

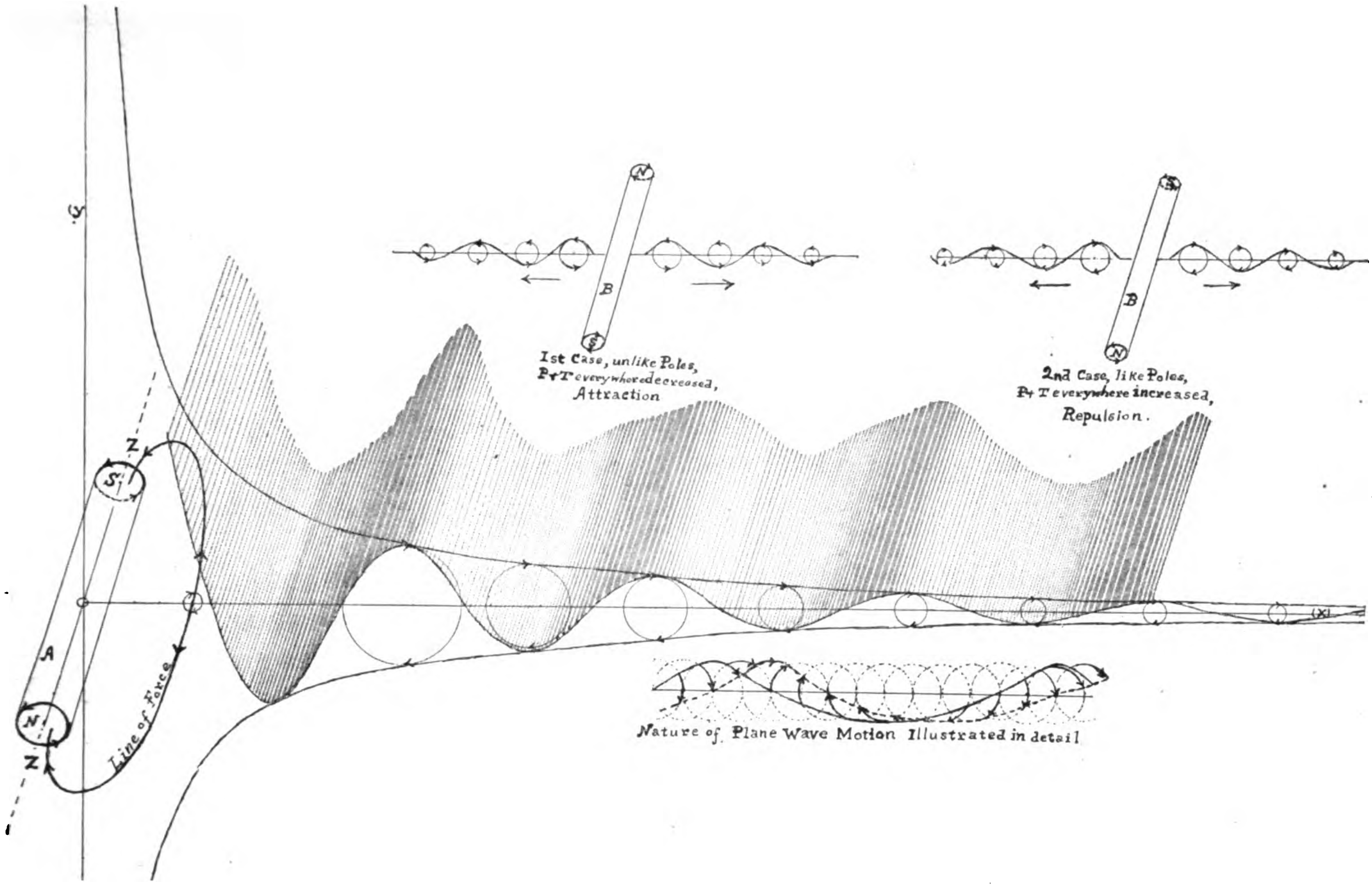

Fig. 1. Electrodynamic Wave-Theory of Magnetic Attraction and Repulsion.

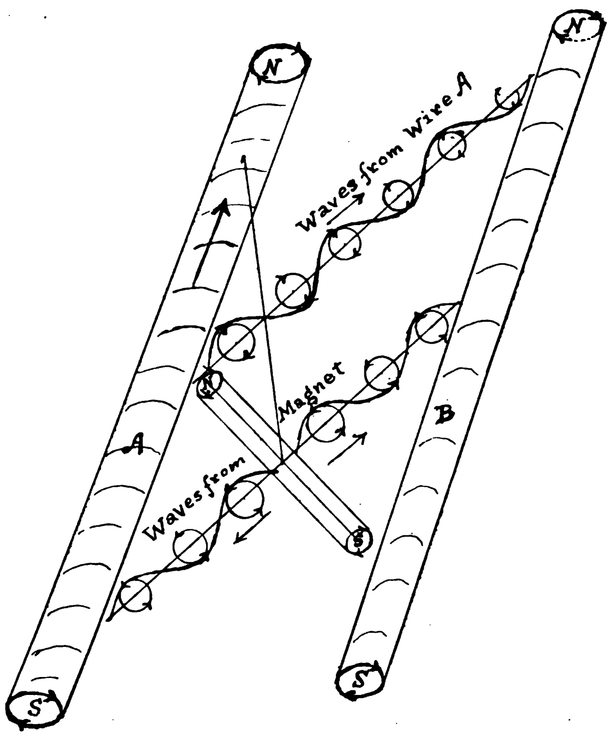

Fig. 2. New Theory of Oersted's and Arago's Experiments, 1819-20, and of Induction.

rotations of the elements in these waves will be right handed, as seen from the eastern side of the southern end of the wire, owing to the well known direction of the rotations in the earth's magnetic field.

2. Now let particles of *soft iron filings* — which we may suppose cylindrical in form, as if made by cutting up soft iron wire of small diameter — be brought near the copper wire. It is evident that the electrodynamic waves proceeding from the wire will induce magnetism in these filings; one end will become a north pole and the other a south pole.

3. The south pole of any little cylinder will take a position similar to that of the needle just described in OERSTED's experiment. And, just as in that celebrated experiment, these little temporary magnetic needles will be *attracted* to the wire — the south end of one fitting on to the north end of another, till the whole mass of iron filings forms an envelope about the copper wire. If not too thick and heavy this sheath of temporary magnets will cling to the wire so long as the current flows.

4. But let the current cease, and it is observed that the filings will lose their magnetism, and drop off. This also follows from the theory of OERSTED's experiment described above. For the *attraction* only takes place when the little filings under the waves from the wire become magnets. The instant a filing becomes a magnet, it sends out waves of its own, like those from the needle in OERSTED's experiment; these little waves tend to decrease the stress in the medium due to the waves from the copper wire. And then *Attraction* necessarily follows.

5. *As the filings are very small cylinders, they are feeble as magnets, and their magnetic waves are sensible only at very small distances.* Hence in ARAGO's experiment it usually is necessary to dip the copper wire into the iron filings to get them to adhere. Yet as we see from the foregoing figure the theory is identical with that of OERSTED's experiment, except that the *little magnets are electro-magnets*, depending on the current, but so small that they have sensible wave power only at very short distances.

6. Accordingly, it appears that a correct theory of OERSTED's experiment is the first desideratum for explaining ARAGO's experiment. Up to the present time it seems that no satisfactory explanation of these two celebrated experiments has been forthcoming. By the adaptation of MAXWELL's theory of *stresses* in the Medium to the new electrodynamic wave-theory this long standing difficulty appears to be completely overcome.

VIII New Theory of the Circular Whirls of Iron Filings about a Wire Bearing a Steady Current

The phenomena presented by the circular whirls of iron filings about a conducting wire has never been satisfactorily explained, although spoken of as showing a sketch of the *lines of force* in the surrounding magnetic field.

1. From the above theory of Oersted's and Arago's experiments, we perceive that just as the wire attracts a needle, or small electromagnet in Arago's experiment, so also will the waves about the wire make electromagnets out of the iron filings, whether they be near the wire or further away. The filings line up, with the north pole of one joined to the south pole of another, and are thus winnowed by the electrodynamic waves into helices, which are almost concentric circles about the conducting wire. The filings are heavy, and not drawn to the wire unless very near, as their *magnetic fields* are minute in extent.

2. *Two little electromagnets in the same line thus exert mutual attraction at their adjacent (unlike) poles; but on the sides two such magnets, further apart, in different circles, offer slight repulsion, because there the adjacent poles are like, as shown in the accompanying Figure 4.*

3. The result therefore is repulsion of the circles for one another, as was observed by Faraday for his lines of force generally. Indeed he noticed that they tend to shorten themselves, and to separate, and we now see why this tendency exists.

4. As the electrodynamic waves recede from the wire they react on the filings, by induced waves, as before explained. The winrows of iron filings are thereby inevitably drawn nearer and shortened, just as the needle in Oersted's experiment and the filings in Arago's experiment are brought as near the wire as possible. Yet owing to the repulsion, due to the presentation of similar poles in adjacent winrows, the lines of force inevitably tend to separate and appear as distinct winrows. The neighboring lines probably repel each other slightly, but at the same time tend to shorten themselves to the maximum extent.

5. This apparently gives a complete explanation of the observed properties of Faraday's lines of force. It only remains to add that in the case of a magnet the *shortening and tension* of the lines of force is towards the *two poles*, as shown in the writer's unpublished memoir of 1914, and in Fig. 1 above.

6. In the case of waves from currents on the wires, the lengths of the lines of force naturally are a *minimum.* Magnetic waves proceed from the atoms,

and the lines of force re-enter at the magnetic poles, the *tension* making them also of the shortest possible length. On the other hand *pressure*, in the direction at right angles to the lines of force, due to rotations of the wave elements, makes the medium tend to expand itself to the *maximum*. This simple reasoning appears to open to our mental vision a perfect picture of the processes going on in the Magnetic Field.

It should be noted that the position taken by the needle in OERSTED's experiment shows that the lines of force about a conducting wire are not exact circles, as MAXWELL believed, but helices, somewhat closely wound, and thus roughly resembling circles.

EXPLANATION OF INDUCTION AND OF THE INDUCED CURRENT BASED ON CHANGE IN THE WAVE ACTION

Having given above a new theory of Magnetic Induction, based on wave action, we add here a simple theory of the Induced Current.

1. When waves from wire A, Fig. 2, are increasing, their action in passing wire B also produces a current — the *induced current* — which ceases as soon as current A becomes steady.

But this only shows that the *waves* from A have become *uniform* in their action on wire B. Let current A be interrupted or cut down, and there will be an immediate response in B, in the *contrary direction.*

2. This indicates that the aether in and about B was stressed by the steady flow of waves, and springs back by elastic reaction the moment current A is cut down or stopped, which shows the perfect elasticity of the medium and its instant response to change. It thus appears that by the theory of waves we have a perfectly simple explanation of the *Induced Current.*

The electrodynamic wave-theory shows how this occurs, and thus makes clear the nature of the *reaction* for developing the Induced Current, which it seems has not been clearly understood heretofore.

3. As the *induced current* always resists the original current, flowing in the contrary direction, this tendency shows that it is due to the *elastic reaction* of the waves, when the wave-development or flow changes. We thus obtain a *very convincing proof of the wave-theory;* and the explanation is so simple that anyone can grasp it.

IX In Magnets the Waves Emitted by the Atoms Start as Flat Wavelets in Planes Normal to the Magnetic Axis, but as the Rotations of the Wave Elements Give Increase of Pressure Normal to the Lines of Force, Balanced by Tension Along these Lines, There Necessarily Results from this Equilibrium Between the Pressure and Tension a Maximum Tendency to Lateral Spreading of the Rotating Filaments, and Hence a Crowding of the Vorticose Aether into the Poles.

After what has been shown in regard to the waves proceeding from a Magnet, the above conclusion is almost obvious. For it is well known that if we start plane waves, in a fluid such as water, and let them issue through an orifice with sharp walls, the wave disturbance tends to spread laterally, on either side of the orifice. This spreading to the side was carefully discussed by NEWTON, and is illustrated by the accompanying figure from the *Principia*, (Lib. II, Section VII, Prop. XLI, p. 359, Glasgow edition, 1871).

Now it is obvious that if such a lateral spreading as is here indicated by NEWTON can take place in such a fluid as water, when agitated by steady waves — which is easily verified by observation — then it is evident that a similar lateral spreading will tend to take place in the filaments of vorticose aether about a magnet. Water waves passing through an opening, like that above drawn by NEWTON, have both longitudinal and transverse motion, and the periodic oscillations of the particles are in planes normal to the fluid surface. The periodic oscillations making up the wave motion are around lines parallel to the *axes of the waves* here drawn: which correspond exactly with the FARADAY Lines of Force in the Magnetic problem. Therefore in Magnetism the similar periodic oscillations of the particles of the aether will be around the FARADAY Lines of Force; and the vorticose aether will experience a crowding towards either pole of the Magnet, from the equilibrium of the pressure and tension incident to the Magnetic Waves proceeding from each atom. The necessary result, in the maintenance of the equilibrium, will be a reaction extending the wave along the axis, and, after the filament curves around, crowding the vorticose aether in at either pole of the Magnet, as announced in the above proposition.

X The Equilibrium Between Pressure and Tension Gives the Side-spreading of Waves from Magnets, and thus is the Cause of the Motion along the Lines of Force.

1. The integral developed by WHITTAKER (*Monthly Notices*, 1902, p. 619) in his theory of flat wavelets for explaining the Gravitation of a particle is

$$V = \frac{2}{\pi r}\int_0^\infty \frac{\sin y\, dy}{y} = \frac{1}{r}\ ; \tag{11}$$

and obviously this will also hold for atomic waves in the planes of their equators.

2. Now from each atom wavelets originally flat, parallel, and normal to the Magnetic axis are traveling outward through all space (x, y, z). But as in passing through an orifice the water waves tend to spread to the side, — so also will the aether waves tend to be prolonged under the *tension* in the medium. And in the equilibrium between the *pressure* and *tension* there necessarily is a tendency at every point of the line of force to extend the vorticose aether motion sidewise. And in this continued balance between *pressure* and *tension* the result is an adjustment of the lines of force to minimum length for the given stress of the medium, which is determined by the sum total of the waves from all the atoms.

3. Obviously it is not by chance that the lines of force are flattest at the side of a magnet. This form of the vorticose filaments necessarily results from the extension of motion under the very nature of the system of waves incessantly emitted. By the extension of the filaments, due to *tension* in the lines of force, the vorticose aether tends to lengthen at every point parallel to the axis of the Magnet; thus even at the end of the bar, there is a spreading beyond, under the tension of the vorticose filaments, and the lines therefore curve gently around, and enter towards the pole.

4. Imagine an infinite series of narrow orifices along the sides of a magnet to allow of the emission of waves and their side-spreading, as in the waves of water treated by Newton in the *Principia;* then we may easily see how the medium about the magnet reacts to make *tension* along the lines of force. Such centers of wave-generation in the medium must be imagined to exist throughout the bar of the magnet, while narrow apertures are imagined at the surface of the bar, in all directions.

5. As the waves emerge through these narrow orifices, with increase of *pressure* normal to the axis, the reaction will give *tension* incident to the spreading of the vorticose motion sidewise: the fluid therefore has a *tension* in that direction, and the lines of force necessarily are made as short as possible, under the *pressure* of the medium in planes normal to the axis of the magnet.

XI The Electrodynamic Waves Propagated from Magnets Normal to the Lines of Force Are Flat, with Motion of Aether Particles both Longitudinal and Transverse, Exactly like that of Waves in Still Water; and as these Longitudinal Waves Travel only with the Velocity of Light, It Follows that the AETHER Does not Vibrate as an Incompressible Elastic Solid.

In his celebrated Article on the *Wave Theory of Light*, *Ency. Brit.*, 9th edition, pp. 422 and 446, 1887, Lord Rayleigh shows that *an isotropic elastic solid* may transmit two kinds of waves:

1. *Transverse Waves* depending on *rigidity or resistance* to shearing motion, as layers of the solid slide over one another, the velocity of transmission depending on the power of recovery from a state of shear.

2. *Compressional Waves, depending on alteration of volume.* Lord Rayleigh cites Green's conclusion that the aether is incompressible; so that if α, β, γ be the component displacements, the dilatation

$$\delta = \frac{d\alpha}{dx} + \frac{d\beta}{dy} + \frac{d\gamma}{dz} = 0\,. \tag{12}$$

Otherwise, as in Sound, these *longitudinal waves* would become sensible. But as such waves are not observed, in Light, it is concluded that the *longitudinal waves have infinite velocity*, owing to the incompressibility of the medium.

In the foregoing discussion we have seen that in Magnetism, the electrodynamic waves normal to the lines of force are flat, with motions of the particles or elements in rotation, which are both longitudinal and transverse. It is also a fact, well determined by experimental research, like that of Hertz, that electromagnetic waves, as Maxwell first predicted in 1864, travel only with the velocity of light. *Thus the part of the wave which is longitudinal* travels with the *same speed as the rest of the wave disturbance.* Accordingly, the older view of Green, that the longitudinal wave may have infinite velocity, is contradicted, and must be wholly abandoned. Hence it follows that the aether does not vibrate as an incompressible elastic solid, but must be held to be a compressible medium of high elasticity.

3. In his celebrated Article on the *Wave Theory*, p. 451, Lord Rayleigh gives the following lucid account of *Magnetic Rotation:*

"The possibility of inducing the rotatory property in bodies otherwise free from it was one of the finest of Faraday's discoveries. He found that, if heavy glass, bisulphide of carbon, etc., are placed in a magnetic field, a ray of polarized light, propagated along the lines of magnetic force, suffers rotation. The laws of the phenomenon were carefully studied by VERDET, whose conclusions may be summed up by saying that in a given medium the rotation of the plane for a ray proceeding in any direction is proportional to the difference of magnetic potential at the initial and final points. In bisulphide of carbon, at 18° and for a difference of potential equal to unit C. G. S., the rotation of the plane of polarization of a ray of soda light is .0402 minute of angle."

"A very important distinction should be noted between the magnetic rotation and that natural to quartz, syrup, etc. In the latter the rotation is always right handed or always left handed with respect to the direction of the ray. Hence when the ray is reversed the absolute direction of rotation is reversed also. A ray which traverses a plate of quartz in one direction, and then after reflexion traverses the same thickness again in the opposite direction, recovers its original plane of polarization. It is quite otherwise with the rotation under magnetic force. In this case the rotation is in the same absolute direction even though the ray be reversed. Hence, if a ray be reflected backwards and forwards any number of times along a line of magnetic force, the rotations due to the several passages are all accumulated. The non-reversibility of light in a magnetized medium proves the case to be of a very exceptional character, and (as was argued by THOMSON) indicates that the magnetized medium is itself in rotatory motion independently of the propagation of light through it."

4. *Without in the least intending it, this account shows in the clearest manner the true nature of Magnetism, which really depends on flat waves in planes normal to the lines of force.* Such waves are illustrated in the accompanying diagram, Fig. 1, discussed in III above. *These flat waves explain both magnetic attraction and repulsion, by stresses in the medium, due to the rotations about the lines of force, which* MAXWELL *treated of as far back as 1861, and Lord* KELVIN *in 1856.*

Accordingly, contrary as it is to traditional opinion, there is nothing in modern Science to contradict the existence of *these flat waves, of which the magnetic rotations are the elements.* Since the velocity is that of Light these flat longitudinal waves do not correspond to the incompressible elastic solid theory, and become in fact an *experimentum crucis* against it.

5. As shown in the detailed illustration of plane wave motion, Fig. 1, regular waves on still water are propagated by similar flat oscillations, or rotations of the particles around mean positions. If such waves be maintained steadily from any source, the rotations of the fluid elements will be incessant, — as observed

by FARADAY for Magnets, 1845, in the experiment on the rotation of polarized light, and in the immemorial phenomenon of the tides of the sea (cf. AIRY's *Tides and Waves*, 1845) depending on the return of the Sun and Moon. *From the foregoing theory it follows that such flat waves in the Aether do actually proceed from the atoms of all magnets, and the persistence of the Faraday lines of force shows that they remain flat at all distances.*

The power of Magnetism is thus extended to infinity, but only with the velocity of light, and without any change in the nature of the waves. Accordingly it follows from the above reasoning of Lord RAYLEIGH and the similar reasoning of Lord KELVIN in the Article *Elasticity*, *Encyc. Brit.*, ninth edition (pp. 824–5), that GREEN's inference of an infinite velocity for the longitudinal wave is contradicted by well established observations in Electrodynamics, which apply also to Magnetism.

6. Therefore, in the propagation of Magnetic Waves the aether certainly does not behave as an incompressible elastic solid. There can be no doubt that the whole Wave-Theory of Light will require corresponding revision. Already in his celebrated Article on the *Wave-Theory*, written in 1887, Lord RAYLEIGH shows that as a whole the analogy with a solid is not very satisfactory, the theory being of doubtful applicability. Perhaps the present line of thought will give some definite grounds on which a consistent reconstruction may be attempted.

XII THE ELECTRODYNAMIC WAVE-THEORY POSTULATES WAVES FLAT IN THE EQUATORS OF THE ATOMS, AND THEREFORE OF A NEW TYPE

Not the least of the advantages of this simple and harmonious Wave-Theory is that it completely wipes away the *mechanical difficulties* which so beset MAXWELL in 1861, as described in II above, and which Lord KELVIN and other modern physicists have been so wholly unable to overcome that in his *Mathematical Theory of Electricity and Magnetism*, 1915, p. 485-6, Dr. JEANS says:

"We have even obtained formulae for the stresses and energy in the ether. But it has not been possible to proceed any further and to explain the existence of these stresses and energy in terms of the ultimate mechanism of the ether . . ." "We do not know how the aether behaves, and so can make no progress towards explaining electromagnetic phenomena in terms of the behaviour of the ether."

In view of the nature of the electrodynamic waves emitted from magnets, which are flat in planes normal to the lines of force, we see that there is everywhere minimum friction and resistance, and an essential similarity between these waves and those observed on the surface of still water. As applied to the aether the waves are thus of a new type. In the planes of the equators of the atoms, the motion is that of a flat wavelet. On either side of the equator, the motion becomes that of a helix, with right handed or left handed rotation, according to the side from which it is viewed.

But it should be distinctly pointed out that the movement of the vector is not wholly perpendicular to the direction of the ray — as held in the tradition handed down and still taught in the Wave-Theory of Light — but has associated with its advance the fraction of the original periodic longitudinal motion in the equator determined by the cosine of the atomic latitude of the point of observation. Thus if the amplitude of the flat wave in the equator be a, the longitudinal displacement of the particles of the aether is

$$\xi = a \cos \left\{ \frac{2\pi}{\lambda} (Vt - x) + \alpha \right\} , \tag{13}$$

and on either side of the equator the longitudinal displacement becomes $\xi' = \xi \cos \varepsilon$, where ε is the latitude of the point of observation in reference to the equator of the atom emitting the wave.

How different is the teaching handed down in the Wave-Theory of Light may be seen from the account given by MAXWELL, *Treatise on Electricity and Magnetism*, §816.

MAXWELL's argument is not at all conclusive, but is satisfied equally well by waves of the type here outlined. Thus if the mutual inclination of the planes of two polarized rays be i, we shall have for the relative transverse motion at any point

$$\eta' = \eta \cos \epsilon \cos i = a \sin \left\{ \frac{2\pi}{\lambda} (Vt - x) + \alpha \right\} \cos \epsilon \cos i \ , \tag{14}$$

This vanished when $i = \left.\begin{matrix} 90^\circ \\ 270^\circ \end{matrix}\right\}$, and otherwise fulfills the recognized laws of polarized light, which does not interfere when the planes of polarization are at right angles to each other.

It will simplify matters to remember that polarized light also has longitudinal motion, owing to the rotation of its elements as the flat wavelet advances; in fact such light is much like the magnetic waves normal to the lines of force. Hence the magnetization of a body consists in *polarizing its waves* by turning the atoms into parallel planes, so that the waves everywhere conform and vibrate in concert. The waves produce stresses in the medium, and the natural vibrations of the atoms, so well known in *Light*, are easily extended to *Magnetism*, owing to the connections established by FARADAY and ZEEMAN. In fact the present investigation gives a new foundation for the theory of the ZEEMAN effect, as well as that long ago discovered by FARADAY.

This simple Theory of Magnetism so well explains the phenomena of Nature that the scrupulous philosopher will hesitate before denying its truth. The magnetic waves considered are longer than the waves of light and heat, and thus readily go through all bodies. Perhaps it is well to recall the relative lengths of the electrodynamic waves now known:

1.	The smallest wave length measured	0.00001 cm.
2.	Maximum of chemical action in solar spectrum which is near the extremity of the visible spectrum	0.00004
3.	The yellow *D* line of sodium	0.000059
4.	Red extremity of visible spectrum	0.00007
5.	Maximum energy in solar spectrum	0.00008
6.	Longest wave-length measured	0.0025
7.	Shortest electrical oscillation yet measured	0.6
8.	Waves used in Radio-Telegraphy { Calling	95200
	Working	240000

This table shows the almost infinite range of the waves already measured. In many cases they are very short, and we can not see them with the eye or feel them with the touch; yet the invention of the galvanometer and bolometer has given us the most refined methods of testing the existence and laws of action of these electrodynamic disturbances. In the next Bulletin we hope to deal with Cosmical Magnetism, including the periodic fluctuations in the Earth's Magnetism depending on the actions of the Sun and Moon.

But from what is here shown it will be obvious that the heavenly bodies emit electrodynamic waves of similar length to those emitted from artificial and natural magnets. The study of Magnetism from the new point of view will thus deserve the attention of the astronomer, the geometer, and the natural philosopher.

Being among the most familiar cases of Attraction in Nature, Magnetism naturally appeals to us as one of the most accessible to study and observation. It is only after we have studied the new theory of Magnetism, in its cosmical aspects, that we can profitably approach the greater mystery of the *Cause of Universal Gravitation.*

T. J. J. SEE

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RESULTS OF RESEARCHES

ON THE

ELECTRODYNAMIC WAVE-THEORY OF PHYSICAL FORCES

BULLETIN NO. 3

DISCOVERIES IN COSMICAL MAGNETISM

By T. J. J. SEE

I INTRODUCTORY REMARKS

At the outset of Sir GEORGE AIRY'S *Treatise on Magnetism,* which was written in 1870, and dedicated to Sir JOHN HERSCHEL, the late distinguished Astronomer Royal begins the discussion as follows: "In ordinary observation, magnetism is scarcely known except as existing in iron and especially in steel, and as related in some obscure manner to the Earth. But there is reason to believe that it is one of the most extensively diffused agents in nature."

He then treats of the leading phenomena of magnetism as observed in iron, steel and other substances, including the magnetism of the Earth. And at length, having shown that the Sun and Moon also exert magnetic influence, Sir GEORGE AIRY terminates his thoughtful work with this interesting confession: "On the whole, we must express our opinion that the general cause of the Earth's magnetism still remains one of the mysteries of cosmical physics." (p. 220).

On perfectly general principles it almost necessarily follows that the cause of the Earth's magnetism must be mysterious, so long as we are without a clear and definite physical theory of the ordinary magnetism of iron and steel, as set forth in BULLETIN NO. 2. Yet this recent comprehensive statement by Sir GEORGE AIRY, thirty-two years after GAUSS' work on the mathematical theory of Terrestrial Magnetism, 1838, is of more than passing interest. For in the interval since the epoch of GAUSS the organizing influence of HUMBOLDT had led to the widest studies of the magnetism of the globe in both hemispheres; and in this same period MAXWELL had made a most comprehensive survey of the whole field of Electricity and Magnetism, and included in his celebrated *Treatise* a thoughtful discussion of *Terrestrial Magnetism,* (Book III Chapter VII).

It is true that in this subject MAXWELL did not reach any new results of significance, yet he recognized clearly the work to be done, when he declared that "The field of investigation into which we are introduced by the study of terrestrial magnetism is as profound as it is extensive." (§474).

The secular changes in the Magnetism of the Earth, and the periodic fluctuations depending on the Sun and Moon strongly appealed to MAXWELL'S curiosity, but he could only "conclude that we are not yet acquainted with one of the most powerful agents in Nature, the scene of whose activity lies in the inner depths of the Earth, to the knowledge of which we have so few means of access."

In the *Preface* to his *Treatise*, Feb. 1, 1873, MAXWELL pays a glowing tribute to the progress of Terrestrial Magnetism under the leadership of GAUSS. He says that the electromagnetic speculation originated by GAUSS and carried on by WEBER, RIEMANN, FRANZ NEUMANN, CARL NEUMANN, LORENZ, etc., "is founded on the theory of action at a distance, but depending either directly on the relative velocity of the particles, or on the gradual propagation of something whether potential or force, from the one particle to the other."

This view of action as something propagated in time was before GAUSS in 1835; for it is so stated in the celebrated letter to WEBER, March 19, 1845 (cf. GAUSS' *Werke*, Band V, p. 629). But as GAUSS was not able to obtain the desired form of expression, he put the matter aside temporarily, and, as he did not resume it, thus left to WEBER the formulation of the Fundamental Law of Electrodynamic Action:

$$F = \frac{mm'}{r^2}\left\{1 - \frac{1}{c^2}\left(\frac{dr}{dt}\right)^2 + \frac{2r}{c^2}\frac{d^2r}{dt^2}\right\}, \tag{1}$$

where c is the velocity of propagation, and r the distance between the bodies m and m'.

It is a remarkable fact that although WEBER'S law was formulated in 1846, nine years before the death of GAUSS, neither this great mathematician nor WEBER nor any of their numerous followers — not even MAXWELL, who studied Magnetism so closely — took any step for disclosing to our mental vision the existence of electrodynamic waves upon which magnetic action depends. And whilst WEBER'S law is correct, the latter terms involving both the induction and the change of the induction under varying relative motion — yet this equation in the form of a *working representation* — the *construirbare Vorstellung* sought by GAUSS in 1835 — conveyed to us no definite physical meaning, prior to the outline of the Electrodynamic Wave-Theory in BULLETIN No. 2.

It is now hoped that the Electrodynamic Wave-Theory of Magnetism there developed may give us a new view of Terrestrial Magnetism and of Cosmical Magnetism in general. Instead of a vague theory of action at a distance we have a simple explanation of Magnetism and of Magnetic Attraction under the action of waves propagated from the atoms. In the case of the Earth this means waves of such length that they traverse the body of the globe; for GAUSS found that Terrestrial Magnetism depends upon the matter within the depths of the globe as well as near the surface.

We have verified the validity of AMPÈRE's theory of Magnetism from a new point of view, and found that in common magnets the atoms are lined up in parallel planes, so that their waves act together in concert, and yield the maximum power of attraction. In a body like the Earth or Sun only a small fraction of the atoms are thus mutually parallel, in respect to an axis, so as to yield magnetic poles, by their concerted vibrations. Accordingly cosmical magnetism naturally is feeble.

But if Gravitation also be regarded as an electrodynamic action, it is obvious that the wave action in respect to the mass of atoms of a heavenly body, which are arranged in haphazard fashion, should under their combined vibrations give a *mean action which is central,* in accordance with the observed law of Nature. This explains why gravitation is central and does not present the *duality of powers* noticed by AIRY as characteristic of Magnetism. (*Treatise on Magnetism,* p. 10.)

Thus the study of Cosmical Magnetism will throw light not only on the magnetism of the Earth and Sun, but also on the connection existing between Terrestrial Magnetism and the gravitational forces which control the motions of the planets in their orbits. These forces are due to electrodynamic wave action propagated in right lines with the velocity of Light.

In order to unfold this new view of many related natural phenomena it is necessary to proceed step by step. And we may look forward to a complete unification of the forces involved only after we have traversed extensive groups of phenomena heretofore apparently standing in isolation, and thus utterly bewildering to the natural philosopher.

II GAUSS' MATHEMATICAL THEORY HARMONIZED WITH THE ELECTRODYNAMIC WAVE-THEORY OF TERRESTRIAL MAGNETISM

In his *Algemeine Theorie des Erdmagnetismus,* 1838, GAUSS has deduced general expressions for the Earth's magnetism, and given complete formulæ for calculating the component forces at any point of the Earth's surface. The usual resolutions of the forces are

$$
(2) \quad \left\{
\begin{aligned}
X &= H \cos \delta = -\frac{1}{a}\frac{dV}{dl}, \\
Y &= H \sin \delta = -\frac{1}{a \cos l}\frac{dV}{d\lambda}, \\
Z &= H \tan \theta = \frac{dV}{dr};
\end{aligned}
\right.
$$

where V is the Magnetic potential, H the horizontal force, δ the magnetic declination, θ the dip, l the latitude of the place of observation, on a spherical earth of radius a, and λ the longitude, and r the distance from the earth's centre.

Gauss takes $d\mu$ for the element of the magnetism at (r_0, u_0, λ_0), within the Earth, and finds the action on a point in space (r, u, λ) distant ρ from $d\mu$:

$$
(3) \qquad V = \int \frac{d\mu}{\rho} = \int \frac{d\mu}{\sqrt{r^2 - 2rr_0\,(\cos u \cos u_0 + \sin u \sin u_0 \cos (\lambda - \lambda_0) + r_0^2}}
$$

Accordingly with these spherical co-ordinates the function V may be expanded into a converging series of solid spherical harmonics, involving sines and cosines of u and λ.

"The foundation of our investigation," says Gauss, "is the assumption that the terrestrial magnetic force is the combined action of the magnetized particles of the body of the earth," (p. 6). He conceives the magnetism to be due to the separation of the fluids, but adds that Ampère's conception of galvanic currents circulating about the atoms would serve equally well to explain the phenomena, which are thus somewhat independent of the particular hypothesis involved.

By calculating the total magnetic moment of the globe Gauss finds that the Earth has 8,464 trillion times more magnetism than the one pound standard bar magnet used in his experiments. There would thus be required in the Earth's interior 8,464 trillion of such magnets, all having parallel magnetic axes, to produce the magnetic action of the Earth in external space.

With uniform distribution throughout the whole body of the globe this would amount to very nearly eight bar magnets, more exactly 7.831, for each cubic meter of the Earth's matter. Gauss expresses surprise at this result, and adds that the more the bar magnets lack in parallelism, the stronger would have to be the average magnetization of their parts, in order to produce the actual total magnetic moment for the Earth shown by observation.

If the mean density of the Earth be taken at 5.5, the average cubic meter of the matter will weigh 5,500 kilograms. And as GAUSS' eight one pound bar magnets would thus weigh 4 kilograms, we may compute that the fractional part of the Earth acting as magnetized bars with parallel axes is $5{,}500/4 = 1/1375$. The magnetism of the Earth is therefore exceedingly powerful, owing to the large size of our planet, yet the intensity is feeble compared to that of some artificial magnets.

It thus appears that a large fraction of the Earth's mass has the property of saturated bar magnets; and hence we perceive that the Earth's magnetic field ought to exert a strong influence over very considerable regions of space. The magnetic field is thus everywhere well defined.

On general principles we may be quite sure that the Earth is not filled with an artificial distribution of magnetized steel bar magnets having an average density of eight pounds per cubic meter throughout the globe. We are rather to view this average portion of the Earth's mass (1/1375) as having magnetic qualities in virtue of the atoms lined up in parallel planes, just as in AMPÈRE's theory of Magnetism.

In that case we may regard the magnetism of the Earth as due to the waves proceeding from these atoms in parallel planes. If the waves penetrate through the globe, all the observed results will follow, and we shall have a complete theory of the Earth's magnetism, based on the Electrodynamic Wave-Theory.

This modern Wave-Theory of Terrestrial Magnetism is illustrated by the accompanying figure, in which the directions of the rotations of the wave elements are shown at every point of the globe. To get a view of the waves going out of the Earth at any point, it suffices to extend the rotations here shown, till they can be connected into waves of the type discussed in BULLETIN No. 2.

Accordingly, the Electrodynamic Wave-Theory gives a satisfactory explanation of GAUSS' theory of Terrestrial Magnetism. And nothing now remains but to examine the conditions underlying that theory, and then to apply the theory to the elucidation of particular phenomena. Heretofore such phenomena as the Aurora and Magnetic Storms have almost completely bewildered the investigator. Owing to the lack of connection with any common cause, it has been impossible for the investigator to open up an effective line of attack on the puzzling problems presented. Thus it is not remarkable that Cosmical Magnetism has been a subject of the utmost obscurity, and the greatest natural philosophers have labored in vain to penetrate the mystery of the underlying physical forces. Even to the luminous mind of GAUSS the Aurora Borealis always presented a *puzzling appearance — eine räthselhafte Erscheinung.* (p. 50.)

III The Theory of a Uniformly Magnetized Sphere Shows that the Magnetism of the Earth Depends upon the Combined Action of the Magnetic Matter Throughout the Globe

The magnetism of the Earth is roughly that of a uniformly magnetized sphere, as we see by simple calculation.

1. If l, m, n be the direction cosines of the direction of magnetization at any point, we have for components of the magnetization due to the element $I dx dy dz$

$$I \left\{ l \frac{\partial}{\partial x}\left(\frac{1}{r}\right) + m \frac{\partial}{\partial y}\left(\frac{1}{r}\right) + n \frac{\partial}{\partial z}\left(\frac{1}{r}\right) \right\} dx dy dz \tag{4}$$

And by integration we obtain the potential of the whole body at the external point p

$$\Omega = \iiint I \left\{ l \frac{\partial}{\partial x}\left(\frac{1}{r}\right) + m \frac{\partial}{\partial y}\left(\frac{1}{r}\right) + n \frac{\partial}{\partial z}\left(\frac{1}{r}\right) \right\} dx dy dz \tag{5}$$

2. By introducing the components of magnetization at (x, y, z)

$$A = Il , \qquad B = Im , \qquad C = In , \tag{6}$$

we obtain an expression in terms of the magnetic components A, B, C:

$$\Omega = \iiint \left\{ A \frac{\partial}{\partial x}\frac{1}{r} + B \frac{\partial}{\partial y}\left(\frac{1}{r}\right) + C \frac{\partial}{\partial z}\left(\frac{1}{r}\right) \right\} dx dy dz \tag{7}$$

3. Let x', y', z' denote the co-ordinates of the external point p, so that

$$\left\{ \begin{aligned} \frac{1}{r} &= \left[(x - x')^2 + (y - y')^2 + (z - z')^2\right]^{-\frac{1}{2}} \\ \frac{\partial}{\partial x}\left(\frac{1}{r}\right) &= -\frac{\partial}{\partial x'}\left(\frac{1}{r}\right), \text{ etc.} \end{aligned} \right. \tag{8}$$

Then, the expression (7) may be written

$$\Omega = -\left(A \frac{\partial}{\partial x'} + B \frac{\partial}{\partial y'} + C \frac{\partial}{\partial z'}\right) \iiint \frac{1}{r} dx dy dz ; \tag{9}$$

since changes in x, y, z do not affect the terms A, B, C, and the operators $\frac{\partial}{\partial x'}$, $\frac{\partial}{\partial y'}$, $\frac{\partial}{\partial z'}$ here placed outside of the signs of integration.

4. Let V denote the potential of a uniform distribution of electricity of volume density unity throughout the region occupied by the magnet. Then by the usual expression for the potential we have for any external point p

$$V = \int\int\int \frac{1}{r}\, dxdydz \tag{10}$$

5. And therefore (9) becomes

$$\Omega = -A\frac{\partial V}{\partial x'} - B\frac{\partial V}{\partial y'} - C\frac{\partial V}{\partial z'} \tag{11}$$

Now let l', m', n' be the direction cosines of the outward drawn normal to the magnet at any element dS of its surface, and we may write the potential at the external point:

$$\Omega = \int\int (Al' + Bm' + Cn')\frac{1}{r}dS = \int\int \frac{I\cos\theta\, dS}{r} \tag{12}$$

For by (6) we have

$$Al' + Bm' + Cn' = I(ll' + mm' + nn') = I\cos\theta\ , \tag{13}$$

where θ is the angle between the direction of magnetization and the outward normal to the element dS of surface.

6. If the uniformly magnetized body be a sphere, of magnetic intensity I, equation (10) reduces to the volume integral for a sphere of radius a, acting on a point distant r,

$$V = \tfrac{4}{3}\pi\, a^3 \left(\frac{I}{r}\right) \tag{14}$$

7. When the magnetization is taken to be in the direction of the axis of x we have to calculate the intensity by taking the differential coefficient of the potential in respect to x:

$$\Omega = -I\frac{\partial V}{\partial x} = \tfrac{4}{3}\pi\, a^3 I\left(\frac{x}{r}\right) = \tfrac{4}{3}\pi\, a^3 I\,\frac{\cos\theta}{r^2}\ , \tag{15}$$

since $\cos\theta = \frac{x}{r}$.

8. Accordingly it appears that the potential at any external point is the same as that of a magnetic particle of moment $\frac{4}{3}\pi a^3 I$ at the centre of the sphere.

But as applied to the Earth this result has no well defined physical meaning. For since the Electrodynamic Wave-Theory shows that magnetism is due to the action of waves, it is natural to hold that these waves can proceed only from the magnetic matter of the Earth, and therefore not from that at the centre only, but from magnetic elements scattered throughout the mass of the globe, and everywhere acting in concert, owing to the parallelism of their atomic planes.

9. Thus any hypothesis that the magnetism of the globe can rest on the properties of the matter located in a particular region is inadmissible. For although GAUSS showed that mathematically the external magnetic potential might result from an unlimited number of internal distributions of magnetic density, yet the physical conditions now imposed by the Electrodynamic Wave-Theory exclude every distribution not consistent with the laws of the internal density of the Earth.

10. For if Magnetism be due to wave action, it must depend on the laws of the internal density of the globe for the required parallel atomic waves; and as the planes of the atoms in cosmical masses must follow the laws of chance, the magnetic field about such bodies can only be determined by the laws of internal density for all the matter. Thus we see that no other view can be held than that of GAUSS, who justly assumed that the terrestrial magnetic force represents the combined action of the magnetized particles of the body of the Earth.

IV THE RELATION BETWEEN "MAGNETIC STORMS" AND THE SUNSPOT CYCLE EXPLAINED BY ELECTRODYNAMIC DISTURBANCES INCIDENT TO MASS-MOVEMENTS IN THE SUN

1. Hoping to find an intra-mercurial planet in transit over the Sun's disc, HEINRICH SCHWABE of Dessau in 1826 began to make systematic daily counts of sunspots, and after seventeen years of patient watching (1843) was able to recognize in these fluctuating phenomena a regular periodicity of about ten years' duration. This is the sun spot cycle of which subsequent investigators have fixed the period more accurately at about 11.1 years.

2. In 1851-52 four independent investigators — Dr. JOHN LAMONT of Munich, General EDWARD SABINE of London, Dr. ALFRED GAUTIER of Geneva, and Dr. RUDOLF WOLF of Berne, — each recognized and aided in establishing a cycle in the fluctuations of Terrestrial Magnetism, also having a period of about ten years. It was found to have the same length as SCHWABE'S Sunspot cycle, and agreed

with it phase for phase over more than a century. Subsequent investigators have fixed this magnetic period also at 11.1 years.

3. The phases are shown to be in such perfect agreement with those of the sunspot cycle that it has long been regarded as certain that the two phenomena — Sunspots and "Magnetic Storms" — are physically connected and mutually dependent on a common cause. The problem presented to the natural philosopher has been to find the physical cause underlying both phenomena.

4. The accompanying graphical illustration of the sunspot cycle and associated "Magnetic Storms," from YOUNG's *General Astronomy*, represents the results of the researches of Dr. RUDOLPH WOLF of Zurich, who long studied the relations between these phenomena.

5. The agreement between the frequency of sunspots and the average daily range of the fluctuations in the horizontal component of the Earth's magnetism is seen to be complete. *This means that when sunspots are most frequent, the range of the daily fluctuations of the Magnetic needle augments, and vice versa.* And this dependence is found to be established by records extending over many sunspot cycles, since the year 1770, with some records running as far back as 1680.

6. Accordingly, it cannot be denied that a physical connection exists between sunspots and "Magnetic Storms." The question then arises: What is the nature of this connection? Many investigators have sought the physical cause for such a relationship, but as yet no one has succeeded in finding one which is rational and satisfactory.

The explanation now put forward was developed in 1914, but has not heretofore been published, although communicated to one of the oldest and most learned of the European Scientific Societies in 1915.

7. It is now shown that Gravity is an electrodynamic action, depending on waves traveling with the velocity of Light. The minor terms of WEBER's law provide for the induction and the change of the induction, with variation of distance. From the form of this law, we see that relative motion will involve induction* between different parts of the system. And this will apply not only to the orbital motion of the Earth, but also to internal mass-movements in the Sun.

* This effect of *relative motion* is largely analogous to that resulting from DÖPPLER's Principle, but in addition to the change in the wave-length, there are waves developed which may be parallel and have a common direction, as in Magnetism and in the Induced Current.

8. If a solar prominence of large size is ejected, so that a considerable mass of matter is in relative motion, as respects the Earth, it is obvious that the Sun's magnetic field will be disturbed; and as the Sun's magnetic field exerts an influence on the Earth's magnetic field, through the infinite system of interpenetrating waves, there will thus arise a trembling of our magnetic needles while the waves due to the solar disturbance are passing the Earth.

9. Not only will the solar disturbance directly derange the Sun's magnetic field and thus affect the Earth's field, but also send irregular gravitational electrodynamic waves to the Earth. The passing of these irregular electrodynamic waves through the Earth — combining as they do at every point with the waves due to the atoms which produce the Earth's magnetic field — will indirectly disturb the Earth's magnetic field, because of the mere irregularity of the wave commotions going on.

10. In this way we explain the greater amplitude of the daily range of the magnetic needle during the maximum of a sunspot period. For the more violent the solar outbreaks, the less damping of the waves from the matter above the reversing layer, and the more violent and intense the wave commotions passing the Earth; and thus the greater range in the amplitude of the daily fluctuations of the magnetic needle. Hence the exact correspondence between the curves for the "magnetic storms" and the sunspot cycle, as shown by WOLF's investigations.

11. It might be objected that solar prominence outbreaks are not always followed by magnetic storms on the Earth, and that the theory thus proves too much. Our theory does not, indeed, require this, but only specifies such response in the Earth's magnetic field when the electrodynamic waves passing the Earth are so irregular and develop such irregular induction in the air as to produce unsymmetrical magnetic stress in the Earth's field, so that the needle swings to and fro, to wider amplitudes than usual. *It is clear that not all solar outbursts would produce such terrestrial effects.* In many cases the matter in motion would have little or no *relative motion*, as respects the Earth — being largely normal to the Earth's radius vector. In other cases the outbursts would produce only minor derangement of the lines of force — or axes of wave propagation — in the Sun's magnetic field, and thus exert little effect on the magnetic field of the Earth.

12. Accordingly, the fluctuation in the average daily range of the Magnetic Needle should have only a very general connection with the curve of the sunspot cycle—not a direct connection in all details, as some have supposed should occur. The WOLF curves are therefore in satisfactory agreement with the Electrodynamic

Wave-Theory. *It is very interesting to notice that this Theory would explain Sunspots themselves (with corresponding extension of the Corona) as eruptive outbreaks, due to the Combined Inductive Action of the Planets and Comets, under* WEBER'S *Law, the cycle sometimes exceeding the period of Jupiter, and sometimes falling short of it, owing to the modifying influence of the other Planets.* This cause of sunspots was suggested, in the unpublished Memoir of 1914, as an extension of the researches of HERTZ, 1880, "On Induction in Rotating Spheres." The analogy is very striking, and the proof has seemed more and more conclusive, but the subject of the physical cause of sunspots is beyond the scope of the present BULLETIN.

V W. GRYLLS ADAMS' RESEARCHES OF 1881 ON "MAGNETIC STORMS" IN VARIOUS PARTS OF THE EARTH SHOW THAT THESE COMMOTIONS ARE SIMULTANEOUS AS IF DEPENDING ON MASS-MOVEMENTS IN THE SUN

1. Having now shown why the *amplitude of the average daily range* of the horizontal component of the magnetic needle is greater* at sunspot maximum and smaller at sunspot minimum than during the years between maximum and minimum, it only remains to prove by terrestrial observations that such wave commotions actually come from the Sun. If they do, these abnormal tremblings of the needle should occur almost simultaneously throughout the globe.

2. The accompanying plate is from the *Philosophical Transactions* of the Royal Society for 1881, which contains an important paper by the late Professor W. GRYLLS ADAMS, brother of the celebrated J. C. ADAMS, theoretical discoverer of Neptune. It will be noticed that ADAMS has here set to common Greenwich Mean Time the records of magnetic disturbances taken at many stations in all parts of the world. The suddenness of the changes in several instances is remarkable. The tremors affect all the components of the Earth's Magnetism, within a few minutes of the same instant, throughout the globe. This record shows that there is a magnetic wave commotion going on in every part of the globe, as if it originated from disturbances in the Sun, and had been propagated to the Earth with the velocity of light.

3. It is from just such "Magnetic Storms" as this that Dr. RUDOLPH WOLF derived his curve of correspondence between "Magnetic Storms" and the sunspot cycle, as given in the preceding section. The nature of the phenomenon is thus made clear, and the physical connection firmly established.

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* Because of the *Induction Effects*, described above.

4. Probably this reasoning will be convincing to most persons, but if there be those who doubt our conclusions they may be referred to the account of the spectroscopic and magnetic observations taken by Professor C. A. YOUNG's eclipse party, Aug. 3-5, 1872, at Sherman, Colorado. Certain eruptions in the Sun gave bright lines in the spectrum and were simultaneous with tremblings of the magnetic needle independently noted by the magnetic observer at Sherman, and afterwards found to have been noted also at Greenwich, England, five thousand miles away.

5. As YOUNG and the magnetic observers worked quite independently of one another (cf. *The Sun*, 1902, pp. 166-168) this coincidence in the visual appearance of *spectral bright lines*, when incandescent matter is raised above the *reversing layer*, and the coming on of a notable "Magnetic Storm" is not accidental. It really shows the true physical connection between "magnetic storms" and the eruptions in the solar atmosphere.

6. It should be noted that just as the FRAUENHOFER lines become bright when the emitting matter is raised above the reversing layer, so also will the magnetic waves from the Sun's atoms be of increased intensity when the emitting matter is above the reversing layer. This increased freedom from damping, and the relative motion of the disturbed matter causes a readjustment in the infinite systems of waves, with a trembling of the needle on the Earth, which may be recorded on the magnetographs.

7. As an observational proof that magnetic waves regularly come from the Sun, yet may have their induction effects damped and reduced in intensity by clouds and similar terrestrial objects, we refer to the well-known observations by Professor NIPHER (*Transactions of the Academy of Sciences*, of St. Louis, 1913-16). These observations are so definite that they admit of not the slightest doubt.

8. If the clouds may exert an appreciable influence in reducing atmospheric induction and thus damping these magnetic waves, it follows that the Earth's shadow at night will be still more effective; yet the greater wave commotions in the Sun are powerful enough to give a conspicuous terrestrial effect in the Aurora, "Magnetic Storms," and "Earth Currents," as explained below.

VI The Physical Cause of the Aurora Borealis

The Aurora Borealis is a very striking phenomenon, but remarkably obscure as to the physical cause on which it depends. In order to get at the probable cause of the Aurora, we point out a series of interesting facts.

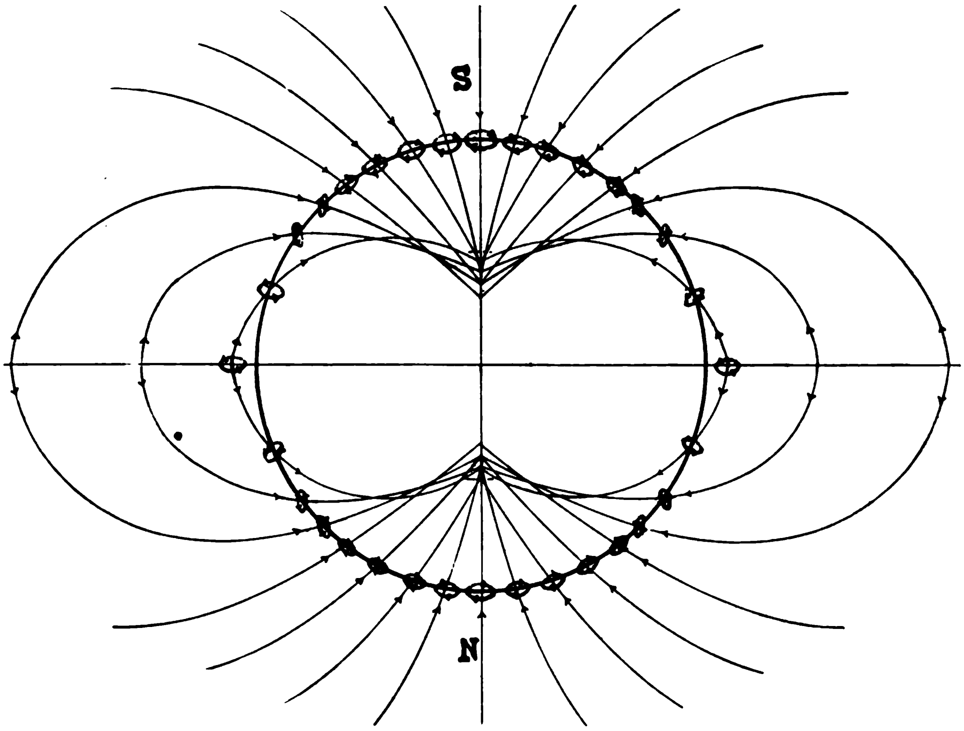


FIG. 1. Graphical Representation of GAUSS' Theory that Terrestrial Magnetism depends on the combined action of the magnetized particles of the body of the Earth.

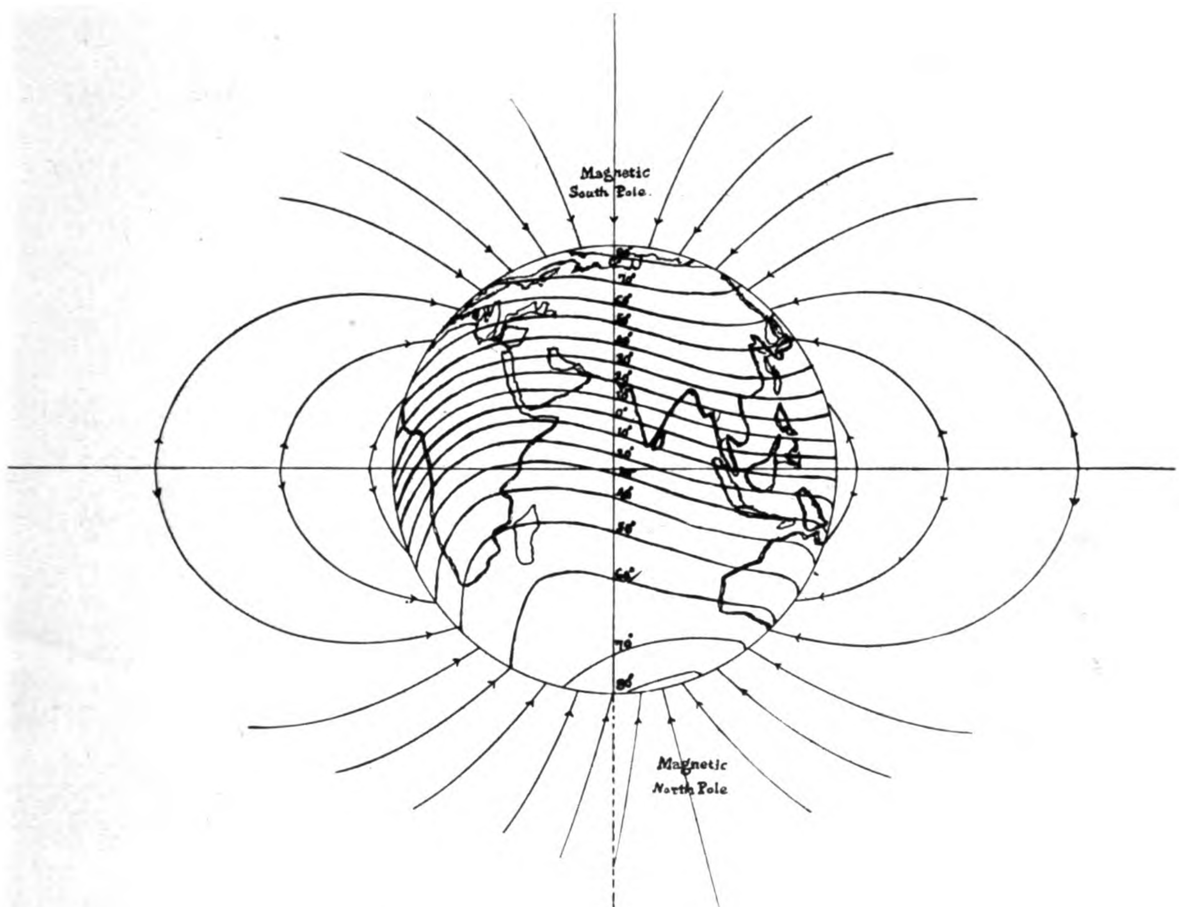


FIG. 2. The Magnetic Field about the Earth, showing contour lines of equal Dip.

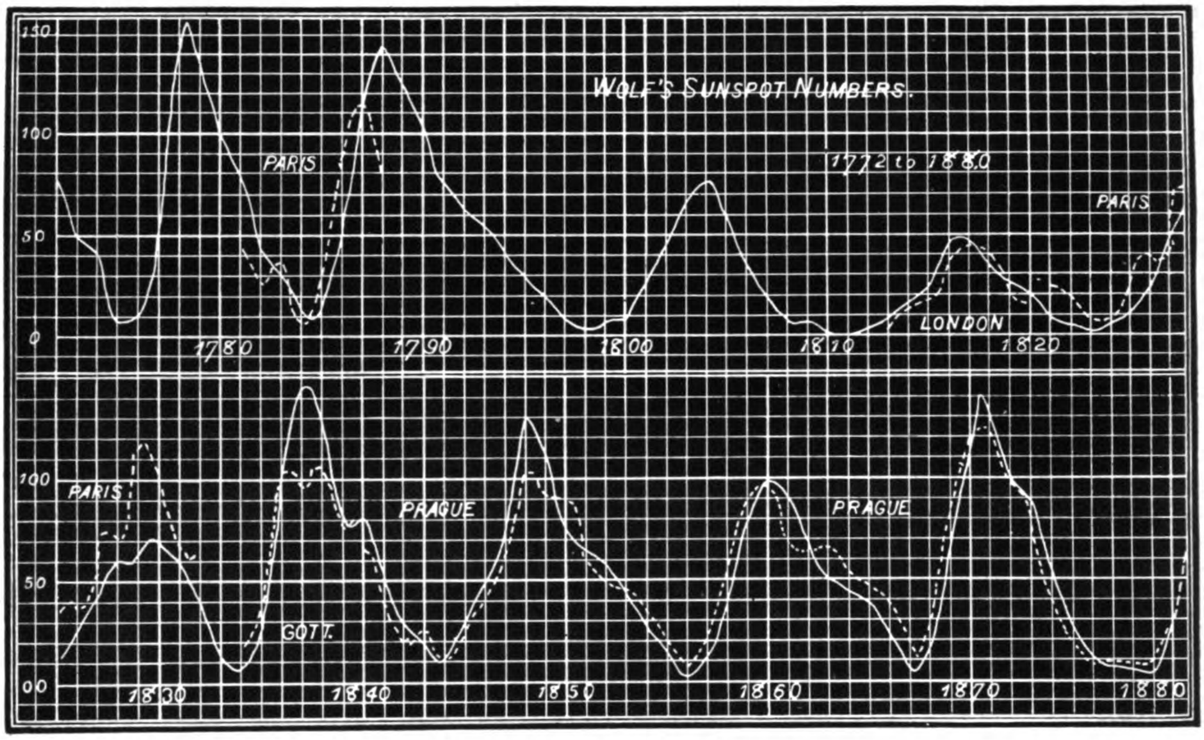


FIG. 3. WOLF'S Sun-Spot Numbers, with associated "Magnetic Storms."

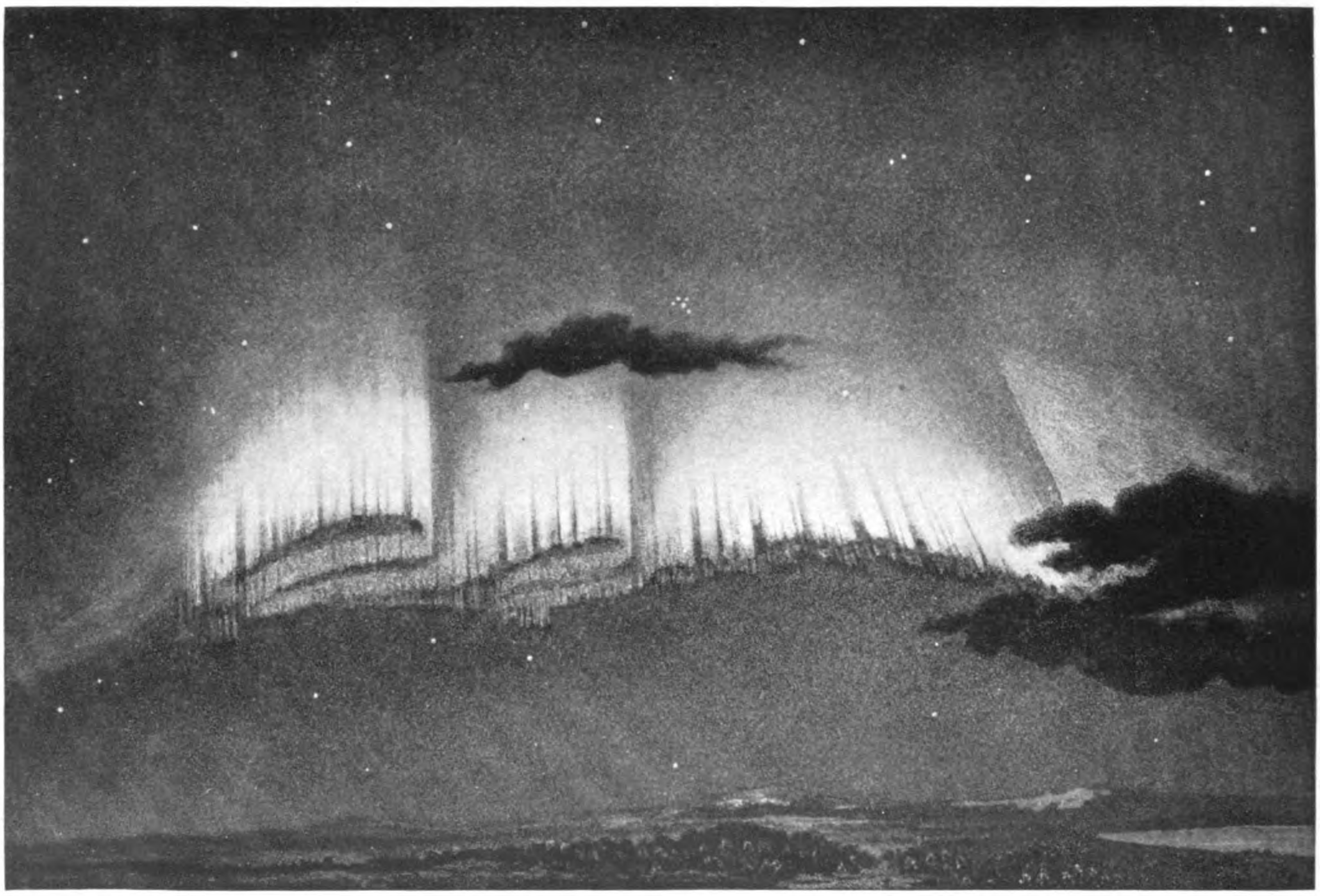
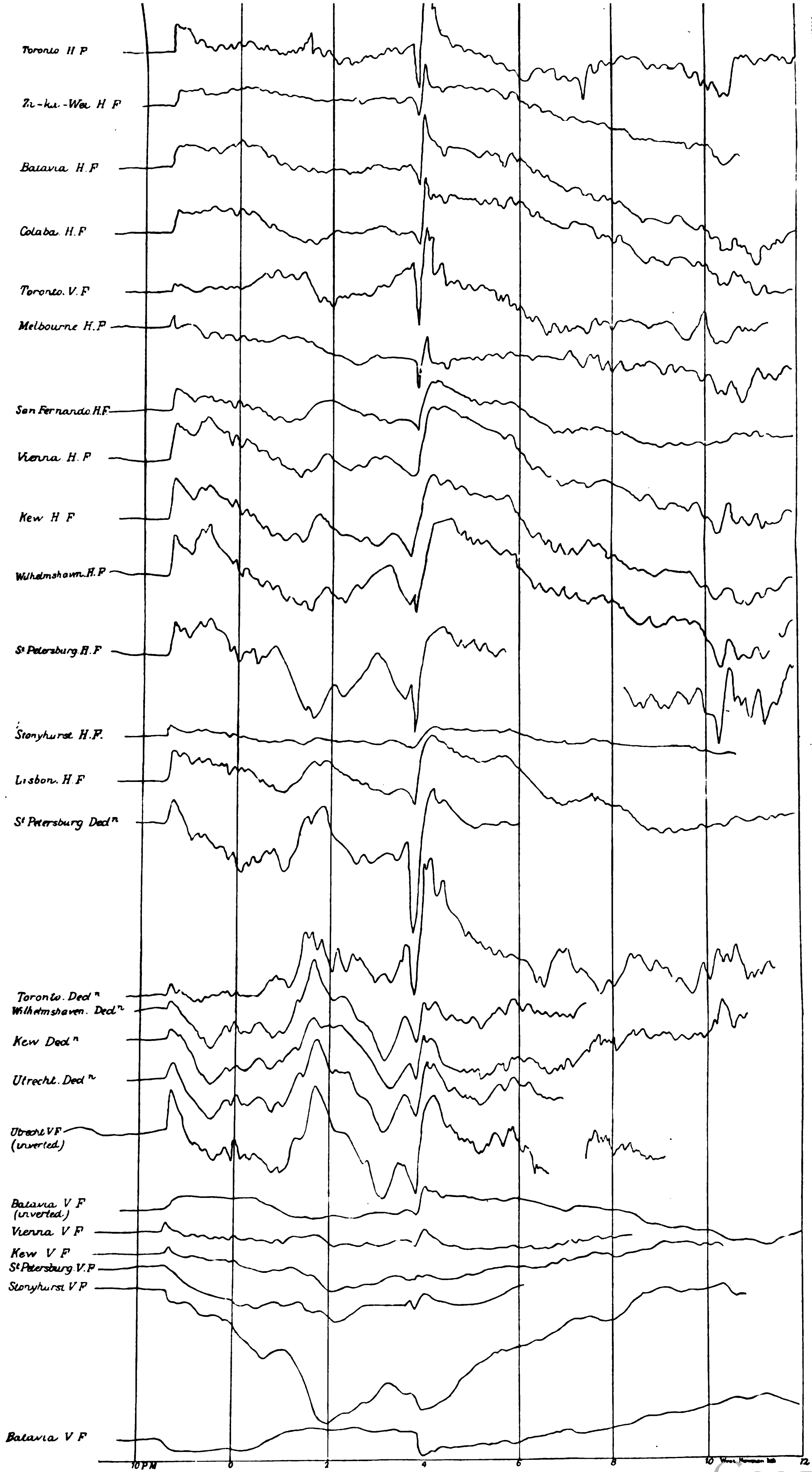


FIG. 4. A typical view of the Aurora Borealis, showing extensive curtain, with ribbon folds at the lower border, and stars visible through the illumination. An Aurora of this general aspect was observed by the author while measuring double stars at the Washburn Observatory, Madison, Wisconsin, Sept. 29, 1895.



Adams

FIG. 5. W. GRYLLS ADAMS' Record of Simultaneous Disturbances of the Magnetic Needle throughout the World, in the great "Magnetic Storm" of June 25, 1885.

Phil. Trans. 1892. A. Plate 8

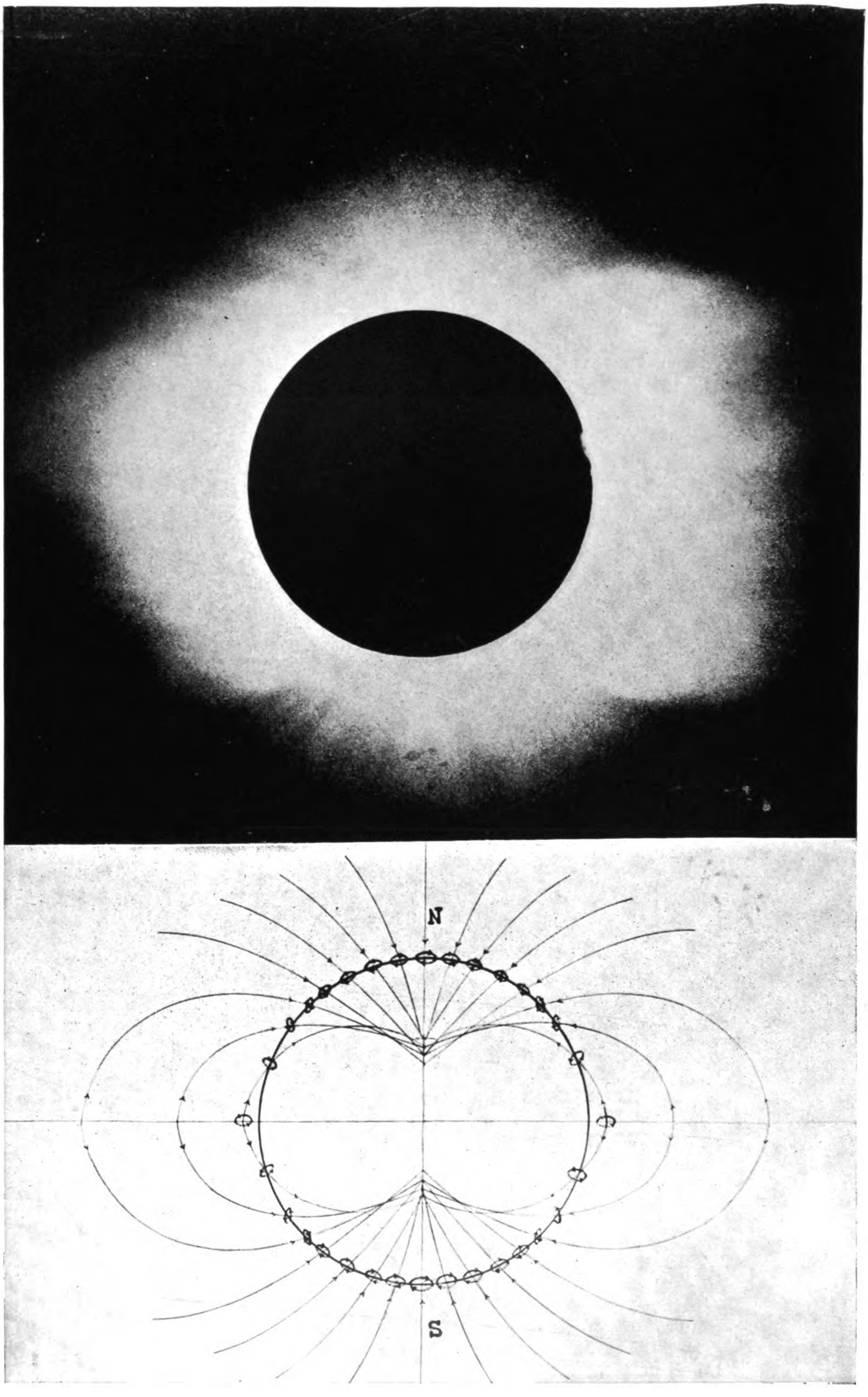


FIG. 6. Comparison of the observed Rays in a Typical Solar Corona with the Lines of Force in GAUSS' Theory. In his studies on the Motion of *Mars*, 1609, KEPLER concluded that the Sun and Planets are Magnetic (*Opera Omnia*, III, p. 37), and the evidence has since become decisive.

1. The coincidence of the rays of the Aurora with the direction of the magnetic needle. This is one of the most striking proofs of the relationship between the Aurora and "Magnetic Storms," which are also observed to recur in similar periodic cycles. So long ago as 1714 Dr. Edmund Halley regarded the Aurora as a magnetic phenomenon (*Phil. Trans.*, 1714-16. No. 341). Naturally modern data were not then available to lend such general support to the theory as might establish its truth.

2. When a great "Magnetic Storm" occurs it usually is felt throughout a large part of the globe, and the Aurora frequently is visible in different countries, and over wide areas of continental extent, sometimes in both hemispheres. It is found that the "Magnetic Storms" are associated with "Earth Currents," or electrical disturbances so strong that occasionally the telegraph lines may be operated by them without the use of the regular batteries.

3. The "Earth Currents" associated with "Magnetic Storms" and Aurora Borealis may be most easily explained by supposing that there is electric induction in the globe due to passing electrodynamic waves incident to mass-movements in the Sun. This induction would generate galvanic currents in closed circuits — such as telegraph and cable lines — and also produce intensified waves from the atoms of magnetic matter such as iron, steel, and similar substances.

4. Now Faraday showed that all substances are magnetic, and thus the result of the passing of waves incident to mass-movements in the Sun would involve the whole globe in a ceaseless magnetic tremor. And owing to the unequal conductivity of the globe these moving waves would thus lead to an apparent development of free electricity. In the atmosphere the electric disturbances incident to the passing waves would take on the form of light; for the incessant wave flow, with the resulting electric inductions on large masses of air and clouds, would lead to slight discharges of free electricity through the highly rarified air, as in the Geissler tube experiments.

5. It is impossible to doubt that when the air is traversed by such waves, of surging electrical mass oscillations, the disturbances or stresses produced would generate light in some form. The Geissler tube experiments show not only the general type of these oscillations, but also their enormous variety, with changes of color, which are also characteristic of the Aurora. This is one of the surest proofs that the Aurora is an electrical oscillation with visible discharge due to the passage of electrodynamic waves through the higher regions of the Atmosphere.

6. An additional proof not only that the Aurora is an electrical phenomenon but that it depends on commotions in the Sun may be gathered from the periodicities in the two phenomena.

(*a*) The Aurora has the 11-year cycle characteristic of sunspots and "Magnetic Storms."

(*b*) The semi-annual period in the Aurora, with greatest Auroral Activity in March and September, and least activity in June and December, seems to indicate that this fluctuation depends on the spots in the equatorial region of the Sun which are most directly exposed to the Earth in March and September.

(*c*) There is also a periodicity in the Aurora of 25.93 days, which corresponds so closely to the period of the Sun's mean axial rotation, that we are compelled to believe it depends on the return of spotted areas of the Sun to our meridian, and thus disturbs the electrical equilibrium of the globe in this average period. The researches of MAUNDER (*Monthly Notices*, 1904) confirm this result from another point of view.

CARRINGTON found from the observation of sunspots at the solar equator a period of 24.9 days, but that in latitude 30°, the period was lengthened to about 26.4 days. Spectroscopic measurements of the Sun's rotation period by DUNER, CREW, and other observers agree closely with CARRINGTON'S periods from sunspots. As the area of maximum solar activity is somewhat nearer the equator than 30° heliocentric latitude, we may take the Auroral period of 25.93 days as the time of the mean solar rotation.

7. Now suppose wave commotions to occur from bodily movements of matter in the Sun. The magnetic field of the Sun will thereby be disturbed, and a reaction generated in the Earth's magnetic field. The resulting solar wave commotion will travel towards the Earth with the velocity of light; and the magnetic waves incessantly receding from the Earth will also be set into irregular motion and the normal calm of the magnetic field of the Earth will be disturbed.

8. As the magnetic potential of the globe is greatest towards the magnetic poles, in accordance with HUMBOLDT'S law of the increase in higher latitudes (cf. *Cosmos*, Vol. I, p. 179, BOHN'S Translation) it will follow that when the wave commotion incident to the outbreaks in the Sun are violent enough to elicit luminosity in our rarified atmosphere — as in electric discharge through the rare gases of a Geissler tube — the light will appear on our northern horizon and move rapidly southward along the lines of force of the Earth's magnetic field.

9. For the wave commotion will naturally have a rise in intensity to a maximum, then a decline; and the luminosity will first appear in the region of greatest magnetic potential furthest north; and as the maximum approaches, the luminosity will travel southward to the fields of smaller magnetic potential. And as the disturbance declines, the streamers of the Aurora may drop back to the northern horizon.

10. This is exactly what is frequently observed in the celebrated phenomenon of the Aurora Borealis. We thus conclude that an Aurora is due to passing magnetic waves incident to internal commotions in the Sun. HUMBOLDT gives the following description of a typical Aurora, based largely on ARGELANDER's observations at Königsberg:

"Low down in the distant horizon, about the part of the heavens which is intersected by the magnetic meridian, the sky which was previously clear is at once overcast. A dense wall or bank of cloud seems to rise gradually higher and higher until it attains an elevation of eight or ten degrees. The colour of the dark segment passes into brown or violet; and stars are visible through the cloudy stratum, as when a dense smoke darkens the sky. A broad brightly luminous arch, first white, then yellow, encircles the dark segment; but as the brilliant arch appears subsequently to the smoky gray segment, we cannot agree with ARGELANDER in ascribing the latter to the effect of mere contrast with the bright luminous margin." . . .

"The luminous arch remains sometimes for hours together flashing and kindling in ever-varying undulations, before rays and streamers emanate from it, and shoot up to the zenith. The more intense the discharges of the northern light, the more bright is the play of colours, through all the varying gradations from violet and bluish white to green and crimson. Even in ordinary electricity excited by friction the sparks are only coloured in cases where the explosion is very violent after great tension. The magnetic columns of flame rise either singly from the luminous arch, blended with black rays similar to thick smoke, or simultaneously in many opposite points of the horizon, uniting together to form a flickering sea of flame, whose brilliant beauty admits of no adequate description, as the luminous waves are every moment assuming new and varying forms. The intensity of this light is at times so great, that LOWENÖRN (on the 29th of June, 1786) recognized the coruscation of the polar light in bright sunshine." (*Cosmos*, Vol. I pp. 189-190 — BOHN's Translation).

11. From this description, which corresponds with the results of my own observations, it seems clear that the well-known *curtain appearance* is due to the

escape of slight charges of free electricity under the varying electrical potential produced by the passing of electrodynamic waves incident to bodily disturbances in the Sun. The crown sometimes developed in the Aurora is a particular form of this luminosity so projected as to make a central canopy high in the heavens.

12. Accordingly, it appears that all the leading phenomena of the Aurora Borealis may be easily explained by the present Electrodynamic Wave-Theory. The result is a harmonious view of many puzzling phenomena of Terrestrial Magnetism. It accords with WEBER's fundamental law of Electrodynamic Action, and confirms HALLEY's suggestion of 1714 that the Aurora is a magnetic phenomenon.

13. It has long been recognized that *in its geographical distribution the Aurora hovers about the cold poles of the Earth* — doubtless because, as EXNER showed, the fall of the atmospheric electrical potential is some *fifteen times* greater in the zones of the *cold poles* than at the terrestrial equator. An electric phenomenon depending on this fall of the potential, like the change of the Induction, under the varying influence of the Sun's Electrodynamic Waves, is thus more intense in winter and in high latitudes than in summer and in the equatorial regions of the globe.

14. This view of the Aurora is borne out by the very delicate photographic researches of Dr. SLIPHER recently recorded in *Lowell Observatory Bulletin No. 79*, showing by actual observation that from June to November, 1916, there was *almost persistent Aurora* in the sky near the horizon. It was recorded satisfactorily even in Moonlight, but appeared plainest near the horizon, because this region of the air is of greatest depth. The strength of the luminosity is increased by this depth, because on the photographic plate the integral effect of the radiation accumulates, when the electrical potential is changing in any layer, under the progress of the Sun's diurnal influence.

15. The Aurora is therefore much more widely distributed geographically, and a much more persistent phenomenon than was previously believed. Obviously its full extent and persistence can be ascertained only by the sensitive plate of the modern camera. The more spectacular classic phenomenon becomes visible to the eye chiefly under the cold climates of the polar regions, and fluctuates in cycles corresponding to the electrodynamic wave agitations in the Sun, as above set forth.

VII Kreil of Prague, 1841, and John Allen Broun, 1845, Independently Detect a Lunar Influence upon the Magnetism of the Earth, which Is Semi-Diurnal and Otherwise Follows the Laws of the Tides: the Researches of Lloyd and Airy Confirm the Existence of a Lunar Magnetic Tide, but Lloyd Misinterprets It

In 1841 Kreil of Prague laid before the Bohemian Society of Sciences evidence of slight semi-diurnal changes in the magnetism of the Earth depending on the influence of the Moon. He thus anticipated Schwabe's discovery of sunspot periodicity by two years, and preceded Lamont's discovery of the corresponding cycle in Terrestrial Magnetism by ten years; so that the new lunar magnetic inequality was unexpected, and appeared the more remarkable because the period was only half a lunar day.

Shortly after Kreil's discovery the same fact of a semi-diurnal inequality in the Earth's magnetism depending on the Moon was independently noted by the English investigator John Allen Broun, 1845, and fully confirmed by the researches of Sir Edward Sabine.

When attention was directed to the effect of the changing distance of the Moon from the Earth, both Sabine and Broun found the variation is greater for perigee than for apogee. According to Broun's careful analysis the mean ratio of the apogee effect to the perigee effect is as 1 to 1.24 nearly. Looking into the cause of this difference Broun notices that the distance of the half orbit near apogee is to that about perigee very nearly as 1.07 to 1. He then remarks that the cube of 1.07 is 1.23 nearly; hence it follows that the mean ranges of the curves for the two halves of the lunar orbit are in the ratio of the inverse cube of the distances from the Earth, as in the Theory of the Tides of the Sea.

Lloyd's Misleading Discussion of the Direct Magnetic Influence of the Sun and Moon upon the Diurnal Variations of the Earth's Magnetism

In the *Philosophical Magazine* for March, 1858, Dr. Humphrey Lloyd, Professor of Natural Philosophy in the University, and subsequently Provost of Trinity College, Dublin, has a learned discussion, "On the Direct Magnetic Influence of a Distant Luminary upon the Diurnal Variations of the Magnetic Force at the Earth's Surface," which was afterwards reprinted as the concluding chapter of his "*Treatise on Magnetism*," London, 1874 (Longmans, Green & Co.). Dr. Lloyd's *Treatise* is a very useful work and doubtless has been of great service to Science, but we shall here direct attention to certain false conclusions in the article of 1858, which apparently misled MAXWELL (*Treatise on Electricity and Magnetism*, edition, 1892, §474, Vol. II. p. 136) and his successors in dealing with problems of Terrestrial Magnetism.

LLOYD develops his analysis at some length, and shows from the equations that the magnetic effect of the distant body "consists of two parts, one of which is *constant* throughout the day, while the other varies with the hour angle of the luminary." Although each of the parts varies inversely as the cube of the distance, of the magnetic body, he concludes that the variable part depending on the hour angle of the luminary will give rise to a *diurnal inequality*.

Since the lunar variation has two maxima and two minima of nearly equal magnitude in the twenty-four lunar hours, LLOYD concludes that "*the phenomena of the diurnal variation are not caused by the direct magnetic action of the Sun and Moon.*" (p. 238.)

A just criticism of LLOYD's analysis may be made, in that he retained the hour angle θ, in his expressions, (p. 235) instead of the angle 2θ which occurs in the expressions for the tide-generating potential, (Cf. Sir GEORGE DARWIN's Article *Tides*, *Ency. Brit.*, 9th edition, §7, p. 357); but as LLOYD doubtless did not suspect that there could be such a thing as a *magnetic tide*, probably this oversight was unavoidable in the state of the subject existing more than half a century ago. LLOYD closes his discussion by calling attention to STONEY's investigation (*Philosophical Magazine*, Oct., 1861) showing that the maximum effect of the Moon, supposed equally magnetic with the Earth bulk for bulk, upon a declination magnet would be less than one-tenth of a second.

The following tables embody the results of LLOYD's discussion of the magnetic observations at Dublin.

Owing to the above error of analysis, LLOYD could not interpret these results, holding that the observed diurnal variations, with two maxima and two minima of nearly equal magnitude in twenty-four lunar hours, are not caused by the direct magnetic action of the Moon. He was thus misled, and in turn misled MAXWELL (*Treatise*, §474) and his successors.

LUNAR INEQUALITY OF THE EASTERLY FORCE AT DUBLIN (LLOYD)

Lunar Hours	Summer Lunations	Winter Lunations	Year
−12	−0.19	−0.09	−0.14
−10	−0.21	−0.08	−0.15
− 8	−0.06	+0.02	−0.02

Lunar Inequality of the Easterly Force at Dublin (Lloyd) — *Continued*

Lunar Hours	Summer Lunations	Winter Lunations	Year
− 6	+0.09	+0.09	+0.09
− 4	+0.12	+0.13	+0.13
− 2	+0.08	−0.01	+0.03
0	−0.06	−0.09	−0.08
+ 2	−0.04	−0.07	−0.05
+ 4	+0.05	−0.02	+0.01
+ 6	+0.17	+0.08	+0.12
+ 8	+0.06	+0.07	+0.07
+10	0.00	−0.05	−0.03

Lunar Inequality of the Northerly Force at Dublin (Lloyd)

Lunar Hours	Summer Lunations	Winter Lunations	Year
−12	−0.06	+0.01	−0.03
−10	−0.01	+0.02	+0.01
− 8	+0.08	+0.07	+0.07
− 6	+0.04	+0.05	+0.04
− 4	−0.02	0.00	−0.01
− 2	−0.03	+0.04	+0.01
0	−0.01	+0.02	+0.01
+ 2	0.00	+0.05	+0.08
+ 4	+0.08	−0.03	+0.03
+ 6	−0.01	−0.03	−0.02
+ 8	−0.10	−0.12	−0.11
+10	−0.08	−0.07	−0.08

Lunar Inequality of the Vertical Force at Dublin (Lloyd)

Lunar Hours	ζ	Lunar Hours	ζ
−12	+0.05	0	+0.02
−10	+0.00	+ 2	+0.02
− 8	0.00	+ 4	−0.01
− 6	−0.02	+ 6	−0.06
− 4	−0.07	+ 8	−0.01
− 2	+0.10	+10	+0.01

How remarkable an argument LLOYD has here developed to establish the existence of a magnetic Tide in the Earth may be readily seen by anyone familiar with Sir GEORGE DARWIN'S discussion of the Path of a Pendulum as deflected from the vertical by the tidal forces of the Moon, (*Tides and Kindred Phenomena of the Solar System*, edition, 1898, p. 112). We reproduce here, for comparison, DARWIN'S diagram of the path of the pendulum, when the Moon is north latitude 15°, and the observer in North latitude 30°.

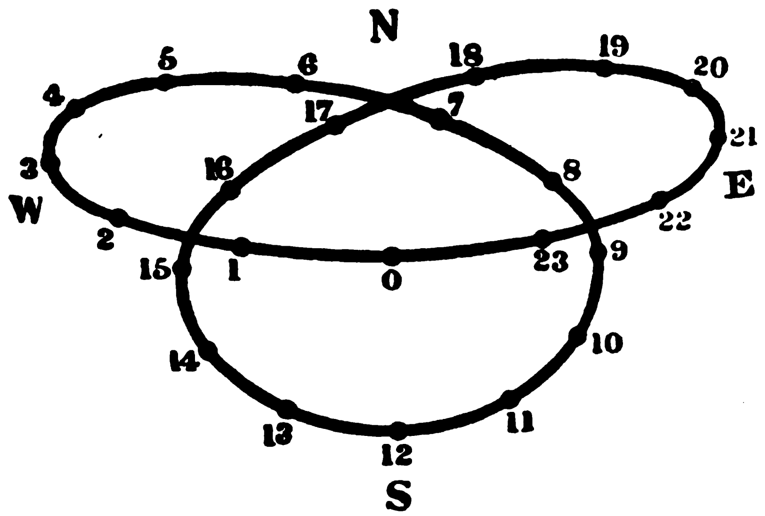


FIG. 7. DARWIN'S Diagram of the semi-diurnal movements of a pendulum

The lunar hours are indicated by the numbers marked along the successive positions of the pendulum.

The comparison however is somewhat incomplete, because DARWIN'S diagram gives the total forces which deflect the pendulum, while the analysis of LLOYD gives only the *components* into which the Lunar Inequality of the Earth's Magnetism usually are divided.

SIR GEORGE AIRY ALONE RECOGNIZED THE LUNAR MAGNETIC INFLUENCE AS A TRUE MAGNETIC TIDE

It appears that among all his eminent contemporaries Sir GEORGE AIRY alone correctly interpreted this inequality, which he says is "a true lunar tide of magnetism, occurring twice in the lunar day, and showing magnetic attraction backward and forward in the line from the Red Sea to Hudson's Bay." (*Treatise on Magnetism*, 1870, p. 206.)

AIRY adds that the lunar forces are considerably less than those which follow the law of solar hours; the mean diurnal solar inequality being about 1/600 of the horizontal force, while the lunar is about 1/12000. According to this, the solar influence is approximately twenty times more powerful than that depending on the Moon. This will cause no surprise in view of the magnetic waves now shown to proceed from certain fields about the sunspots, and to come to us in the form of undamped vibrations when the prominences project above the reversing layer.

We have seen that this ebb and flow in the magnetism of the globe has been actually observed, but its semi-diurnal character has been misinterpreted by LLOYD and several other eminent investigators.

Now NEWTON's law of Gravitation will explain the ebb and flow of the sea, but not the ebb and flow of the Earth's magnetism discovered by KREIL and JOHN ALLEN BROUN. *It is necessary to explain both phenomena,* for in his First Rule of Philosophy NEWTON says that we are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances. And thus, according to NEWTON's Rule, we are obliged to adopt WEBER's Electrodynamic Law, to explain the *Magnetic Tide,* by wave action in reference to poles and propagated in time, thus giving what GAUSS called a *construirbare Vorstellung* of the established phenomena of Nature.

VIII THE SERIES OF PERIODIC FLUCTUATIONS IN THE MAGNETISM OF THE EARTH SHOWN TO DEPEND ON THE SUN AND MOON, AND EXPLICABLE BY THE INFLUENCE OF THEIR COSMICAL MAGNETISM

To convince ourselves that there are numerous fluctuations in the Earth's Magnetism depending on the Sun and Moon, we need only read attentively the well-known discussion of *Terrestrial Magnetism* by the late Professor BALFOUR STEWART, in the Article *Meteorology, Encyclopedia Britannica,* 9th edition.

1. It is there shown that there is a solar diurnal variation of declination changing slowly through the year, but with the form of the curve depending on the latitude of the place of observation. There is also brought out the semi-annual inequality or difference from the whole year's mean of the two half yearly means.

2. Among the recognized cosmical periods established by investigators are:
   (*a*) A yearly period,
   (*b*) A period of 11 years — the sunspot cycle,

(*c*) The 18.6 year period — which is the cycle of the Moon's nodes,

(*d*) A period of a Lunar Synodic Month, 29.53 days,

(*e*) A period of 25.93 days, the Sun's mean rotation period,

(*f*) A semi-diurnal magnetic tide depending on the Sun; and also a smaller but very definite semi-diurnal magnetic tide depending on the Moon, discovered by KREIL at Prague in 1841, and independently detected by JOHN ALLEN BROUN, 1845, as set forth above.

3. All these periodic changes are small compared to the residual constant of the Earth's magnetism, which changes but slowly or not at all. Thus as GAUSS first showed in 1838, the body of the magnetism of the globe depends on the particles within the Earth. The small periodic variations, on the other hand, arise mainly from the external influence exerted by the Sun and Moon, through their systems of magnetic waves.

4. It is commonly stated that the daily variation of the Earth's magnetism is not such as could be explained by the magnetism of the Sun itself. (cf. JEANS, *Mathematical Theory of Electricity and Magnetism,* p. 402.) As shown in the foregoing section, this original view of LLOYD is based on a false premise. It seems certain that with this correction of LLOYD's mode of analysis, the daily variation in the Earth's magnetism may be fully explained. Thus the difficulties encountered by MAXWELL, CHREE, STEWART, SCHUSTER, JEANS, and others entirely disappear.

5. This result confirms JOHN ALLEN BROUN's discovery that the diurnal variation depending on the Moon follows very accurately the law of the inverse cube of the Moon's distance. BROUN remarks that "the ratio of the Moon's mean distance from the Earth in the half orbit about apogee is to that in the half orbit about perigee nearly as 1.07 to 1; as the cube of 1.07 is 1.23 nearly, we see that the mean range of the curves for the two distances are in the ratio of the inverse cubes of the Moon's distance from the Earth, as in the theory of the tides." (STEWART's Article *Meteorology, Ency. Brit.,* 9th edition, p. 179.)

As BROUN had observed the Lunar magnetic effects to be as 1 to 1.24, and SABINE had found similar results, he naturally regarded the verification of this tidal law in the lunar semi-diurnal variation as very important. With the above correction of LLOYD's error of analysis, this result of BROUN shows conclusively that all the diurnal effects observed can be explained by the magnetism of the Sun and Moon.

6. It is not strange therefore that in his celebrated Article on *Terrestrial Magnetism,* §139, BALFOUR STEWART recognized that as the Moon's magnetic

influence follows as nearly as possible BROUN'S law of the inverse cube of the distance from the Earth, it is impossible to refrain from associating this magnetic influence either directly or indirectly with something having the type of tidal action. STEWART points out that AIRY found a similar semi-diurnal inequality depending on the Sun in the Greenwich records, and A. ADAMS found corresponding "Earth Currents" to be induced in the crust of the globe at the corresponding hours.

CONCLUSION

After this review of many mysteries in Cosmical Magnetism, it only remains to add that as the Electrodynamic Wave-Theory fully accounts for the periodic fluctuations in the magnetism of the Earth, it must be held to rest on true laws of Nature. Otherwise it would not be possible to explain so many observed periodic changes in the magnetism of the globe. Heretofore these variations have seemed so utterly mysterious as to completely bewilder the natural philosopher.

It is now clearly established that Magnetism is due to electrodynamic waves, which travel in free space with the velocity of light, but are propagated less rapidly through solid masses.

Any view that Gravitation is an isolated force, entirely disassociated from the other forces of Nature, is wholly untenable. WEBER'S law explains how electrodynamic action is propagated in time by means of the waves we have shown to underly Magnetism in general.

As Magnetism is a form of attraction to poles while Gravitation is central, the two actions are related, and it is obvious that the waves from the atoms of the heavenly bodies should give rise to both Cosmical Magnetism and Universal Gravitation.

T. J. J. SEE

Starlight on Loutre,
Montgomery City, Missouri, June 6, 1917.

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THEORY OF THE TRANSMISSION OF PHYSICAL FORCES, BASED ON THE PROPAGATION OF ELECTRODYNAMIC WAVES IN RIGHT LINES OR IN CURVES CONFORMING TO FERMAT'S LAW OF MINIMUM PATH AND THE PRINCIPLE OF LEAST ACTION.

By T. J. J. SEE

I Introductory Remarks

In his celebrated *History of the Inductive Sciences*, Volume I, Whewell shows that the *chief conditions prerequisite for the development of true Sciences are clearly defined ideas, which are appropriate to the phenomena of Nature, steadily applied to well determined facts of observation.* It was the neglect of these conditions that caused the failure of the physical sciences among the Greeks.

He also shows that during the Middle Ages there were enormous *Stationary Periods,* sometimes amounting to more than a thousand years, during which no real advance was made, owing to the *indistinctness of ideas* then prevalent among the Arabian commentators on Greek Science, and among European Schoolmen long accustomed to the mere polish of scholastic erudition and the unprofitable subtleties of dialectic rather than real research in physical science such as would have appealed to Archimedes, Newton or Gauss.

In view of the deep impression made by Whewell upon his own age, through his luminous exposition of the methods of Archimedes and Newton, one would not have anticipated a reappearance of such indistinctness of ideas, because this would be more appropriate to the epoch of the Arabians than to our own time. Least of all should we have expected the reappearance of these indistinct ideas in Astronomy, the history of which, for thousands of years, is so well known and so instructive to the investigator. Yet, wonderful to relate, this unexpected event has happened!

We refer to the *mystical speculations* of EINSTEIN in his *General Theory of Relativity*. He proposes a modification of the law of Gravitation, but does it in such patch-work fashion, that it leaves Gravitation without appropriate connection with the recognized laws of Electrodynamics, and thus introduces into Nature the most complete discontinuity.

As Electrodynamic Action rests on universal experience in Electricity and Magnetism, it cannot be that we have one kind of attraction in such cases, and a totally different kind of attraction for the heavenly bodies. This would be like the practice of inventing special aethers for special needs, which MAXWELL justly denounces as unphilosophic (*Treatise on Electricity and Magnetism*, 3d edition, Vol. II, §761, p. 431).

As regards EINSTEIN's Theory some of his English followers seem to have overlooked what BOTTLINGER points out in his Munich Prize *Inaugural Dissertation*, p. 48, (Troemer's Universitäts Buchhandlung, Freiburg, 1912) namely, that recently the view was prevalent among physicists that Gravitation is an Electrodynamic phenomenon, but no positive results have attended the efforts to conceive Gravitation as an Electrodynamic Action, and Professor EINSTEIN found Gravity so fundamentally different from Electrodynamics that more recently he had quite turned away from an Electrodynamic explanation.

Thus having failed to connect this chief force of Nature with Electrodynamics, as he long sought to do, EINSTEIN developed a made-to-order *General Theory of Relativity*, couched in such seductive analysis that it received some favor in Germany, was thence transplanted to Holland, and finally welcomed in England — notwithstanding the fact that the underlying ideas clearly are inconsistent with the *sound physical basis* considered essential by both NEWTON and WHEWELL, but now overlooked by certain mathematicians in the Royal Astronomical Society.

For example, it is stated by Professor DE SITTER in his authoritative review of EINSTEIN's Theory (*Monthly Notices*, Oct., 1916, p. 702) that Gravitation is "not a force" but a "property of space."

It would appear that the present author's well known aversion to such *mysticism* as this must be shared also by the more experienced investigators in England; for on January 28, 1917, one of the most eminent of the Cambridge mathematicians wrote as follows:

"I wish the perihelion of Mercury could be resolved similarly (to the new work on the Lunar Fluctuations). Otherwise we have an unlimited number of ingenious kinds of relativity on our hands; which will be remarkable for self-contradiction of the principle that everything is relative."

It is just such confusion as this that I am laboring to get rid of.

In the *Observatory* for December, 1916, the present writer has made some criticisms which should be emphasized. It is pointed out that Professor DE SITTER of Leiden dwells on the *analysis* of EINSTEIN, but quite ignores the *physical problem* of explaining the stupendous forces at work.

"1. In the *Aether of Space*, pp. 112-126, Sir OLIVER LODGE shows from the recognized laws of mechanics that the pull of the Earth on the Moon is equivalent to the breaking strength of a steel column 400 miles in diameter, or a forest of five million million weightless pillars each a square foot in cross section — the tenacity of the steel being thirty tons to the square inch."

"2. In the same way Sir OLIVER LODGE shows (p. 130) that the pull of the Sun on the Earth is equivalent to the tenacity of a forest of weightless steel pillars each eleven inches in diameter, acting on every square foot of a hemispherical section of the globe — the steel again having the breaking strength of thirty tons to the square inch."

"Do these impressive results indicate that gravitation is 'not a force' but only a 'property of space'?"

"NEWTON was well aware of the difficulty of explaining gravitation, and although he was unable to assign the cause of this force, he very properly says in the *General Scholium* to the *Principia* (1713) that it must proceed from a cause which penetrates to the very centres of the Sun and planets without suffering the least diminution of its force. If it were a 'property of space', how could it be definitely and accurately directed to the centres of the Sun and planets? It obviously is so clearly an influence exerted by *matter* that one is astonished at this talk of a 'property of space'.* Such discussion shows the extent to which purely mathematical reasoning may be misapplied by those who ignore appropriate physical considerations."

"In this connection the reader might be referred to WHEWELL'S *History of the Inductive Sciences*, Vol. I, Chapters I and II, where he shows the cause of the failure of the physical science of the Greeks: namely, that although they were keen observers of nature and reasoned very acutely, *their ideas were not appropriate to the facts.* No mischief is more wide spread in modern science than

---

*If gravity were a "property of space," there would be no reason why the attraction should increase with the mass, and thus NEWTON'S law of direct proportionality to mass would be violated, so as to change entirely the recognized foundations of Physical Science.

reasoning on false premises, though ARISTOTLE warned us against this practice 2,200 years ago, and NEWTON emphasized the same caution again and again."

"It is the belief of many experienced investigators that the whole Doctrine of Relativity rests on a false basis, and will some day be cited as an illustration of foundations laid in quicksand. Dozens of books have appeared on the subject. Thus our problem to-day is not merely to discover Truth, but to discover a way out of an ensnaring mesh of errors."

"In view of these facts even the most aetherial pure mathematician ought to realize the impossibility of successfully treating gravitation from the analytical point of view, by merely transforming the equations, and ignoring the physical conditions which alone enable us to understand these stupendous forces of Nature. From a study of these problems running over many years, I venture to think that the chief difficulty consists not in Analysis or its transformation — since we know the law of the attractive force — but in discovering some *physical agency* by which we can consistently explain the enormous forces actually operative in gravitation."

The accompanying diagram of the Newtonian theory of a central force of attraction constantly directed towards the Sun's centre and thus curving the

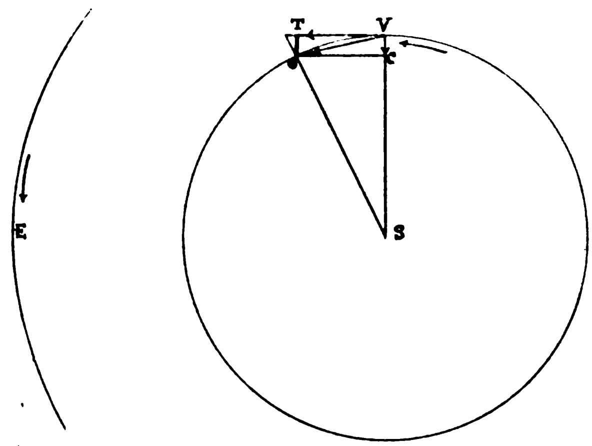

FIG. 1. The Newtonian Theory of the Central Force directed to the Sun's centre, incessantly acting on a planet and curving the path at every point of the orbit.

planet's motion at every point of the orbit will make the central forces discussed in the above extract from the *Observatory* a little more intelligible than they otherwise would be. *The stupendous power of these forces and the exactness with which they are directed to the Sun's centre must be clearly borne in mind by those who wish to form ideas appropriate to the facts of Nature. Moreover, if the Force of Gravitation is to be done away with, so also must the Force of Magnetism; yet we do not hear* *of such a theory of Magnetic Relativity, and it is doubtful if* EINSTEIN, DE SITTER *and their associates will propose one.*

In view of the surprising fact that so learned a mathematician as Dr. J. H. JEANS (in the *Observatory* for January, 1917, pp. 57-58) attempts to reply to the above criticisms, and speaks of FERMAT'S condition, it seems advisable to make known part of my investigations into that problem and others relating to the *path of transmission of gravitational force*, whether in right lines or in curves which conform to FERMAT'S Law of minimum path, in accordance with the Principle of Least Action. We shall therefore outline without further delay certain geometrical conditions, with the appropriate physical foundations, which underly the Electrodynamic Wave-Theory of Universal Gravitation, as applied to the *Fluctuations* of the Moon's Mean Motion.

II ELECTRODYNAMIC WAVES PROPAGATED IN FREE AETHER WITH THE VELOCITY OF LIGHT THE PHYSICAL CAUSE OF THOSE ACTIONS IN NATURE WHICH INVOLVE THE PRINCIPLE OF LEAST TIME.

In BULLETIN No. 2 we have given a direct and simple proof that Magnetism is due to the action of plane waves originating in the atoms and propagated throughout all space. *Two magnets mutually attract when the rotations of the elements in the waves from one magnet undo the rotations of the elements in the waves from the other magnet: for this decreases the* STRESS *in the medium, and it then contracts just as a stretched mass of India rubber does when released.*

It was one of the most remarkable of FARADAY'S experimental discoveries that all bodies are magnetic. Thus it is established that all bodies emit waves of the type described in BULLETIN No. 2. *And if the waves from a second body are so directed as to decrease the* STRESS *in the medium between that and the first body, then the two masses will mutually attract. Upon this general theory rests the discovery of the cause of Magnetism, of Electrodynamic Action, and of Universal Gravitation.*

In the sixth BULLETIN we shall examine with more detail, and enter with all due rigor into the cause of Universal Gravitation. *It will be shown that the Medium suffers decrease of stress and is therefore under tension between two heavenly bodies, while beyond them there is increase of stress and therefore increase of pressure, both influences operating to hold the bodies together. The increase of tension between the masses and the increase of pressure beyond them fully explains Universal Gravitation, through mutually interpenetrating waves emitted from the atoms.* Thus the geometrical and physical laws of wave action require rigorous investigation.

The theory of *physical forces* developed in these Bulletins is so fundamentally different from that heretofore current, if indeed any uniform conception of forces can be said to have been held by investigators, that it is necessary to lay a secure foundation for the superstructure of the argument to follow. Thus we have to develop a line of reasoning of some length; yet in the end we shall be compensated for our labors by a clearly defined view of the invisible mechanism underlying the processes of the physical Universe. Heretofore this view has been hazy and so securely hidden from our imagination as to be utterly bewildering to both the geometer and the natural philosopher.

(*i*) *The simple case of the reflection of Light from a plane mirror.*

We shall first illustrate these operations in the case of *reflected light*, because this case is the simplest, and has the advantage that it clearly illuminates some of the deepest mysteries of Nature.

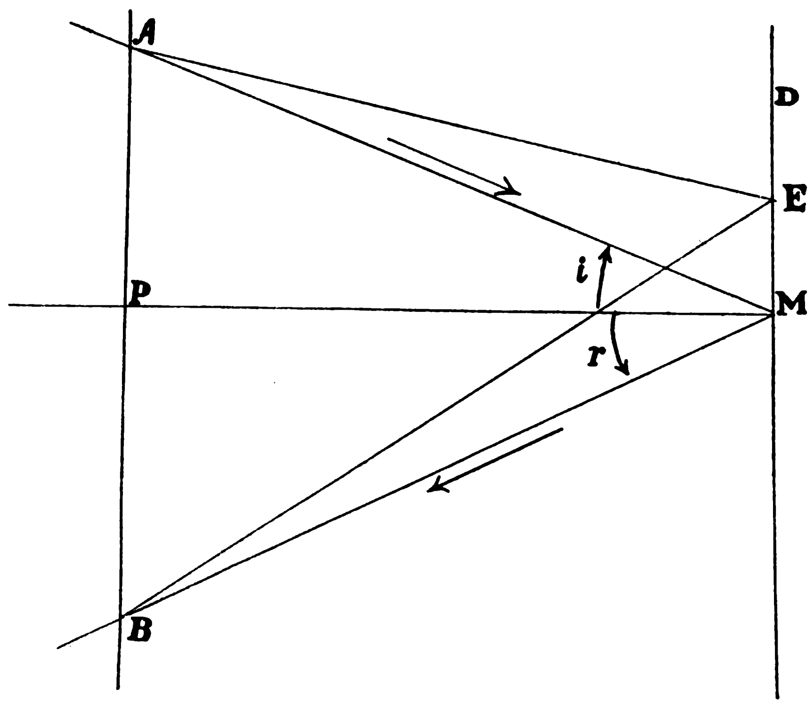

FIG. 2. Illustration of the reflection of a ray of light from a plane mirror, in the path of *Least Time*, along the perimeter of an Isosceles Triangle.

It was an early discovery of the Greek natural philosophers, implied in the Geometrical Theorems of EUCLID, about 300 B.C., but on physical grounds ascribed to PTOLEMY by LAPLACE, that in the case of a plane mirror the bent line formed by the incident and reflected rays of light are shorter than any other bent line with the same extremities, having its point of bending in the plane of reflection in the mirror. On account of the geometrical reasoning to follow this important observed fact is illustrated by the accompanying figure.

Now what is the real cause of this apparent propagation of the light in the *path of Least Time?* We shall try to answer this question briefly, in accordance with valid geometrical and physical principles.

1. In EUCLID'S *Elements*, Book I, Propositions 10-12, (Todhunter's edition, MacMillan & Co., 1903) we see how the Greek mathematicians reasoned in regard to the bisection of a line AB by means of a circle described from the centre with radius $CF = CG$, and the perpendicular CH at the mid-point of the circular arc FDG.

2. One of EUCLID'S celebrated axioms is that a straight line is the shortest distance between two points. He also shows that a perpendicular is the shortest distance from any point as C to a right line, as AB; and proves that any right line drawn from C to AB but not meeting it in the perpendicular at H is longer than CH. He does this by the use of the simple rule and compass; and his conclusion is so obvious that no elaborate demonstration is required.

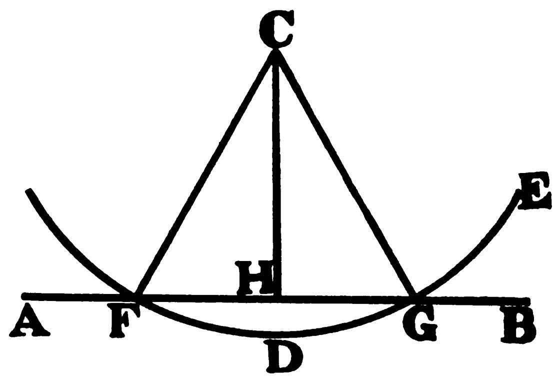

FIG. 3. EUCLID'S Geometrical Theorem of the perpendicular as the shortest distance from a point to given line, and the isosceles triangle which may be erected at equal distances from the base of the perpendicular.

3. Now in the above figure of the ray of light reflected from the mirror, AMB corresponds to EUCLID'S isosceles triangle FCG. Therefore AMB is an isosceles triangle, and MP is the perpendicular from the reflecting point M in the plane of the mirror. Consequently by geometrical construction the angles of incidence and of reflection necessarily are equal.

4. In regard to the *physical cause* of this equality of angles, it is obvious that it can only rest on the *elastic rebound of the waves, at the surface of the mirror, due to the physical properties of the aether when the motion is retarded. This medium is elastic, and disturbances in it are therefore reflected according to the recognized law of action and reaction in ponderable bodies of elastic constitution.*

5. A few familiar illustrations will suffice: (*a*) When a boy bounces a rubber ball on the smooth pavement, the descending and ascending angle of the ball's motion are equal. (*b*) When a glass, stone, or metal marble is thrown against a smooth, highly rigid slab, the angle of the rebound is noticed to be equal to that of the descent, and the same law holds for the reaction of all elastic solids in collision. (*c*) So also for arrested *fluid movements*, as when waves in water are reflected from a smooth wall. Here we may easily observe that the angle of incidence is equal to the angle of reflection. This law of Nature is indeed *universal,* and rests on the *physical property of the more or less perfect elasticity of form and volume.*

6. Now since the *propagation of light* involves the *transmission of waves in the aether*, this medium evidently will react if it is *elastic*, and the forward motion is *obstructed,* as at the surface of the mirror.

7. It is shown *geometrically*, in the reasoning above cited from EUCLID, that the shortest distance from M to AB is the perpendicular MP; and as the points A and B are equidistant from the base of the perpendicular P they too are equally distant from M, so that $AM = MB$.

8. Of all the triangles erected on AB as a base, and with vertices in the parallel line MD, in the plane of the mirror, the *isosceles triangle* has the *shortest perimeter.** Consequently any triangle not isosceles, as AEB will have the sum of the sides $AE + EB$ longer than $AM + MB$. Hence AMB is the path of the *shortest time.*

9. This *result* was known to the Greek mathematicians in the time of EUCLID, 300 B.C., but the *cause* of it has not been fully agreed upon even in our own time. Hence we have entered with some detail into the circumstances of the motion of light.

*The celebrated problem of perimeters was first treated by the Greek Geometer ZENODORUS, in the second century B.C. In general the perimeter is less the nearer the approach of the figure to that of a regular polygon, whether made up of 3 or n sides, so that an Isosceles triangle has the same property.

10. With these geometrical principles before us, we now ask what, then, is the *physical cause* of so wonderful a result? It was long thought by philosophers to be an indication of a tendency to simplicity and economy in Nature. But from the above reasoning we conclude that it rests on two distinct grounds, as follows:

(*a*) *The very perfect elasticity of the aetherial medium. This gives a physical basis for the reflection of wave disturbances which cannot proceed in a right line, and necessarily makes the angle* $r = i$.

(*b*) *As the reflection, on the same physical grounds, requires the reflected waves to follow the other equal side of an isosceles triangle, the geometrical path* AMB *necessarily is the shortest possible. And since light is confined to this isosceles triangle, by the physical properties of the elastic aether, it must on purely geometrical grounds be propagated in the Least Time.*

(*ii*) *The case of Light bent out of its rectilinear path by Refraction.*

The above discussion is sufficient for the simple case of aether waves undergoing reflection from a plane mirror; but in the case of *refraction*, more complex phenomena arise, and it is necessary to consider the path of the light as it goes through media which bend the rays from a rectilinear course.

This problem was first critically examined by the French mathematician FERMAT (1601—1665) whose researches on maxima and minima are considered by many modern geometers to be the earliest germs of the Differential Calculus. When FERMAT tried to trace out this supposed economy of Nature, in the more complex problem of *refraction*, he encountered considerable difficulty, because in his time the relative velocities in the refracting media were not yet established by observation.

About 1620, SNELLIUS had discovered the law of refraction, which DESCARTES, 1637, put in the form now used:

$$\sin i = n \sin r.$$

It was soon found that if the two lengths or indices be measured along the incident ray prolonged and the refracted ray, they have a common projection in the plane of refraction.

In order to reconcile these phenomena with the acknowledged propagation in *least time*, supposed to follow from natural laws since the time of the Greek philosophers, FERMAT was led to conclude from considerations of simplicity that the relative velocities are inversely as the refractive indices, and therefore that the velocity of light is diminished on entering the denser medium, in which the path was found by observation to approach the perpendicular.*

FERMAT believed that the total time of propagation would be found by taking the sum of the products of the length of path of the incident light by its velocity, in the first medium, l_1v_1, and of the length of path of the refracted light by its velocity in the second medium, l_2v_2.

Indeed, FERMAT'S mathematical treatment showed this compound sum, $l_1v_1 + l_2v_2$ to be less, for a path in a plane refraction, than that which would result if the light went by any other than its actual path. Hence resulted FERMAT'S condition of *minimum path,* which has since become so celebrated in physical science. Thus he was enabled to reconcile his mathematical discovery that in the actual path the sum $\tau = l_1v_1 + l_2v_2$ is a *minimum,* with his cosmological notion of propagation in *Least Time,* recognized in simple reflection since the age of the Greek geometers.

For a time these results of FERMAT'S researches were attacked by DESCARTES and his followers, but after the discovery of the Calculus they were zealously and effectively defended by LEIBNITZ. HUYGHENS was among the first to recognize ROEMER'S discovery of the velocity of light (1675), and thus he too was led to adopt FERMAT'S conclusions of a velocity inversely as the index of the medium, and of *minimum time of propagation.*

But as NEWTON'S widely current emission theory was not reconcilable with FERMAT'S results, the theorem of shortest *time* was abandoned by many. It was at this stage that MAUPERTUIS proposed in its stead the *celebrated Principle of Least Action.* This new principle was afterwards much elaborated by the researches of EULER and LAGRANGE.

(*iii*) MAUPERTUIS' *Principle of Least Action.*

MAUPERTUIS' principle of *Least Action* as formulated by LAGRANGE, is stated thus:

(1) $$A = \int_0^t \sum_{i=0}^{i=i} m_i v_i ds_i$$

*Cf. TAIT, Article, *Light, Encyc. Brit.,* 9th edition, Vol. XIV, p. 598.

This means that in any independent system, the integral of the actions between $t = 0$, $t = t$, is found by summing up the *products of mass by velocity, by element of curvilinear space traversed.*

MAUPERTUIS tried to reconcile the results of FERMAT and NEWTON, by showing that the course chosen by light corresponds to the *least possible action,* though not always to the *least possible time.* The introduction of the product of masses by velocity, in the above formula, was required to compose the difficulties of NEWTON's corpuscular theory of light, and first occurs in LAGRANGE's researches on Dynamics.

Among the several great mathematicians of the preceding age EULER was pre-eminently NEWTON's intellectual successor. And as the Swiss geometer became attached to MAUPERTUIS during his residence at St. Petersburg and Berlin, and was agreeably impressed with the new principle of *Least Action,* he employed his own great mathematical powers to show that the law of *Least Action* applies to motion in all curves described by points under the influence of central forces, such as planets and comets revolving about the Sun, and of heavy bodies falling to the Earth in the *curve of least time* (Brachistochrone), which had also long engaged the attention of NEWTON, LEIBNITZ, and the BERNOULIS.

In EULER's method of proof he takes the curve actually described and compares it with another having the same extremities, but differing from it indefinitely little in shape and position, yet such as may be imagined described by a neighboring point with the same law of velocity. Then, applying the term *action* to the integral of the product of the velocity and element of the curve, he finds that the difference of the two neighboring values of this *action* will be indefinitely less than the greatest linear distance (itself indefinitely small) between the two adjacent curves. Sir WM. R. HAMILTON, whose discussion we have here followed, calls this unchanging result *Stationary Action.*

LAGRANGE extended this theorem of EULER to the motion of a system of bodies which act in any manner on each other, the *action* being formed of the sum of the masses by the velocity-space integrals employed by EULER, so as to give the LAGRANGE-HAMILTON form:

$$A = \int_0^t \sum_{i=0}^{i=i} m_i V_i \frac{ds_i}{dt} dt \tag{2}$$

In tridimensional space the velocity along the arc of any curve of double curvature becomes

(3) $$\frac{ds_i}{dt} = \sqrt{\left(\frac{dx_i}{dt}\right)^2 + \left(\frac{dy_i}{dt}\right)^2 + \left(\frac{dz_i}{dt}\right)^2}$$

And therefore

(4) $$A = \int_0^t \sum_{i=0}^{i=i} \left(m_i V_i \sqrt{\left(\frac{dx_i}{dt}\right)^2 + \left(\frac{dy_i}{dt}\right)^2 + \left(\frac{dz_i}{dt}\right)^2} \right) dt = \int_0^t \left(\sum_{i=0}^{i=i} m_i V_i^2 \right) dt$$

This formula therefore represents the integral for the preservation of the Kinetic Energy or *Vis-Viva.*

III The Principles of Least Action and of Varying Action Deducible from the Conservation of the Kinetic Energy under Physical Forces Due to Waves

We have just seen that in any *conservative system,* such as an isolated star cluster, in steady motion but without collisions and in space free of a resisting medium, the *changes of the squared velocities,* in a given time $t - t_0$, mutually balance up, and within the whole system entirely disappear; so that

(5) $$\sum_{i=0}^{i=i} m_i V_i^2 = \int_0^t \left(\sum_{i=0}^{i=i} m_i V_i \frac{ds_i}{dt} \right) dt$$

In other words: Given the system of $i + 1$ bodies moving with velocities v_i at the epoch t_0, they will at the epoch t also be moving with other velocities v_i of such a nature that although individual increases and decreases occur, yet in the whole system the squares multiplied by the masses mutually compensate, so that the action becomes

(6) $$A = \int_0^t \left(\sum_{i=0}^{i=i} m_i V_i \frac{ds_i}{dt} \right) dt$$

And therefore the *variation of the Action* vanishes:

(7) $$\delta A = \delta \int_0^t \left(\sum_{i=0}^{i=i} m_i V_i \frac{ds_i}{dt} \right) dt = \delta \left(\sum_{i=0}^{i=i} m_i V_i^2 \right) = 0$$

Now whilst the *variation of the Action* vanishes in a *conservative system,* it does not do so in a *non-conservative one.* Yet when work is done the change in the Kinetic Energy of the system in any time, t_0 to t, depends only on the relative distances of the masses at the beginning and end of that time. *The changes in the mutual distances is purely a problem of mutual action, under the attractive forces exerted by the bodies, along right lines joining their centres, and hence the absurdity of the claim made by Einstein and his followers that Gravity is not a "force," but a "property of space."*

For it is obvious that the *paths of the motions* follow from the *initial co-ordinates* x_i, y_i, z_i, and the *velocities* $\frac{dx_i}{dt}$, $\frac{dy_i}{dt}$, $\frac{dz_i}{dt}$, and the *accelerations* produced by the *mutual action* of the bodies of the system in the interval t_0 to t. The *conservation of areas* holds even with *internal collisions,* but the *vis-viva* only for a *conservative system.*

In the *force function* introduced by LAGRANGE the *mutual potential* of the $i + 1$ bodies taken in pairs $m_i\, m_j$ is formulated thus:

$$\left.\begin{aligned} U = \frac{m_0 m_1}{\Delta_{0,1}} + \frac{m_0 m_2}{\Delta_{0,2}} + \frac{m_0 m_3}{\Delta_{0,3}} + \dots + \frac{m_0 m_n}{\Delta_{0,n}} \\ + \frac{m_1 m_2}{\Delta_{1,2}} + \frac{m_1 m_3}{\Delta_{1,3}} + \dots + \frac{m_1 m_n}{\Delta_{1,n}} \\ + \frac{m_2 m_3}{\Delta_{2,3}} + \dots + \frac{m_2 m_n}{\Delta_{2,n}} \\ \vdots \qquad\qquad \vdots \\ + \frac{m_{n-1} m_n}{\Delta_{n-1,n}} \end{aligned}\right\} \begin{aligned} & i = 0,\ j = 1 \ \dots\ j = n. \\ & i = 1,\ j = 2 \ \dots\ j = n. \\ & i = 2,\ j = 3 \ \dots\ j = n. \\ & \vdots \\ & i = n - 1,\ j = n. \end{aligned} \tag{8}$$

$$= \sum_{i=0}^{i=i} \sum_{j=1}^{j=n} \frac{m_i m_j}{\Delta_{i,j}} \tag{9}$$

And since

$$\Delta_{i,j}^2 = (\xi_i - \xi_j)^2 + (\eta_i - \eta_j)^2 + (\zeta_i - \zeta_j)^2 \tag{10}$$

we have

$$\frac{\partial \Delta_{0,1}}{\partial \xi_0} = \frac{\xi_0 - \xi_1}{\Delta_{0,1}}, \quad \frac{\partial \Delta_{0,2}}{\partial \xi_0} = \frac{\xi_0 - \xi_2}{\Delta_{0,2}}, \quad \dots \frac{\partial \Delta_{0,n}}{\partial \xi_0} = \frac{\xi_0 - \xi_n}{\Delta_{0,n}} \tag{11}$$

And the corresponding differential equations for the actions of the other bodies upon m_0 become:

$$
(12)\qquad \begin{cases} m_0 \dfrac{d^2\xi_0}{dt^2} = m_0 m_1 \dfrac{\xi_1 - \xi_0}{\Delta_{0,1}{}^3} + m_0 m_2 \dfrac{\xi_2 - \xi_0}{\Delta_{0,2}{}^3} + \ldots + m_0 m_n \dfrac{\xi_n - \xi_0}{\Delta_{0,n}{}^3} \\[2ex] m_0 \dfrac{d^2\eta_0}{dt^2} = m_0 m_1 \dfrac{\eta_1 - \eta_0}{\Delta_{0,1}{}^3} + m_0 m_2 \dfrac{\eta_2 - \eta_0}{\Delta_{0,2}{}^3} + \ldots + m_0 m_n \dfrac{\eta_n - \eta_0}{\Delta_{0,n}{}^3} \\[2ex] m_0 \dfrac{d^2\zeta_0}{dt^2} = m_0 m_1 \dfrac{\zeta_1 - \zeta_0}{\Delta_{0,1}{}^3} + m_0 m_2 \dfrac{\zeta_2 - \zeta_0}{\Delta_{0,2}{}^3} + \ldots + m_0 m_n \dfrac{\zeta_n - \zeta_0}{\Delta_{0,n}{}^3} \end{cases}
$$

Similarly for the action of the other bodies on the mass m_1, etc.

These familiar equations (12) show that the *actions* of the bodies always are *mutual;* and therefore no change can be made in the component velocities of one body without corresponding reactions on the component velocities of the other bodies.

It is obvious therefore that *changes of velocities* due to inter-actions *necessarily are mutual.* It is only under interpenetration of waves that stress of the medium is developed, and the magnitudes of the changes produced in the energy of these masses depend upon the *forces at work,* under the stresses operating in the action of the attracting bodies to which any single mass may be subjected within the system.

Accordingly the changes within the system, depending on the mutual actions of its component masses, necessarily will be such that the Action is in accordance with the LAGRANGE-HAMILTON Equation

$$
(13)\qquad A = \int_0^t \left(\sum_{i=0}^{i=i} m_i v_i \frac{ds_i}{dt} \right) dt = \int_0^t \left(\sum_{i=0}^{i=i} m_i v_i^2 \right) dt
$$

And therefore, in a *conservative system,* free of collisions and *devoid of friction*

$$
(14)\qquad \delta A = \delta \int_0^t \left(\sum_{i=0}^{i=i} m_i V_i^2 \right) dt = 0
$$

Thus any system necessarily pursues the path of *Least Action* under its own forces; for this merely conserves the *vis viva.* If the bodies be started with the velocities v_i and co-ordinates x_i, y_i, z_i, the changes in the velocities due to the *mutual wave action* of the bodies of the system necessarily will be such as to conform to *the forces which are appropriate to the given co-ordinates.*

No other changes are dynamically possible. Consequently the system always conserves the *vis viva,* and hence automatically pursues the path of *Least Action;* and therefore for any interval, t_0 to t, we have the above integrals (13) and (14). But starting at any epoch t_0 it is obvious that

$$A = \int_0^t \left(\sum_{i=0}^{i=i} m_i V_i^2 \right) dt = \int_0^t C dt \,, \qquad C = \text{a constant}, \tag{15}$$

and therefore for any given interval of time $t - t_0$ there is no change in the *Action.*

From the point of view of the *conservation of energy,* we may write T for the kinetic energy, V for the potential energy; and then for the total energy of the system we have

$$H = T + V = \frac{1}{2} \sum_{i=0}^{i=i} m_i \left\{ \left(\frac{dx_i}{dt} \right)^2 + \left(\frac{dy_i}{dt} \right)^2 + \left(\frac{dz_i}{dt} \right)^2 \right\} + V \,. \tag{16}$$

And as the total energy is constant for any system subjected only to the mutual actions of its parts, it is obvious that we shall have $\delta T = -\delta V;$ therefore

$$\sum_{i=0}^{i=i} m_i \left(\frac{d^2 x_i}{dt^2} \delta x_i + \frac{d^2 y_i}{dt^2} \delta y_i + \frac{d^2 z_i}{dt^2} \delta z_i \right) + \delta V = 0 \,. \tag{17}$$

Accordingly this equation defines any variation in the potential energy which may occur as due to variations in the co-ordinates.

The integral therefore

$$\delta A = \delta \int_0^t \sum_{i=0}^{i=i} \left(m_i V_i \frac{ds_i}{dt} \right) dt = 0 \tag{18}$$

expresses HAMILTON'S *Stationary Condition.*

HAMILTON *calls the changes of the Action due to variation of path, with the same terminal co-ordinates, Varying Action.*

Let W denote the *work function,* and T the kinetic and V the potential energy; then we have

$$L = T + W = T - V \,. \tag{19}$$

But W and V are functions of the co-ordinates, not of the velocities; and therefore if AB be the actual trajectory, CD the neighboring path, with the same terminal points and times of description, we obviously have identical action in the two paths

$$\int_{CD} L dt - \int_{AB} L dt = 0 \tag{20}$$

which shows that the *Action is Stationary.*

This result is an extension of the researches of EULER *and* LAGRANGE, *and known as* HAMILTON'S *Principle. It implies that the motion in the same time is one of Least Action, along paths appropriate to the conservation of the kinetic energy, and thus combines* FERMAT'S *Principle of Least Time with* MAUPERTUIS' *Principle of Least Action.*

All these results really depend on the *conservation of the kinetic energy*, and thus are referable to *wave action* propagated in *least time. In any conservative system, the conservation of the Kinetic Energy is obvious.* This is all that *Least Action* implies; and it is traceable to physical forces directed to the masses mutually, and due to the interaction of infinite systems of waves in the aethereal medium, everywhere under tension and therefore propagated in the *least time*, and thus more rapidly in free space than through ponderable bodies, involving resistance to the progress of the disturbances.

Accordingly, if such an attractive force as Universal Gravitation be due to electrodynamic wave action, as shown by the writer's researches of 1914–17, it will follow that the disturbances due to such waves, pursuing paths of *least action, in least time*, — but with the adjacent path offering unequal resistance — will necessarily suffer refraction and dispersion, perhaps absorption, while traversing the solid matter of the heavenly bodies.

The Sun's gravitational attraction on the Moon, therefore, will be decreased in and near the shadow of the Earth, giving that very "expenditure or absorption of energy somewhere in the solar system" postulated by NEWCOMB for explaining the Moon's *Fluctuations*, in his posthumous memoir of 1912.

Accordingly whilst there is conservation of energy in the Universe, this conservation is not perfect in the temporary operations of a single system involving the mutual interaction of parts under forces propagated through solid masses, in which the electrodynamic waves may be refracted and dispersed, when bent from a rectilinear path in transmission. It was in the hope of unfolding this view of the mutual actions of the heavenly bodies, when these actions are somewhat obstructed, by the interposition of some of these masses in the rectilinear paths of the mutual electrodynamic wave action, that we have dwelt at length on the laws of Dynamics.

We cannot assume the ordinary formulæ for the dynamics of *particles*, without considering the interposition of *ponderable masses of finite dimensions* in the line of mutual action for wave disturbances traveling with the velocity of Light, and therefore suffering both refraction and dispersion, perhaps absorption, of part of the wave energy. But as the dimensions of the heavenly bodies are very small compared to the immense distances which separate them, there is reason to anticipate that the modification of the gravitational forces in propagation through the masses should be very small.

It is not wonderful therefore that this refraction, dispersion and perhaps absorption of gravitational energy long escaped attention; and is only now beginning to be placed on a sound mathematical and physical basis, and verified by observations drawn from the motion of the Moon.

IV Application of Hamilton's Principle to the Wave-Theory of Light, and Thence to Electrodynamic Waves in Electrodynamics and Universal Gravitation

In the Wave Theory of Light the *time of passage* from one point to another, as was found by Fermat, rests on the *principle of least time.* Accordingly this *time of transmission* should thus fulfill Hamilton's *Stationary Condition.* We restrict our present considerations to the case of single refraction in a non-homogeneous medium; for if it applies to this case perfectly, it will obviously apply to the more complicated phenomena of light which occur in Nature, and also to the Electrodynamic Waves which give rise to the attractions observed in Electrodynamic Action and Universal Gravitation.

In the problems of refractions, we have to consider the *curved path* of the light; and the measure of *curvature* is defined by the equations:

$$\left.\begin{aligned} s &= a\phi\,, \\ \text{curvature} &= \frac{d\phi}{ds} = \frac{1}{a}\,, \end{aligned}\right\} \tag{21}$$

where ϕ is the angle between the osculating tangent planes, and ds is the element of the curve, and a the *radius of curvature,* for the osculating circle passing through three consecutive points.

For any path in space, the *curvature* is

$$\frac{1}{\rho} = \sqrt{\left(\frac{d^2x}{ds^2}\right)^2 + \left(\frac{d^2y}{ds^2}\right)^2 + \left(\frac{d^2z}{ds^2}\right)^2} \tag{22}$$

And the direction cosines of the radius of curvature

$$\rho\frac{d^2x}{ds^2}\,,\qquad \rho\frac{d^2y}{ds^2}\,,\qquad \rho\frac{d^2z}{ds^2} \tag{23}$$

In a single refracting medium the velocity of light at any point (x, y, z) is the same whatever be the direction of the ray; and thus the velocity of the disturbance depends only on the co-ordinates of the point, and some characteristic of the waves, such as their length.

If τ be the time of passage, ds the element of the path, and v the velocity of light, we have as the integral which must fulfill HAMILTON'S *stationary condition:*

$$\tau = \int \frac{ds}{v} \tag{24}$$

Accordingly, by the *Calculus of Variations,*

$$\delta\tau = \int \frac{d\delta s}{v} - \int \frac{ds\,\delta v}{v^2} \tag{25}$$

If λ be the wave length, it is obvious that the velocity would be defined by the functional relation

$$v = f\,(\lambda, x, y, z) \tag{26}$$

the form of the function f depending on the arrangement of the parts of the medium.

Making use of this value of v in (25), we obtain

$$\delta\tau = \int \frac{dx\delta x + dy\delta y + dz\delta z}{vds} - \int \frac{ds}{v^2}\left(\frac{dv}{d\lambda}\delta\lambda + \frac{dv}{dx}\delta x + \frac{dv}{dy}\delta y + \frac{dv}{dz}\delta z\right) \tag{27}$$

$$= \left[\frac{1}{v}\left(\frac{dx}{ds}\delta x + \frac{dy}{ds}\delta y + \frac{dz}{ds}\delta z\right)\right] - \delta\lambda\int \frac{1}{v^2}\frac{dv}{d\lambda}ds - \delta x\int \frac{1}{v^2}\frac{dv}{dx}ds - \delta y\int \frac{1}{v^2}\frac{dv}{dy}ds - \delta z\int \frac{1}{v^2}\frac{dv}{dz}ds \tag{28}$$

The last three integrals of (27), under HAMILTON'S *stationary condition,* vanish, because the fixed terminal points make δx, δy, δz each equal to zero. The rest of the expression depends on the terminal points of the path, and on the wave length only.*

These conditions therefore lead to four equations

$$\frac{\delta\tau}{\delta x} = \frac{1}{v}\frac{dx}{ds}; \qquad \frac{\delta\tau}{\delta y} = \frac{1}{v}\frac{dy}{ds}; \qquad \frac{\delta\tau}{\delta z} = \frac{1}{v}\frac{dz}{ds}; \qquad \frac{\delta\tau}{\delta\lambda} = -\int \frac{1}{v^2}\frac{dv}{d\lambda}ds\,. \tag{29}$$

Now the tangent to the curved path ds is defined by the three differential direction cosines, fulfilling the condition

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*Cf. TAIT, Article *Light, Encyclopedia Britannica,* 9th Edition.

$$\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 + \left(\frac{dz}{ds}\right)^2 = 1 \tag{30}$$

And therefore if we square and add the first three equations of (29) we shall obtain

$$\left(\frac{\delta\tau}{\delta x}\right)^2 + \left(\frac{\delta\tau}{\delta y}\right)^2 + \left(\frac{\delta\tau}{\delta z}\right)^2 = \frac{1}{v^2} \tag{31}$$

(*i*) HAMILTON'S *Characteristic Function defined.*

In 1823, when only eighteen years of age, HAMILTON obtained insight into his method, and gradually introduced the consideration of a *characteristic function* A defined by the following differential equation for a single particle of unit mass,

$$\delta A = \left[\frac{dx}{dt}\delta x + \frac{dy}{dt}\delta y + \frac{dz}{dt}\delta z\right] - \left(\frac{dx_0}{dt}\delta x_0 + \frac{dy_0}{dt}\delta y_0 + \frac{dz_0}{dt}\delta z_0\right) + t\delta H\ , \tag{32}$$

where H is the constant of the total energy $H = T + V$.

If the moving particle be entirely free, the seven variables in the right member of (32) are independent of one another; and thus the *characteristic function* A fulfills the following remarkable differential equations:

$$\left.\begin{array}{ll} \dfrac{\partial A}{\partial x} = \dfrac{dx}{dt}\,, & \dfrac{\partial A}{\partial x_0} = -\dfrac{dx_0}{dt}\,, \\ \dfrac{\partial A}{\partial y} = \dfrac{dy}{dt}\,, & \dfrac{\partial A}{\partial y_0} = -\dfrac{dy_0}{dt}\,, \\ \dfrac{\partial A}{\partial z} = \dfrac{dz}{dt}\,, & \dfrac{\partial A}{\partial z_0} = -\dfrac{dz_0}{dt}\,, \\ & \dfrac{\partial A}{\partial H} = t\,. \end{array}\right\} \tag{33}$$

Therefore we have at once

$$\left(\frac{\partial A}{\partial x}\right)^2 + \left(\frac{\partial A}{\partial y}\right)^2 + \left(\frac{\partial A}{\partial z}\right)^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 = v^2 = 2(H - V) \tag{34}$$

$$\left(\frac{\partial A}{\partial x_0}\right)^2 + \left(\frac{\partial A}{\partial y_0}\right)^2 + \left(\frac{\partial A}{\partial z_0}\right)^2 = \left(\frac{dx_0}{dt}\right)^2 + \left(\frac{dy_0}{dt}\right)^2 + \left(\frac{dz_0}{dt}\right)^2 = v_0^2 = 2(H - V_0) \tag{35}$$

(*ii*) *Remarkable Properties of* HAMILTON'S *Characteristic Function, applicable alike to the propagation of Light, Electrodynamic Action and Universal Gravitation.*

Since the *Characteristic Function A* satisfies the partial differential equation:

$$\left(\frac{\partial A}{\partial x}\right)^2 + \left(\frac{\partial A}{\partial y}\right)^2 + \left(\frac{\partial A}{\partial z}\right)^2 = v^2 = 2(H - V) \tag{34}$$

it follows that the partial differential coefficients with respect to the co-ordinates represent the components of the velocity in a motion possible under the forces whose potential is V. And as V is the potential energy of the system, this result is very remarkable; for it assimilates the propagation of wave disturbances, such as light, and electrodynamic action, to the action of universal gravitation, which also fulfills the same condition.

By partial differentiation of (34) with respect to the co-ordinates we have

$$(36)\quad \left\{\begin{aligned}
\frac{\partial A}{\partial x}\frac{\partial^2 A}{\partial x^2} + \frac{\partial A}{\partial y}\frac{\partial^2 A}{\partial x\partial y} + \frac{\partial A}{\partial z}\frac{\partial^2 A}{\partial x\partial z} &= -\frac{\partial V}{\partial x} = X = \frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right),\\
\frac{\partial A}{\partial y}\frac{\partial^2 A}{\partial x\partial y} + \frac{\partial A}{\partial y}\frac{\partial^2 A}{\partial y^2} + \frac{\partial A}{\partial z}\frac{\partial^2 A}{\partial y\partial z} &= -\frac{\partial V}{\partial y} = Y = \frac{d^2y}{dt^2} = \frac{d}{dt}\left(\frac{dy}{dt}\right),\\
\frac{\partial A}{\partial z}\frac{\partial^2 A}{\partial x\partial z} + \frac{\partial A}{\partial z}\frac{\partial^2 A}{\partial y\partial z} + \frac{\partial A}{\partial z}\frac{\partial^2 A}{\partial z^2} &= -\frac{\partial V}{\partial z} = Z = \frac{d^2z}{dt^2} = \frac{d}{dt}\left(\frac{dz}{dt}\right),
\end{aligned}\right.$$

Also, differentiating in respect to t, we have

$$(37)\quad \left\{\begin{aligned}
\frac{dx}{dt}\frac{\partial^2 A}{\partial x^2} + \frac{dy}{dt}\frac{\partial^2 A}{\partial x\partial y} + \frac{dz}{dt}\frac{\partial^2 A}{\partial x\partial z} &= \frac{d}{dt}\left(\frac{\partial A}{dx}\right),\\
\frac{dy}{dt}\frac{\partial^2 A}{\partial x\partial y} + \frac{dy}{dt}\frac{\partial^2 A}{\partial y^2} + \frac{dz}{dt}\frac{\partial^2 A}{\partial y\partial z} &= \frac{d}{dt}\left(\frac{\partial A}{dy}\right),\\
\frac{dz}{dt}\frac{\partial^2 A}{\partial x\partial z} + \frac{dy}{dt}\frac{\partial^2 A}{\partial y\partial z} + \frac{dz}{dt}\frac{\partial^2 A}{\partial z^2} &= \frac{d}{dt}\left(\frac{\partial A}{dz}\right).
\end{aligned}\right.$$

On comparing equations (36) and (37), we find that

$$\frac{dx}{dt} = \frac{\partial A}{\partial x}, \qquad \frac{dy}{dt} = \frac{\partial A}{\partial y}, \qquad \frac{dz}{dt} = \frac{\partial A}{\partial z} \tag{38}$$

satisfy simultaneously the two sets of equations.

If now we take α, β to be constants which may combine with H to give the complete integral of (34), it follows that the *corresponding path and the time of its description are given by the equations:*

$$\frac{\partial A}{\partial \alpha} = \alpha_1, \quad \frac{\partial A}{\partial \beta} = \beta_1, \quad \frac{\partial A}{\partial H} = t + \varepsilon \tag{39}$$

where α_1, β_1, ε *are three additional arbitrary constants.*

By complete differentiation of (39) with respect to t through the three co-ordinates x, y, z we have at once:

$$\left.\begin{aligned} \frac{\partial^2 A}{\partial x \partial \alpha}\frac{dx}{dt} + \frac{\partial^2 A}{\partial y \partial \alpha}\frac{dy}{dt} + \frac{\partial^2 A}{\partial z \partial \alpha}\frac{dz}{dt} &= 0\,, \\ \frac{\partial^2 A}{\partial x \partial \beta}\frac{dx}{dt} + \frac{\partial^2 A}{\partial y \partial \beta}\frac{dy}{dt} + \frac{\partial^2 A}{\partial z \partial \beta}\frac{dz}{dt} &= 0\,, \\ \frac{\partial^2 A}{\partial x \partial H}\frac{dx}{dt} + \frac{\partial^2 A}{\partial y \partial H}\frac{dy}{dt} + \frac{\partial^2 A}{\partial z \partial H}\frac{dz}{dt} &= 1\,. \end{aligned}\right\} \tag{40}$$

Similar differentiation in respect to α, β, H, respectively, gives:

$$\left.\begin{aligned} \frac{\partial^2 A}{\partial \alpha \partial x}\frac{\partial A}{\partial x} + \frac{\partial^2 A}{\partial \alpha \partial y}\frac{\partial A}{\partial y} + \frac{\partial^2 A}{\partial \alpha \partial z}\frac{\partial A}{\partial z} &= 0\,, \\ \frac{\partial^2 A}{\partial \beta \partial x}\frac{\partial A}{\partial x} + \frac{\partial^2 A}{\partial \beta \partial y}\frac{\partial A}{\partial y} + \frac{\partial^2 A}{\partial \beta \partial z}\frac{\partial A}{\partial z} &= 0\,, \\ \frac{\partial^2 A}{\partial H \partial x}\frac{\partial A}{\partial x} + \frac{\partial^2 A}{\partial H \partial y}\frac{\partial A}{\partial y} + \frac{\partial^2 A}{\partial H \partial z}\frac{\partial A}{\partial z} &= 1\,. \end{aligned}\right\} \tag{41}$$

On comparing these two sets of equations, we find

$$\frac{dx}{dt} = \frac{\partial A}{\partial x}\,, \qquad \frac{dy}{dt} = \frac{\partial A}{\partial y}\,, \qquad \frac{dz}{dt} = \frac{\partial A}{\partial z}\,. \tag{42}$$

And as the first members of these equations represent the components of the velocity of the moving particle, it follows that the second members also represent the same thing. Accordingly the proposition stated after equation (34) above is established, and obviously applies equally to Light, Electrodynamic Action and Gravitation.

After this digression on the properties of HAMILTON's *Characteristic* Function* we resume the equation

$$\left(\frac{\delta \tau}{\delta x}\right)^2 + \left(\frac{\delta \tau}{\delta y}\right)^2 + \left(\frac{\delta \tau}{\delta z}\right)^2 = \frac{1}{v^2} \tag{31}$$

*This celebrated function was invented by HAMILTON for the treatment of *Light*, but if all physical forces depend on waves due to vibrations in atoms, with equatorial planes lying haphazard, or mutually inclined at various angles, — it will apply also to magnetism, gravitation, and all kinds of electrodynamic action. HAMILTON's Characteristic Function is therefore above all a *Wave Function*, equally applicable to all the forces of the Universe, and hence the fulness of this discussion.

And we see that if we can obtain a complete integral of this equation, containing therefore two arbitrary constants α, β, in the form

$$\tau = F(x, y, z, \lambda, \alpha, \beta) ; \tag{43}$$

then the derived equations

$$\left\{ \begin{aligned} \frac{\partial \tau}{\partial \alpha} &= \frac{\partial F}{\partial \alpha}(x, y, z, \lambda, \alpha, \beta) = \alpha' , \\ \frac{\partial \tau}{\partial \beta} &= \frac{\partial F}{\partial \beta}(x, y, z, \lambda, \alpha, \beta) = \beta' , \end{aligned} \right. \tag{44}$$

will represent two series of surfaces, whose intersections give the path of the light in the medium.

As α' and β' are also *arbitrary constants, the four constants* α, β, α', β' *are necessary and sufficient* for the purpose of making the two intersecting surfaces each pass through any two given points $p_0(x_0, y_0, z_0)$ and $p(x, y, z)$.

These Hamiltonian considerations on single refraction in non-homogeneous media show, as was originally found by FERMAT, that the actual path is that of *least time*, as well as of *least action.*

Now the *physical cause* of such action is known to be waves in the highly elastic aether, and propagated with unequal velocities, in different media, according to density, effective elasticity, and wave length. Increase of density, due to the presence of ponderable matter, hinders the progress of the wave of given length, while increase of elasticity under thinning of the matter accelerates it. And in general decreasing the wave length increases the retardation in velocity.

Equiactional surfaces, orthogonal to the path of light, are so distributed that the distances between them, for geometrical reasons, are always inversely as the velocity in the corresponding path.

Now it is clearly shown in BULLETINS 1, 2, 3, that *Electrodynamic Action* is conveyed by waves, traveling in free aether with the velocity of light, and therefore these waves will follow the same general laws as the waves of light. Such a physical cause necessarily takes the path of *least time* and of *least action,* which is also that of *least resistance* to the disturbances of the medium. And as the motions of the planets conform to these principles, the question may properly be asked whether any other cause than Electrodynamic Wave Action could be imagined to produce the attractions of the heavenly bodies.

According to the Newtonian Rules of Reasoning in Philosophy, it is obvious that all other causes would be excluded; but before reaching such an actual conclusion, we shall apply HAMILTON's method to the motions of a planet about the Sun. If the orbit be found to be a path of least action, in the least time, the presumption that Gravitation is a phenomenon of wave action will be confirmed by observed phenomena, and the result may safely be used as the basis of sound Natural Philosophy.

The following outline is based on TAIT's discussion, but the processes in the main are due to HAMILTON, yet also partly due to JACOBI (cf. JACOBI, *Vorlesungen Ueber Dynamik;* and TISSERAND's *Mécanique Céleste*, Tome I, Introduction and Chapter VII). JACOBI was a friend of HAMILTON, greatly interested in his mathematical methods, and in a public address at Manchester, England, at the end of June, 1842, referred to HAMILTON as "le LAGRANGE de votre pays." (Cf. GRAVES' *Life of Hamilton*, Vol. II, p. 388).

V INTEGRATION OF THE DIFFERENTIAL EQUATIONS FOR THE MOTION OF A PLANET ALONG AN ELLIPTIC ORBIT AS A PATH OF LEAST ACTION ILLUSTRATED BY HAMILTON'S METHOD FOR WAVES TRAVELING IN LEAST TIME: THE CASE OF WAVES TRAVERSING A SPHERE MADE UP OF CONCENTRIC SPHERICAL SHELLS OF UNIFORM DENSITY ILLUMINATES THE DECREASE OF GRAVITATION NEAR THE SHADOW OF THE EARTH.

(*i*) *The Elliptic Motion of a Planet illustrates* HAMILTON's *principle of Varying Action, quite as well as the Wave Theory of Light, — thus mathematically justifying the Electrodynamic Wave-Theory of Universal Gravitation.*

In BULLETIN No. 2, Section III, it is shown that in wave motion the *amplitude* $a = \frac{k}{r}$, and the *energy* of wave action varies as the *square of the amplitude*, $E = \frac{k^2}{r^2}$; which corresponds to the law of attraction found by GAUSS to hold for magnets (*Intensitas Vis Magneticæ*, 1833, §21) and by NEWTON for the motion of the planets under Universal Gravitation. These results indicate the adequacy of the Wave-Theory for explaining the attractive forces of Nature, both Terrestrial and Celestial; and the presumption therefore is that all the attractions and motions in Nature conform to the resulting physical laws. Yet there will be some advantage in verifying the inference that this may occur by a rigorous examination of the question from a purely geometrical point of view.

If $V = \frac{M}{r}$ (45) be the gravitational potential due to the Sun's attraction, then $\frac{\partial V}{\partial r} = -\frac{M}{r^2}$ (46) will represent the central force.

But in summing up the total energy of a system the potential energy is always taken with negative sign, $V = -\frac{M}{r}$. Using this value therefore the Hamiltonian partial differential equation (34) becomes*

$$\left(\frac{\partial A}{\partial x}\right)^2 + \left(\frac{\partial A}{\partial y}\right)^2 + \left(\frac{\partial A}{\partial z}\right)^2 = v^2 = 2\left(H + \frac{M}{r}\right) \tag{47}$$

If the plane of xy be taken as the plane of the planet's motion, the term in z disappears, and (47) reduces to

$$v^2 = \left(\frac{\partial A}{\partial x}\right)^2 + \left(\frac{\partial A}{\partial y}\right)^2 = 2\left(H + \frac{M}{r}\right) \tag{48}$$

And in the more convenient polar co-ordinates this takes the well known form

$$\left(\frac{\partial A}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial A}{\partial \theta}\right)^2 = 2\left(H + \frac{M}{r}\right) \tag{49}$$

This equation is satisfied by the values

$$\left\{\begin{aligned} &\frac{\partial A}{\partial \theta} = a\,, \quad \text{a constant}\,, \\ &\left(\frac{\partial A}{\partial r}\right)^2 = 2\left(H + \frac{M}{r}\right) - \frac{a^2}{r^2}\,. \end{aligned}\right. \tag{50}$$

Therefore, since the first of these equations gives

$$A = \int \frac{\partial A}{\partial \theta}\, d\theta = \int a\, d\theta = a\theta \tag{51}$$

and the second leads to

$$\left\{\begin{aligned} &\left(\frac{\partial A}{\partial r}\right)^2 dr^2 = dr^2\left[2\left(H + \frac{M}{r}\right) - \frac{a^2}{r^2}\right], \\ \text{or}\quad &\frac{\partial A}{\partial r}\, dr = \pm\, dr\sqrt{2\left(H + \frac{M}{r}\right) - \frac{a^2}{r^2}}, \end{aligned}\right. \tag{52}$$

we have for the positive value

$$A = \int \frac{\partial A}{\partial r}\, dr = \int dr \sqrt{2\left(H + \frac{M}{r}\right) - \frac{a^2}{r^2}} \tag{53}$$

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*Cf. P. G. Tait, Article *Mechanics*, *Ency. Brit.*, 9th Edition.

Therefore the *general integral* of (48) is

$$A = \alpha\theta + \int dr \sqrt{2\left(H + \frac{M}{r}\right) - \frac{\alpha^2}{r^2}} \tag{54}$$

And the final solution, found by differentiating this expression in respect to α and H becomes

$$\frac{\partial A}{\partial \alpha} = \alpha_1 = \theta - \alpha \int \frac{dr}{r^2\sqrt{2\left(H + \frac{M}{r}\right) - \frac{\alpha^2}{r^2}}} \tag{55}$$

$$\frac{\partial A}{\partial H} = t + \varepsilon = \int \frac{dr}{\sqrt{2\left(H + \frac{M}{r}\right) - \frac{\alpha^2}{r^2}}} \tag{56}$$

These equations involve four constants α_1, α, H, and ε, and the solution is complete (cf. WATSON'S *Theoretical Astronomy*, pp. 43-46). For the equation (55) defines the orbit, and (56) fixes the time in terms of the radius vector.

To verify this inference, we may take two new arbitrary constants defined by the relations

$$\left.\begin{aligned} \frac{M}{\alpha^2} &= \frac{1}{l}, \\ \frac{2H}{\alpha^2} &= \frac{(e^2-1)}{l^2}, \quad l = a(1-e^2). \end{aligned}\right\} \tag{57}$$

Then the equation (55) reduces to the form

$$\alpha_1 = \theta - \int \frac{dr}{r^2\sqrt{(e^2-1)/l^2 + 2/lr - \frac{1}{r^2}}} \tag{58}$$

$$= \theta - \int \frac{dr}{r^2\sqrt{e^2/l^2 - \left(\frac{1}{r} - \frac{1}{l}\right)^2}} = \theta - \cos^{-1}\frac{1}{e}(r^{-1} - l^{-1}) \tag{59}$$

And since $e\cos(\theta - \alpha_1) = \dfrac{l-r}{r}$ we have

$$r = \frac{l}{1 + e\cos(\theta - \alpha_1)}, \tag{60}$$

which is the general polar equation of the conic section referred to the focus as origin.

Again, if we differentiate (55) with respect to r we get

(61) $$d\theta = \frac{a\,dr}{r^2\sqrt{2\left(H + \frac{M}{r}\right) - \frac{a^2}{r^2}}}$$

or, when we have regard to (56), and (57),

(62) $$t + \varepsilon = \int \frac{dr}{\sqrt{2\left(H + \frac{M}{r}\right) - \frac{a^2}{r^2}}} = \frac{1}{a}\int r^2 d\theta = \frac{1}{\sqrt{Ml}}\int r^2 d\theta$$

This is the integral for the areas, which, by KEPLER'S Law, is constant; and therefore properly taken as the measure of the time from a fixed epoch ε, the time of perihelion passage.

(*ii*) *In the elliptic motion of a planet, the Time is measured by the area described around the lower focus, while the Action is measured by the area described about the upper focus.*

For we have

(63) $$\begin{cases} dt = \dfrac{1}{\sqrt{Ml}} \cdot r^2 d\theta = \kappa \cdot v \cdot p\,, \\[2ex] dA = v ds = \dfrac{h}{p} ds\,, \end{cases}$$

where p is the perpendicular distance from the focus upon the tangent to the ellipse, and h is constant.

Now we have also (cf. WILLIAMSON and TARLTON'S *Dynamics*, 1885, §166, p. 174) for the radius vector r' and perpendicular p' from the upper focus

(64) $$p = \frac{b^2}{p'}\,, \quad \frac{p'}{p} = \frac{r'}{r}\,.$$

Therefore we have

(65) $$\begin{cases} dt = \kappa v p\,, \\[1ex] dA = \dfrac{h}{b^2} ds \cdot p'\,. \end{cases}$$

And as $\frac{p}{p'} = \frac{r}{r'}$, we see that while dt depends on the area described about the lower focus, the element of the *action* dA on the other hand, depends on the area described about the upper focus.* This is a very curious result in the planetary theory, and shows that the Time and the Action are in a reciprocal relation as to the origins of their respective areas used as the basis of the measurements.

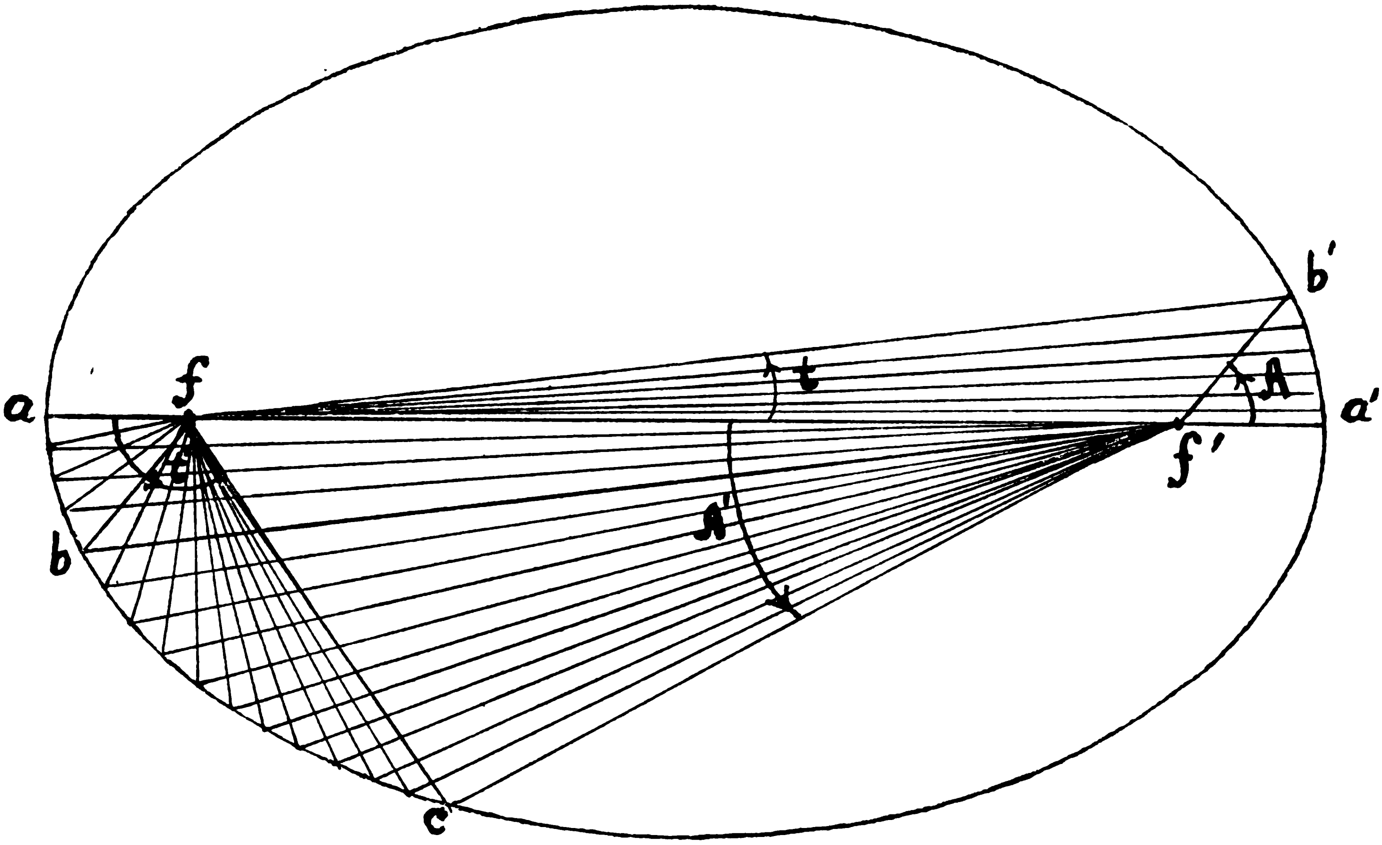

FIG. 4. Illustration of the *Areas* afc and $a'fb'$ described about the occupied focus f, which measure the times t' and t, according to KEPLER'S second Law; and of the *Actions* $a'f'b'$ and $af'c$ described about the empty focus f' in the intervals t and t'. The *Actions are very unequal*, as might be expected from the very unequal intensities of the central forces at perihelion and aphelion respectively, the product vds magnifying the difference in the ratio of the square of the forces.

The integrals between the instants t_0 and t are respectively:

$$\left.\begin{aligned} T &= \int_{t_0}^{t} r^2 d\theta = C(t - t_0)\,, \\ A &= \int_{t_0}^{t} (vds)\,dt\,. \end{aligned}\right\} \qquad (66)$$

*Cf. TAIT, Article *Mechanics*, *Ency. Brit.*, 9th edition.

Here C is the constant of areas, according to KEPLER's second law; and A may be calculated from the areas described about the empty focus as origin.

(*iii*) HAMILTON'S *method of Varying Action applied to Light of uniform wave length propagated through a medium arranged in concentric spherical shells of uniform density.*

We may assume that at a distance r from the centre of the sphere, the light will be traveling in the spherical shell of mass

$$dm = 4\pi\sigma r^2 dr \tag{67}$$

It thus appears that the density varies inversely as r^2, for we have

$$\sigma = \frac{dm}{4\pi dr} \cdot \frac{1}{r^2} \tag{68}$$

But the velocity decreases in denser media, so that it varies in the inverse ratio of the density, or directly as r^2. We may therefore take

$$v = \frac{b^2 + r^2}{c} \tag{69}$$

where b and c are absolute constants, such that at the centre of the sphere, where

$$r = 0,$$

$$v_0 = \frac{b^2}{c} \tag{70}$$

On account of symmetry, the path of any ray necessarily will be in a plane through the centre. Calling this plane that of xy we have

$$\left(\frac{\partial\tau}{\partial x}\right)^2 + \left(\frac{\partial\tau}{\partial y}\right)^2 = \frac{1}{v^2} = \frac{c^2}{(b^2 + r^2)^2} \tag{71}$$

In the more convenient polar co-ordinates this expression becomes

$$\left(\frac{\partial\tau}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial\tau}{\partial\theta}\right)^2 = \frac{c^2}{(b^2 + r^2)^2} \tag{72}$$

Of this equation see seek the general solution. If now we assume, as before,

$$\frac{\partial \tau}{\partial \theta} = \alpha \, , \tag{73}$$

then (72) gives

$$\frac{\partial \tau}{\partial r} = \sqrt{\frac{c^2}{(b^2 + r^2)^2} - \frac{\alpha^2}{r^2}} \tag{74}$$

And as (73) yields the first solution $\tau = \alpha\theta$, the equation (74) leads to

$$\tau = \int dr \sqrt{\frac{c^2}{(b^2 + r^2)^2} - \frac{\alpha^2}{r^2}} \tag{75}$$

Hence the complete solution from (74) becomes the *general solution:*

$$\tau = \alpha\theta + \int dr \sqrt{\frac{c^2}{(b^2 + r^2)^2} - \frac{\alpha^2}{r^2}} \tag{76}$$

The equation of the *path of the light* therefore is

$$\alpha' = \frac{\partial \tau}{\partial \alpha} = \theta - \alpha \int \frac{dr}{r^2 \sqrt{\frac{c^2}{(b^2 + r^2)^2} - \frac{\ ^2}{r^2}}} \tag{77}$$

$$= \theta - \cos^{-1} \frac{b^2 - r^2}{r \sqrt{\frac{c^2}{\alpha^2} - 4b^2}} \tag{78}$$

This gives at once

$$\frac{b^2 - r^2}{r} = \sqrt{\frac{c^2}{\alpha^2} - 4b^2} \cdot \cos(\theta - \alpha') \tag{79}$$

(*iv*) *The Paths of the Rays in Diametral Planes through the Concentric Spherical Shells are Circles.*

Accordingly, we conclude that the above equation represents a series of circles, with the following common property. Let b^2 be the square of the least half chord, through any point as cut by any one of the concentric spherical shells. Then it is shown in Geometry (cf. CHAUVENET'S *Geometry*, Book III, Prop. XVII,

p. 114) that this is equal to the product of the two segments $AO \cdot OA'$ in any other direction whatever,

(80) $$AO \cdot OA' = b^2$$

The points A and A' are *conjugate foci,* in respect to O, the common centre of the concentric spherical shells; for light going from either focus, will return to the other through the paths of the circle specified. In the theory of Light this remarkable result is of great interest in connection with the phenomena of the *Mirage.* In Electrodynamics it opens up a *physical theory* of the decrease of the Sun's gravitation in the shadows of the planets, and thus will throw new light on the *Unexplained Fluctuations of the Moon's Mean Motion first suspected by* NEWCOMB *in 1869, and definitely established by his last Researches in 1909.*

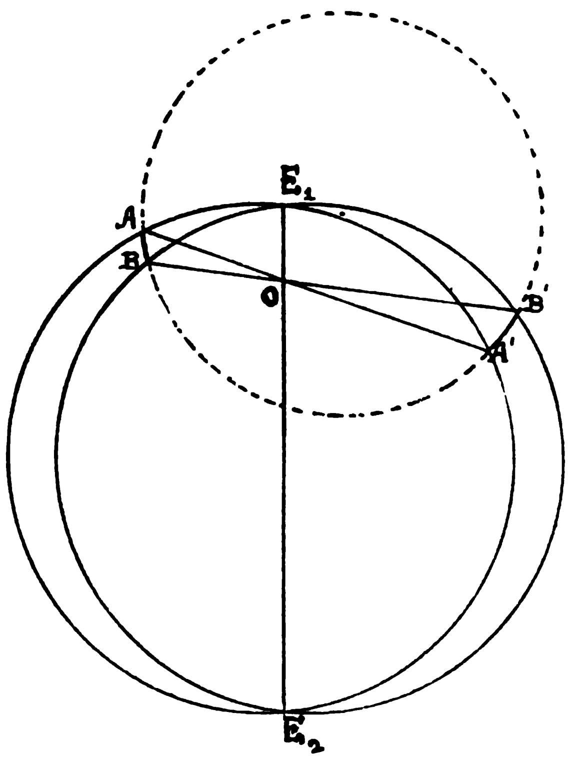

FIG. 5. MAXWELL'S Theory of the Circular Refraction of rays of light in the eye of a fish, which also applies to the Refraction of Electrodynamic Waves in traversing the Earth made up of concentric spherical shells with density increasing towards the centre.

From the above reasoning we see that every ray through a sphere made up of concentric spherical shells of uniform density, but with the density increasing towards the centre, will be confined to a diametral plane, and describe a circle; and a pair of conjugate foci will always lie on a line through the centre of the shells. MAXWELL discovered this remarkable result from the arrangement of the eye of a fish (*Cambridge and Dublin Mathematical Journal,* Vol. XI), but the present discussion follows the analysis given by Tait, in the Article, *Light*, *Encyc. Brit.*, 9th edition.

In the figure let O be the common centre of the spherical shells of equal refractive index; then obviously, every ray from A describes a circle which passes through A', and AOA' is a straight line through the centre. A and A', B and B' are *conjugate foci;* and by Geometry the common property is:

$$\left.\begin{array}{l} AO \cdot OA' = b^2 , \\ BO \cdot OB' = b^2 , \quad \text{etc.} \end{array}\right\} \tag{81}$$

If we suppose an eye situated in the layer at E_1, in the small circle of density σ_1, about the centre, it will receive the light from A along the tangent to the larger circle through AE_1A', and similarly for any other point B, seen along the tangent to the circle BE_1B'.

These images will be *erect*, for they have not yet passed through the *conjugate foci*, A', B'; but if the eye now be turned in the opposite direction, towards A', B', the light will have traversed the longer paths AE_2A', BE_2B', and enter the eye after turning its course through 180° and passing through the conjugate foci A', B', so that *the image will be inverted* and moreover represent the *back of the object.* It is obvious from the figure why this results — namely, the circuitous course to the *opposite* focus produces *inversion*, and the light so proceeding is from the *back* of the objects at AB.

(*v*) *Description of the paths of Gravitational Waves propagated through a Heterogeneous Sphere made up of concentric spherical shells of uniform density.*

If the gravitation of the Sun pass through the Earth, in the form of Electrodynamic Waves, it will obviously suffer some decrease in and near the shadow of the Earth; and thus operate to disturb the Moon during eclipses, as pointed out by Dr. K. F. Bottlinger in his crowned Prize *Inaugural Dissertation*, Munich, 1912 (c. Troemer's Universitäts Buchhandlung, Freiburg, 1912).

The above outline of the Hamiltonian theory of refraction in a sphere made up of concentric spherical shells of uniform density, but increasing in density towards the centre will enable us to see that in passing through the globe the Electrodynamic Waves must necessarily be somewhat diverted from a rectilinear course, and therefore the effect would be to decrease the Sun's gravity on the Moon when near the Earth's shadow.

This new physical theory thus confirms Bottlinger's researches, indicating that the Sun's gravitation is less near the anti-solar point; and the subject therefore becomes one of such high interest in connection with the Lunar Theory that it deserves the most careful investigation.

As the waves are refracted through the Earth, they are partially turned aside in circles of various radii. Within a curved conical area having its vertex at the Earth's centre and extending beyond the Moon's orbit, the gravity of the Sun is decreased. This physical refraction of part of the Gravitational Waves

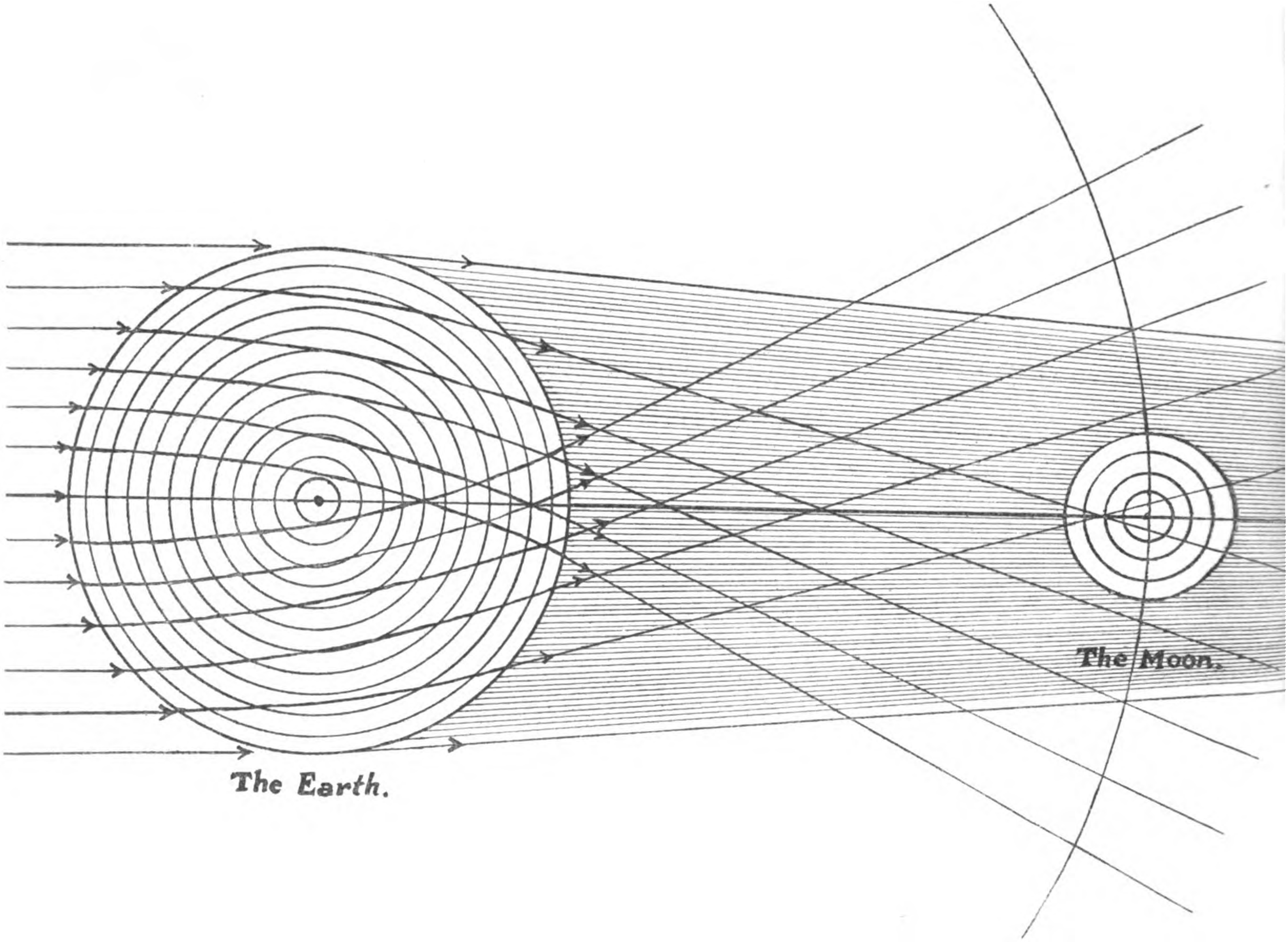

FIG. 6. Illustration of the Refraction of the Electrodynamic Waves from the Sun as they traverse the globe of the Earth, and are partially refracted to either side, before reaching the Moon, so that the Sun's effective gravity is decreased near the shadow of the Earth; and the result is periodic Fluctuations in the Moon's motion depending on the *Saros* and other cycles connected with the movements of the Perigee and Nodes of the Lunar Orbit.

is illustrated roughly by the accompanying figure, which for the sake of simplicity is restricted to uniform wave length, and thus omits the effects of dispersion.

VI Analysis of the Secular Effects of the Circular Refraction, Dispersion and perhaps Absorption of Gravitational Waves Propagated Through the Concentric Homogeneous Spherical Shells Composing the Earth.

(*i*) *Cause of the circular refraction, dispersion and perhaps absorption of rays.*

From the foregoing mathematical theory and the accompanying figure of the law of density in the Earth (*A.N.* 3992) $\sigma = \sigma' \frac{\sin (qx)}{qx}$ (82) we see that in passing through the Earth the course of the gravitational waves will be rectilinear,

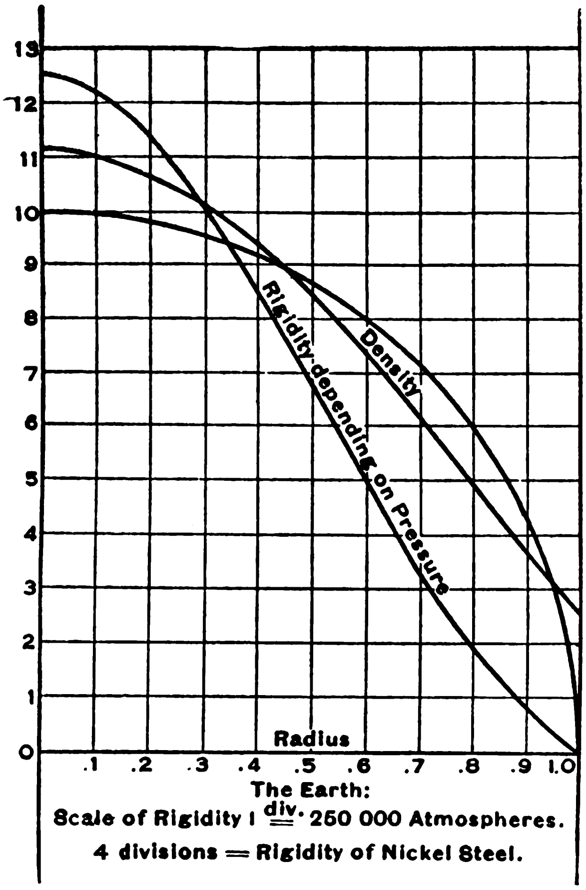

Fig. 7. Curves of Density, Pressure and Rigidity within the Earth, according to Laplace's Law. (*A. N.* 3992.)

except for the theoretical circles described by the waves after they enter the terrestrial globe. Thus the right lines described by the waves from the Sun begin to curve after they enter the Earth. Then, after traversing circular paths about points at considerable but unknown distances from the Earth's centre, the waves again enter free space for a second series of rectilinear paths.

This outline of the paths presupposes that the waves are of uniform length, (so that dispersion may be neglected for the time being) and the density of the Earth is arranged in concentric spherical shells, each one homogeneous, but the series having an increase of density towards the centre. In this case there certainly will be *circular refraction* of the wave paths; also dispersion for different wave lengths; and there may be *absorption* of some of the energy of wave motion.

If σ be the density in any layer of thickness ε the absorption of a ray A_0 by the usual theory becomes

$$A = A_0 e^{-\kappa \int_0^\varepsilon \sigma ds}, \tag{83}$$

where κ is the constant of absorption for the density $\sigma = 1$, ds is the element of the path, and σ may also be a function of s.

Our knowledge of the Earth shows that σ changes steadily, so that the exponential integral $\kappa \int_0^\varepsilon \sigma ds$ must be quite small. Accordingly we may take the first term as a sufficient approximation to the exponential series. This leads to

$$A = A_0 e^{-\kappa \int_0^\varepsilon \sigma ds} = A_0(1 - \kappa) \int_0^\varepsilon \sigma ds = A_0 - A_0 \kappa \int_0^\varepsilon \sigma ds \tag{84}$$

Hence, we may consider that an opposing force arises of strength

$$A_0 \kappa \int_0^\varepsilon \sigma ds \tag{85}$$

and for the case of constant density σ_0, we get

$$A_0 \kappa \int_0^\varepsilon \sigma ds = A_0 \kappa \sigma_0 \Gamma \tag{86}$$

It therefore appears that for a body such as our Moon, in or near the shadow of the Earth, there may be an *absorption of gravitational energy, even in rectilinear transmission;* and also *circular refraction and dispersion of the rays as explained above.* Now the *circular refraction and dispersion* of rays will extend somewhat beyond the Earth's shadow, and as the *rays absorbed* would come not from a mathematical point, but from any part of the Sun's disc, it follows that the absorption, refraction and dispersion would also be diffused over the penumbra as well as the umbra. Yet the conical region in which gravity would be diminished by this thinning out of the Sun's electrodynamic waves would have its base on a circle about the anti-solar point.

The result of this double and perhaps treble decrease in the intensity of the Sun's gravitation, when the Moon is near the shadow of the Earth, would be to cause it to move while passing through this region as if acted upon by a slight *repulsive force, directed from the centre of the Earth.* For at this time the Earth is essentially in line with the Sun, which always exercises the preponderant influence on the Moon's motion.

We here consider only the disturbing influence on the Moon's radius vector, as this alone affects the semi-axis major a and the mean motion n. For this disturbance in the radius vector alone introduces into the Moon's mean motion a term in the mean longitude recurring with each eclipse, and therefore having an accumulative effect which in time might become very sensible to observation.

(*ii*) *Formulæ for expressing the secular Effect of the Refraction, Dispersion and perhaps Absorption of Gravitational Wave Energy.*

If R be the component of the *disturbing force,* in the direction of the radius vector, and the other components vanish, and L be the mean longitude, we have for the perturbation of this element the well known formula (cf. TISSERAND'S *Mecanique Celeste,* Tome I, p. 433)

$$\frac{dL}{dt} = n_0 - \left\{ 2r + \frac{ae\sqrt{1-e^2}}{1+\sqrt{1-e^2}} \cos v \right\} \frac{an}{\nu} R + \int \frac{an}{dt} dt \tag{87}$$

where $\nu = \frac{m'}{1+m}$ the ratio of the Moon's mass to that of the Sun and Earth.

In this expression the disturbing force is an impulse in the direction of the radius vector. To judge of the effect of such a disturbance, we recall that any co-ordinate χ of the place of the body — computed from assumed elements of an elliptic orbit — is defined by the functional relation

$$\chi = F(\pi, \Omega, i, \varphi, M_0, n)\,, \tag{88}$$

and *the variation* given by the equation:

$$\chi' - \chi = \frac{d\chi}{d\pi}\Delta\pi + \frac{d\chi}{d\Omega}\Delta\Omega + \frac{d\chi}{di}\Delta i + \frac{d\chi}{d\varphi}\Delta\varphi + \frac{d\chi}{dM_0}\Delta M_0 + \frac{d\chi}{dn}\Delta n \tag{89}$$

But as the disturbing impulse R is sensible chiefly near the anti-solar point, where the eclipses of the Moon occur, the variations of most of the elements are periodic, and in the long run vanish. On account of the *symmetry of the impulses during a long interval,* in which whole eclipse cycles are completed, we may therefore neglect the variations in π, Ω, i, φ, M_0; and thus there remains only the last terms in (87) and (89), which involve the mean motion or mean longitude.

This integration of the term in the mean motion is to be extended over the period of the eclipses or appulses near the anti-solar point; for after the passage through or near the shadow of the Earth the variation in the mean motion ceases, like the impulses acting on the Moon.

The mean motion is defined by the equations

$$(90)\qquad \left\{\begin{aligned} & n = \frac{k\sqrt{\nu}}{a^{3/2}}\,, \quad \frac{dn}{dt} = -\frac{3}{2}\frac{n}{a}\frac{da}{dt}\,, \\ & -\frac{3}{2}\int \frac{n}{a}\frac{da}{dt}\,dt = +\int \frac{dn}{dt}\,dt\,. \end{aligned}\right.$$

Therefore from equation (87) we have at once

$$(91)\qquad \frac{dL}{dt} - n_0 = \int_0^{\varepsilon} \frac{dn}{dt} = \Delta n\,,$$

in which ε denotes the interval of the eclipse or appulse at near approach to the Earth's shadow in space.

And the change in the mean longitude for many eclipses or appulses follows from the second integration

$$(92)\qquad \Delta L = L - L_0 = \int_0^t \int_0^{\Sigma\varepsilon} \frac{dn}{dt}\,dt\,dt\,,$$

where the limits of the integral are the beginning of the appulse and the origin of the time, $\Sigma\varepsilon$ being the duration of all the appulses.

In order to evaluate this double integral for the change in mean longitude, we notice, as pointed out by Bottlinger, that it is made up of two parts: the double integral during the duration of the eclipses or near approaches to the shadow of the earth, and the expression for the direct changes in the integral

$$(93)\qquad \int_{t'}^{t} \Delta n dt = \Delta n\,(t - t')\,,$$

where t is the end of the appulse.

As we have for any one appulse, viewed as a momentary impulse,

$$\Delta n = \int \frac{dn}{dt} dt = -\frac{3}{2} \int \frac{n}{a} \frac{da}{dt} dt = -3 \int \frac{e \sin v}{a\sqrt{1-e^2}} Rdt \tag{94}$$

(cf. WATSON'S *Theoretical Astronomy*, p. 519) we may regard e, a, v as constant and equal to the values at the middle of the appulse. Therefore

$$\Delta n = -\frac{3e \sin v}{a\sqrt{1-e^2}} \int_0^{\epsilon} Rdt \tag{95}$$

(*iii*) BOTTLINGER'S *method of calculation also adapted to appulses without eclipses.*

BOTTLINGER, whose analysis we have here followed, calls the quantity $\int Rdt$ the impulse, and designates it by J. Hence by means of (93) we may write

$$L - L_0 = -\frac{3e \sin v}{a\sqrt{1-e^2}} J(t - t') \tag{96}$$

And for the accumulated effect of a series of appulses we have for equation (92) the form

$$\Delta L = L - L_0 = -\sum_{i=1}^{i=i} \frac{3e \sin v_i}{a\sqrt{1-e^2}} \cdot J_i(t - t') \tag{97}$$

And as a and e are constant for all eclipses and near appulses, we may write this expression

$$\Delta L = L - L_0 = -\frac{3e}{a\sqrt{1-e^2}} \sum_{i=1}^{i=i} \sin v_i J_i(t - t') \tag{98}$$

In using this formula, we have to substitute individual values for the several eclipses and appulses, and sum up the values after multiplying by the interval $(t - t_i)$. This is not very convenient, and we may shorten the operation by computing the change of mean motion Δn for each eclipse or appulse; then, on summing up the Δn-series we obtain the active deviations of the mean motion between each two eclipses or appulses. The terms of this first summed series each multiplied by $(ti - t_{i-1})$, the interval between two eclipses or appulses, when again summed up, give the sum of the accumulated perturbations in longitude. Accordingly, from equation (94) using $J = \int R dt$ we have

$$\Delta n = n - n_0 = -\frac{3e \sin v}{a\sqrt{1 - e^2}} J \tag{99}$$

And since the integral J always has the same positive fore sign, we find that when $0° < v < 180°$, or the Moon in appulse is going from perigee to apogee, the mean motion n is diminished; but when $180° < v < 360°$, or the Moon in appulse is going from apogee to perigee, the mean motion n is increased. Hence if the Moon is receding from the Earth the appulse retards it, but if it is approaching the Earth the appulse accelerates it.

Proceeding along the line of thought here sketched, BOTTLINGER has calculated the effect of a long series of eclipses by means of the formula

$$\Delta n_{\iota} = -\frac{3e}{a\sqrt{1 - e^2}} \sin v_{\iota} J_{\iota} \tag{100}$$

and tabulated the results (cf. Prize *Inaugural Dissertation*, 1912, pp. 7–9). His method was admittedly approximate, yet he found theoretical periodic oscillations of the Moon's longitude closely agreeing with NEWCOMB's observed data, 1843, 1861, 1880; but departing from the NEWCOMB curve after 1895.

BOTTLINGER points out that his method is inadequate, in that the Moon's orbit is always changing its shape and position, owing to oscillations of the line of apsides, which may amount to $\pm 30°$. This uncertainty may introduce an error of fifty per cent. in the calculated effect of any eclipse or appulse.

In fact, BOTTLINGER did not suspect that appulses could have any effect unless there was an eclipse; and as we now recognize that appulses without contact with the Earth's shadow may exert almost as much influence as eclipses, we see that the whole problem must be more profoundly investigated, at some future time, and in a way to allow for the effects of both eclipses and appulses.

As BOTTLINGER's researches can hardly fail to convince any impartial investigator that in part at least they rest on a true physical cause, we are not now called upon to elaborate the details of the reduction of gravitation near the shadow of the Earth. We may take such a result for granted, and especially in view of the theoretical refraction and dispersion of electrodynamic waves brought to light in the present investigation.

Conclusion

From this investigation of the Transmission of the Force of Gravitational Attraction across space and through the solid masses of the planets, it is clear that the Electrodynamic Wave-Theory will explain all known phenomena. This theory definitely makes known to us both mathematical and physical grounds for holding that a refraction, dispersion and perhaps absorption of gravitational energy must take place when the electrodynamic waves traverse the solid body of a planet like the Earth.

Such deflected wave energy is not lost in the Universe, but there is a slight redistribution of it in space, by refraction and dispersion, and perhaps reduction in intensity by absorption, if it is transformed into different wave lengths; so that the resulting intensity of the Sun's action is decreased near the shadow of the Earth. The Moon thus experiences a partial release from the Sun's control in this region, as if repelled from the Earth by a small repulsive force, which Newcomb concluded in 1909 would be adequate to explain the *Fluctuations*. In Bulletin No. 5 we shall give details to show that this theory is more rigorously verified by exact investigation than Newcomb or any one else heretofore could have believed possible.

The present theory of the Transmission of the Force of Gravity thus fulfills all necessary mathematical and physical criteria, whether the path of action be in free space, or through solid masses. Fermat's condition is complied with by the Sun's gravitation: otherwise the *Lunar Fluctuations* would not be what they are found to be by observation. The Sun's gravitation is therefore transmitted in the form of Electrodynamic Waves, which thus take Fermat's minimum path, under the conditions imposed by the principle of Least Action.

Newcomb's conclusion of 1909, that the tracing of the *Lunar Fluctuations* to a variation in the attraction between the Earth and Moon would involve an expenditure or absorption of energy somewhere in the solar system — which then seemed to him difficult to admit — now becomes in fact one of the most rigorous proofs of the actual *conservation of energy*. It would not be possible for the Sun's gravitational waves to traverse the body of the Earth without a slight change in the field of force in the region beyond, through refraction, dispersion and perhaps absorption; *yet this redistribution of the wave energy does not alter its total amount, though the effective gravitational action on the Moon is decreased.*

If Gravitation were due to any other cause than electrodynamic waves, it seems certain that the resulting action through the solid mass of the Earth could not rigorously fulfill the severe geometrical and physical conditions outlined in the foregoing discussion. But as the Moon's *Fluctuations* actually are observed to fulfill these severe requirements of the Electrodynamic Wave-Theory, it incontestibly follows that this Theory alone accords with the known laws of Nature.

T. J. J. SEE

Starlight on Loutre,
Montgomery City, Missouri, June 6, 1917.

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RESULTS OF RESEARCHES

ON THE

ELECTRODYNAMIC WAVE-THEORY OF PHYSICAL FORCES

BULLETIN NO. 5

THEORY OF THE FLUCTUATIONS OF THE MOON'S MEAN MOTION DEDUCED FROM PHENOMENA: RESULTS EXPLAINED BY THE REFRACTION, DISPERSION AND PERHAPS ABSORPTION OF PART OF THE SUN'S ELECTRODYNAMIC WAVE ENERGY IN PROPAGATION THROUGH THE EARTH.

By T. J. J. SEE

I Introductory Remarks

When the first Bulletin of this series was completed, February, 1917, it did not seem likely that an urgent demand would soon arise for the publication of the details of the author's processes of discovery in regard to the *Fluctuations* of the Moon's Mean Motion. But the persistence of almost total darkness in certain quarters which ought to be sources of Light, and the danger that the withholding of the processes that led to the discovery of the Physical Cause of the Lunar *Fluctuations*, might contribute still further to the diffusion of error in the Physical Sciences, has led to the impression that it would be better to publish without further delay an outline of the chief processes involved.

In entering upon such a course, for the diffusion of ideas appropriate to the facts of Nature, one has to reckon on the probable persistence of Error now entrenched in the seats where Truth ought to prevail. Whatever the outcome of the impending struggle, this disposition to persist in known error, by reason of false pride, probably will not be worse than that witnessed against the author's efforts of 1909-10 to place Cosmogony on a scientific basis, which at length has borne fruit in the reluctant but open recognition that practically all the previous work on this extensive subject was unsound, and that the new cosmogonic processes set forth in the Capture Theory are correct.* Eight years is not long to wait for the triumph of the laws which disclose to us the sublime processes underlying the formation of the Universe.

Thus the present prevalence of error in regard to the problems of the Moon's *Fluctuations* and the cause of Universal Gravitation is not a source of discouragement, but rather an occasion for genuine gratitude that it is possible for any one to shed light on such difficult subjects, heretofore literally veiled in the impenetrable darkness of night. For I am confident that if the present Bulletins be studied by fellow-laborers interested in Truth, they will not dwell on the unavoidable defects of the pioneer treatment of this difficult and endless subject, but will rather seek to improve the imperfect outline now developed in treating of the sublimest of NEWTON's unfinished problems — the Cause of Universal Gravitation.

It usually happens that when a way through rough places is once opened by the pioneer, improvements can be made in the greater leisure of those who follow. For as the attention of the follower is less occupied in opening the way, he naturally is enabled to give more attention to the improvement of the detailed processes. Yet this refinement is never possible till the pioneer work is done; and thus no apology is required for the somewhat audacious course which it was necessary to adopt to find a way out of the wilderness by which we have been surrounded.

If there be others equally daring and more fortunate — *andax fortuna adjustus*, as GAUSS said of LEVERRIER's independent leadership, anticipating ADAMS in the matter of the discovery of *Neptune* — who may add to the discoveries here outlined, I shall heartily welcome such effort. HIPPARCHUS began the investigation of the *Evection*, the first of the Lunar Inequalities, about 140 B. C., but left it to be completed by PTOLEMY nearly three centuries later. In the course of ages some seventy more sensible Inequalities in the Moon's Longitude have been discovered, and at length explained theoretically by the series of eminent mathematical astronomers who have flourished since the epoch of NEWTON.

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*POINCARÉ, DARWIN, JEFFREYS, and many other eminent mathematicians have confirmed the Capture Theory. In the *Monthly Notices* for January 1917, p. 199, Dr. J. H. JEANS concludes that weighed in balances unduly weighted to favor the rotational theory of LAPLACE "the hypothesis has been found wanting."...."we can say with some confidence that the rotational theory cannot explain the origin of the Solar System."

Many readers will remember that in *A. N.* 4308, Feb. 1909, LAPLACE's *hypothesis was not only shown to be wrong, on perfectly unassailable dynamical grounds, but the much more difficult constructive work of developing a valid theory outlined* (cf. *A. N.* 4341, 4343, and my *Researches on the Evolution of the Stellar Systems,* Vol. II, 1910). Just why English thought should so often lag years behind that of the rest of the world is not apparent.

The Lunar *Fluctuations* are the last of the outstanding anomalies in the motion of the Moon, of which the treatment was begun by HIPPARCHUS over 2,000 years ago. If others add to the present efforts, as PTOLEMY did to those of HIPPARCHUS, and thus contribute to the perfection of the Theory of Universal Gravitation, — which should now become possible, in view of the insight acquired into the physical cause underlying this chief force of Nature — sensible discrepancies will quite disappear from our Theories of the Celestial Motions.

But in outlining the new theory of the Lunar *Fluctuations* it becomes advisable to glance at the recent progress of the subject, and above all to point out the steps by which Professor NEWCOMB established the existence of these hitherto *Unexplained Fluctuations.*

PRINCIPAL RESULTS OF NEWCOMB'S LAST RESEARCHES ON THE MOON'S MEAN MOTION, 1909

It is well known that just before his death, July 11, 1909, the late Professor SIMON NEWCOMB, by well nigh incredible effort, was able to bring to a satisfactory conclusion a series of researches on the Motion of the Moon which had seriously occupied his attention for over forty years. This last work of NEWCOMB was published in the *Astronomical Papers of the American Ephemeris and Nautical Almanac*, 1912, Vol. IX, part I, under the title: "Researches on the Motion of the Moon and related Astronomical Elements based on observations extending from the era of the Babylonians until A.D. 1908."

But prior to the appearance of this posthumous Memoir, NEWCOMB had published the chief results of his researches in the *Monthly Notices* of the Royal Astronomical Society for January, 1909. In this paper he states that he had now revised the results of his *Researches on the Motion of the Moon*, 1878, and that altogether the data used in his latest researches cover a period of more than 2,600 years. The observations utilized embrace:

1. The eclipses of the Moon recorded in PTOLEMY'S *Almagest*, observed between 720 B.C. and 134 A.D.

2. The observation of eclipses by Arabian astronomers, between 829 and 1004 A.D.

3. The observations of eclipses of the Sun and of occultations of stars by the Moon, made by GASSENDI, HEVELIUS, and others, between 1620 and 1680.

4. Observations of occultations of stars by the Moon from 1670 to 1908.

The observational material is thus very complete, but naturally not of equally good quality in the different periods. After 1680 the modern records offer a fair degree of precision, but frequent gaps in the observations occur during the last half of the 18th century; and it is only since 1820, and especially 1829, that the records are fairly continuous, and of the increasing accuracy attained in our own time.

The recent exhaustive researches of Professor E. W. BROWN appear to determine with precision the action of every known mass of matter upon the Moon, and seem to prove beyond serious doubt the reality of the large *Unexplained Fluctuations* in the Moon's motion to which Professor NEWCOMB had called attention at various times during the past forty years.

HANSEN's researches in the Lunar Theory had extended over many years, and the resulting Tables of the Moon, published by the British Government in 1857, had been prepared with great care. Accordingly it was confidently expected that they would accurately represent the Moon's motion for an almost indefinite time. Yet after they had been issued only a few years the Moon began to depart from the assigned longitudes so conspicuously as to occasion great surprise among astronomers. It appears that NEWCOMB began the serious study of the Moon's motion about 1868, and in 1869 he first called attention to the inconsistencies between theory and observation.

In 1871 NEWCOMB visited Europe, and with DELAUNAY's active co-operation, searched for several months in the archive records of the Paris Observatory for old observations which would illuminate the motion of the Moon during the two preceding centuries. The search was very successful, and gave NEWCOMB's *Researches on the Motion of the Moon*, 1878, a high degree of precision; so that in 1909 he justly remarks that his latest researches modify but slightly the conclusions reached in the work of 1878.

The abstract of the Memoir of 1912 included in the *Monthly Notices* for January, 1909, is accompanied by Tables, and a diagram showing graphically the effect of the total Fluctuation. The significance of NEWCOMB's last work on the motion of the Moon can scarcely be overrated, because he was the recognized master of the subject, and combined with his practical discussion of observations also the results of BROWN's careful revision of the purely mathematical part of the Lunar Theory.

Taking it for granted that BROWN's exhaustive researches had allowed for the action of all known bodies, and that no unknown bodies exist, NEWCOMB concluded by saying: "I regard these fluctuations as the most enigmatical phenomenon presented by the celestial motions, being so difficult to account for by the action of any known causes, that we cannot but suspect them to arise from some action in nature hitherto unknown."

After discussing the various possible known causes, such as tidal friction, change in the Earth's rotation, etc., which might modify the Moon's motion by amounts not yet calculated accurately, NEWCOMB doubts their adequacy to meet the difficulty, and finally adds: "The preceding suggestions seem to me to include every known cause of action. If we pass to unknown causes and inquire what is the simplest sort of action that would explain all the phenomena, the answer would be — a fluctuation in the attraction between the Earth and Moon. Accepting the law of the conservation of energy, such a fluctuation would involve an expenditure or absorption of energy somewhere in the solar system, which it seems difficult to admit. Precisely what changes of gravitation would be required I have not yet computed; but it seems quite likely that they would be below any that could be determined by experimental methods on the Earth. It would be natural to associate them with the Sun's varying magnetic activity and the varying magnetism of the Earth; but I cannot find that we have any data on this subject which would enable us to base any law upon varying magnetic action. At present I see nothing more to do than to invite the attention of investigators to this most curious subject."

"One general result of the present state of things is that we cannot draw any precise conclusions from a discussion of the Moon's motion in longitude, how refined soever we may make it. For example, it is impossible to derive from observation the accurate coefficient of the 18.6-year nodal inequality in longitude, owing to the varying fluctuation."

"It is also not possible to predict the future motion of the Moon with precision. If we require our ephemerides of the Moon's longitude to be as exact as possible, we must correct the tabular mean longitude from time to time by observations."

II Analysis of the Four nearly Commensurable Cycles Connected with the Recurrence of Eclipses

The principal eclipse cycles, incessantly repeated in theory of the Moon's motion, are the following:

1. The *Saros*, made up of 223 Synodic months = 6585.32 days, discovered by the Chaldeans and used at Babylon for predicting the return of eclipses, in conjunction with the *eclipse year* of 346.62 days.

2. The *eclipse year* of 346.62 days, the average time of the Sun in passing around the heavens from the Moon's node and returning to the same node again as it retrogrades under the Sun's disturbing action in 18.6 years. Nineteen of these eclipse years make 6585.78 days, almost exactly equal to the cycle of the *Saros* given above, which is 6585.32 days.

The difference in these two periods is only 0.46 of a day, and therefore after 18 Julian years 10.82 days ($0^d.46$ less than 19 *eclipse years*) the *Saros* of eclipses is very nearly repeated, except that the location on the terrestrial globe is $0^d.32 = 7^h\,40^m\,48^s$ further west in longitude.

3. The *Nodical or Draconitic month* made up of $27^d.21222$; and thus $242 \times 27^d.21222 = 6585^d.357$. This again is of almost the same length as the 223 synodic months and 19 eclipse years defined in paragraphs 1 and 2 above.

4. The *Anomalistic month* made up of $27^d.55460$; and thus $239 \times 27^d.55460 = 6585^d.549$. Accordingly, after 223 months the Moon not only returns very closely to its original position in respect to the Sun and node, but also in respect to the line of apsides of the Moon's orbit; so that the perturbations near perigee, during the interval of the difference in these two cycles, $6585^d.549 - 6585^d.32 = 0^d.229 = 5^h\,29^m.76$ are so small as to modify but very slightly the return of the cycle of eclipses composing the *Saros*.

Accordingly, these four fundamental Lunar cycles recur in the following periods:

$$
(1)\qquad \left\{\begin{array}{lll}
1. & \text{The } \textit{Saros} = 223 \text{ synodic months} & = 6585^d.32, \\
2. & 19 \text{ eclipse years of } 346^d.62 \text{ each} & = 6585^d.78, \\
3. & 242 \text{ Nodical or Draconitic months of } 27^d.21222 \text{ each} & = 6585^d.357, \\
4. & 239 \text{ Anomalistic months of } 27^d.55460 \text{ each} & = 6585^d.549.
\end{array}\right.
$$

Now the *Saros* = $6585^d.32$ = 18 Julian years 10.82 days, or 18.0293 sidereal years of $365^d.2563582$ (HANSEN). And according to NEISON the period of the circulation of the Lunar Perigee is 8.855 years. In the 10th edition of his *Outlines of Astronomy*, 1869, p. 472, Sir JOHN HERSCHEL uses the period $3232^d.575343$ = 8.85031 Julian years, which is only slightly different from the value cited above.

Accordingly, the forward motion of the Perigee will carry it twice around the heavens in 17.71 years, while the Node revolves in the retrograde direction in 18.6 years. Thus if we call Ω the yearly motion of the Node, and ϖ the corresponding motion of the Perigee, we have

$$
(2)\qquad \begin{aligned}
\Omega &= -19^\circ.35484 = -360^\circ / 18.6\,, \\
\varpi &= +40^\circ.6550 = +360^\circ / 8.855\,.
\end{aligned}
$$

It may be well to notice that the foregoing results, for the chief Lunar cycles, were determined with very remarkable accuracy before the time of HIPPARCHUS, about 140 B.C. Full details of the methods of calculation developed at Alexandria are given in PTOLEMY'S *Almagest*, Book IV; and, as has been justly observed by many eminent authorities, exhibit an astonishing insight into the Theory of the Moon's motion.

In the *Almagest*, Book IV, Chapter 2, PTOLEMY deals with the Moon's Periodic Time, and shows how the Lunar cycles may be deduced from the observations of eclipses recorded at known intervals in years, months and days. From somewhat superficial estimates he says the older observers made the *Saros* interval 6585 1-3 days (that is, 18 solar years, 10 5-6 days). This corresponds to 223 Lunations, 239 returns of the Anomaly, 242 returns of the Latitude, 241 revolutions of the Longitude, with 10 2-3 degrees left over, which the Sun traverses in 18 circuits, wherewith the restoration of the Sun and Moon is complete in respect to the fixed stars.

In order to make these cycles commensurable with whole days, PTOLEMY says they multiply by 3, which gives 19756 days, corresponding to 3 *Saroses*, in 669 Lunations, 717 returns of the Anomaly, 726 returns of the Latitude, 723 returns of the Longitude, with 32 degrees left over (exactly $32°12'36''$), which the Sun completes in 54 circuits (54 Egyptian years and 46 days).

PTOLEMY remarks that by comparing the Chaldean records of eclipses with those observed in his own time HIPPARCHUS found that these periods are not exact; but the commensurability is more precise for a period of 126007 days and 1 equinoctial hour (345 Eygptian years, 82 days and 1 hour). In this period he finds 4267 Lunations completed, 4573 returns of the Anomaly, 4612 circumferences of the ecliptic, less 7½ degrees, which the Sun falls short of completing in 345 circuits, to restore the Sun and Moon to the same situation in respect to the fixed stars. He finds the Moon's period by dividing $126007^d.041666$ by 4267 Lunations, and gets 29.5305939 mean solar days, which agrees with our present value, 29.53059 days.

Already in the early days of the Alexandrian School, it was recognized that in 6585 1-3 days there were 241 revolutions of the Moon in respect to the fixed stars, but only 239 revolutions with respect to the perigee or anomaly. Hence the length of the anomalistic month admitted of accurate calculation; and similar reasoning applies to the duration of the Synodic, the Sidereal, and the Nodical or Draconitic months. The near coincidence of 19 eclipse years with the *Saros* of 223 lunations doubtless was fully understood by the Babylonians, from whom HIPPARCHUS obtained records of the earliest eclipses preserved by PTOLEMY.

By comparing the data of three Lunar eclipses observed at Babylon in the 366th and 367th years of the era of NABONASSAR (*Almagest*, Book IV, Chap. 10) with those of three others accurately observed at Alexandria in the 547th year of the same era of NABONASSAR, HIPPARCHUS was enabled to determine the eccentricity and apogee of the Moon's orbit at epochs separated by 180 years; and to find from the positions of the instantaneous orbits at these distant epochs the rate of the progressive revolution of the orbit in space. The three early Lunar eclipses thus used were those of Dec. 23, 383 B.C., June 18, 382 B.C., and Dec. 12, 382 B.C.; and the three later Lunar eclipses, those of Sept. 22, 201 B.C., March 19, 200 B.C., and Sept. 12, 200 B.C., (cf. MANITIUS' Translation of PTOLEMY'S *Almagest*, Teubner, 1912, Vol. I, pp. 247-252).

For correcting the mean motion of the Moon in Longitude and Anomaly, PTOLEMY followed HIPPARCHUS in using Lunar eclipses recorded at the remotest available epochs, namely those of March 8, 720 B.C., and Oct. 20, 134 A.D. — the total interval involved being 311783 days, 23 1-3 equinoctial hours (cf. MANITIUS, Translation of PTOLEMY'S *Almagest*, Book IV, Chap. VII, Vol. I, p. 235.)

And for the correction of the motion in Latitude PTOLEMY used eclipses of equal magnitudes, at the same node, likewise separated by a great interval of time — namely, that of April 25, 491 B.C., and April 5, 125 A.D. After some corrections he makes the interval between corresponding Latitudes of the Moon to be 224609 days; and thence deduces once for all the periodic motion of the Moon in Latitude.

These brief illustrations of the methods of HIPPARCHUS and PTOLEMY enable us to realize that the cycles of the modern Lunar Theory are merely considerable refinements of those used in the Alexandrian School, which had already carried the determinations to a high degree of accuracy, and developed methods of investigation which will always be used by astronomers.

III DETERMINATION OF THE PERIODS OF THE HITHERTO UNEXPLAINED FLUCTUATIONS OR OUTSTANDING INEQUALITIES IN THE MOON'S MEAN MOTION

From the above data, it follows that the Node will retrograde through 360° in 18.6 years, but in the same time the Lunar Perigee will progress through an angle of $756^\circ.183 = 720^\circ + 36^\circ.183$; so that after an interval of 18.6 years the Perigee is displaced forward by $36^\circ.183$ in respect to the restored Node.

(*i*) *Determination of the period of the 60-year Fluctuation.*

It is very easily shown that owing to the relative magnitudes of these direct and retrograde revolutions the *angular conjunctions* will tend to recur in the regions of 360°, 240°, 120°, like the *actual conjunctions* of the planets *Jupiter* and

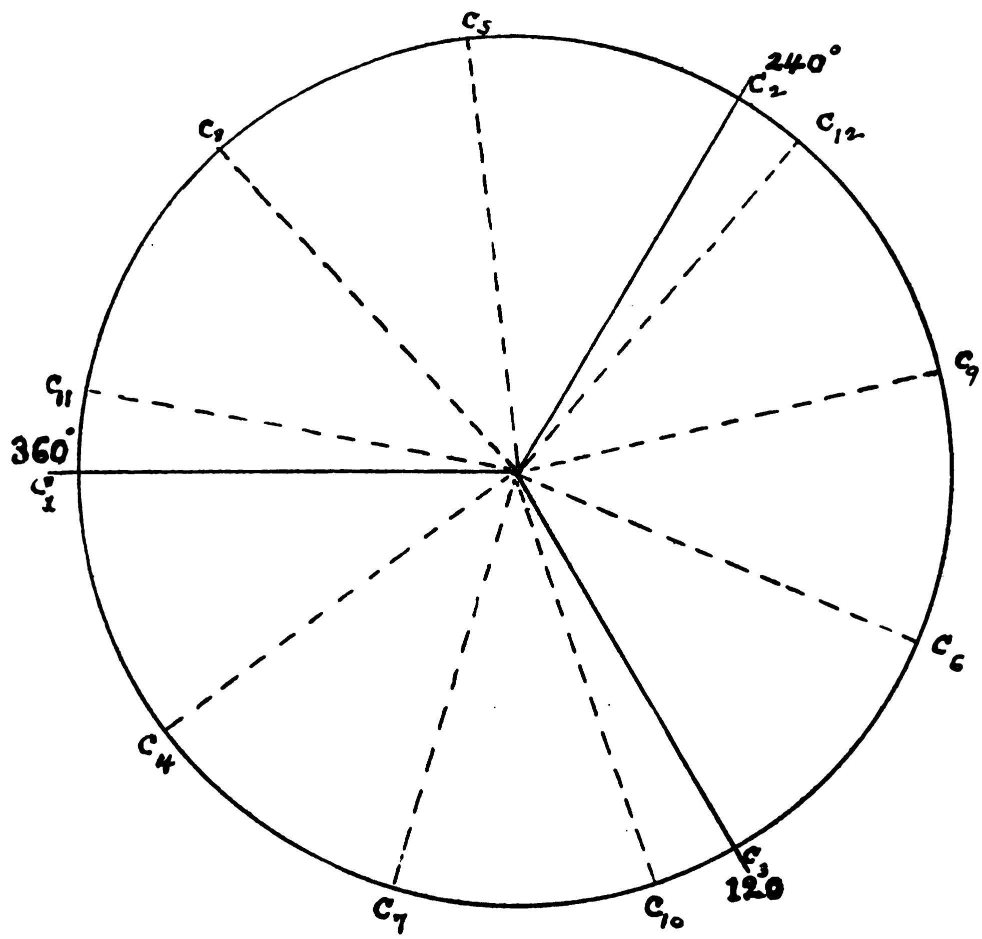

FIG. 1. Illustration of the progress of the Moon's Perigee in respect to the Node, in the 61.7-year Fluctuation, the new theory of which is almost exactly analogous to the celebrated 900-year Inequality in the mean motion of *Jupiter* and *Saturn*, the physical cause of which was discovered by LAPLACE in 1785.

Saturn in the theory of the celebrated 900-year Inequality which was first theoretically explained by LAPLACE in the year 1785. Here, too, as in the theory of *Jupiter* and *Saturn*, the *conjunction lines move forward.* The amount of the displacement is 36°.183 in 18.6 years; and in 3.31648 such periods, 3.31648 × 18.6 years = 61.7006 years, the *angular conjunction* which started out at the angle 360° will revolve forward through 120°, and the cycle of angular conjunctions at all three points will begin over again, exactly as in the Great Inequality of *Jupiter* and *Saturn*. This leads at once to the second Long Inequality in the Moon's Mean Motion, which, without suspecting the cause, NEWCOMB estimated at "60 years, more or less." His judgment of the period was surprisingly accurate; and as he concluded that the coefficient might be about 3″.0, here again his value could be adopted.

(*ii*) *Determination of the Period of the Great Fluctuation in 277.590 years.*

In the case of the *Great Fluctuation* in the Moon's mean motion, of which NEWCOMB estimated the period at about 275 years, the calculation of the period is somewhat similar to that just cited, but also somewhat different. It is physically obvious that the modification of the Sun's gravitation in passing through the body of the Earth will depend on the relative shifting of the line of *angular conjunctions* Node-Perigee.

Now it is easily found by calculation that the angles of the Node-Perigee are in *angular conjunction*, on a line 11°.670 in advance of the original conjunction, after an interval of 17.9971 years. For in this time the Perigee progresses over an arc of $4\pi + 11°.670$, and the Node retrogrades over an arc of $2\pi - 11°.670$, and meet exactly at the *conjunction line* specified.

The problem thus arises to find the interval in which this secular displacement of the angular conjunction line will complete the cycle in the Moon's motion due to the reduction of gravitation near the shadow of the Earth. In each period of 17.9971 years, the node retrogrades through the angle 2π in respect to the shifting mean position of the Perigee, and in the same interval the Perigee progresses through the double of this angle, 4π, in respect to the retrograding mean node; so that on the average their opposite motions amount to 6π in 17.9971 years.

As the physical effect of the reduction of gravity near the shadow of the Earth is the same whether the shifting conjunction line Node-Perigee refer to ascending or descending node, we perceive that this advancing conjunction line need only sweep over the angle π to give the required interval for completing the cycle due to the changes of gravitation near the shadow of the earth.

Now $180° / 11°.670 = 15.422$, and therefore in an interval of 15.422×17.9971 years $= 277.590$ years, the cycle of the changes of gravitation near the shadow of the Earth will be complete.

This is the period of the *Great Fluctuation* in the Moon's Mean Longitude which NEWCOMB estimated at 275 years, from the modern observations, and used in calculating the secular acceleration from the eclipse records extending over 2,600 years since the era of the Babylonians.

The diagram on page 108 presents to the eye a continuous representation of the changes in Node (outside circle) and Perigee (inside circle) during 18 years. At the end of 18 years they both are in conjunction at 1, near the original line of conjunction, 360°, but 11°.670 further forward. In each of these periods of 18 years the Nodes turn to every part of the heavens, so that eclipses occur all around the Earth's orbit, with the Earth and Moon at all possible distances from the Sun. In this interval the lunar Perigee revolves twice, and the Node once; so that the effect of the progression of the Perigee goes through symmetrical phases in respect to the Earth's orbit in 18 years, as shown by the above diagram.

This diagram also illustrates the secular progress of the line Node-Perigee, the restoration to parallelism in this conjunction line, advancing by 11°.670 every 17.9971 years, and requiring 277.590 years for completing the full cycle of a semi-circumference.

We may express this result also by observing that *physically* the decrease of gravitation near the shadow of the Earth will take place with equal effect whether the eclipse be near the ascending or the descending node; and this decrease will always correspondingly affect the Moon's mean longitude. Therefore, the 18-year movement of Node-Perigee conjunction line over the arcs 1, 2, $3, \ldots n$, where $n = 15.422$ at 180°, will comprise all possible combinations of the conjunction line Node-Perigee for modification of the Sun's gravity on the Moon when near the shadow of the Earth.

(*iii*) *Determination of the 18-year period of the Saros cycle.*

The *Saros* cycle is so well known that we need scarcely add that a minor disturbance in the Moon's mean longitude will recur in this period of 6585.32 days = 18.0293 years. *In this period the symmetrical eclipse cycle of 223 Lunations is complete and the eclipses begin to repeat themselves, with the Moon very near the same relative position with respect to the Sun and Node, and also with respect to the line of apsides or Perigee.* This *Saros* cycle of the Chaldeans gives rise to a minor disturbance in the Moon's mean longitude, with period of 18.0293 years, and a coefficient of about 1″.0. It is the smallest of the Moon's sensible *Fluctuations*, yet indicated by the researches of NEWCOMB and BOTTLINGER, and illustrated graphically in BULLETIN No. 1.

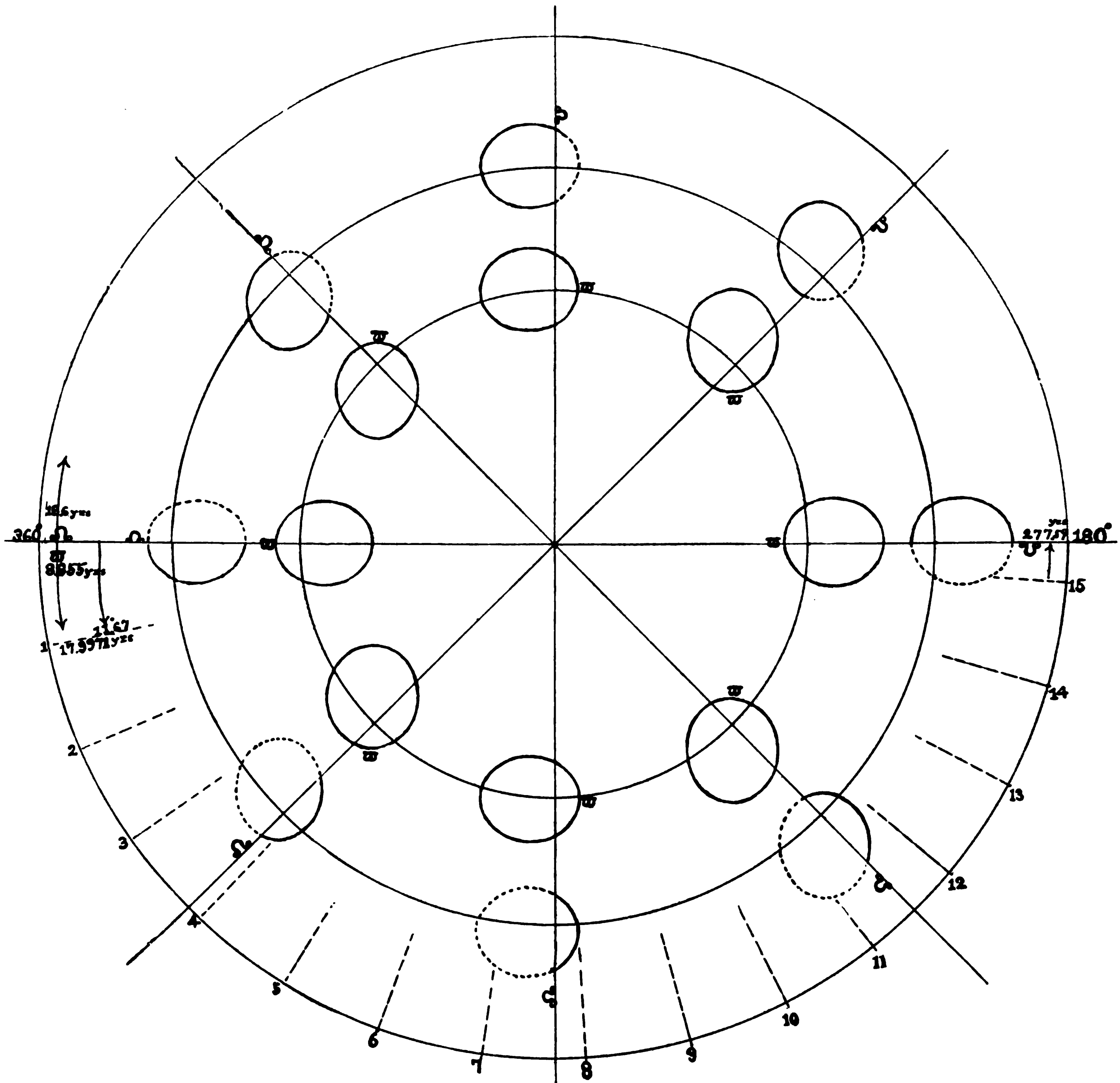

Fig. 2. Illustration of the progress of both Node and Perigee for producing the Moon's Great Fluctuation in 277.59 years. This also is somewhat analogous to the theory of Laplace's Great Inequality in the mean motion of *Jupiter* and *Saturn*. (See p. 107.)

IV CALCULATION OF FORMULÆ FOR THE PRINCIPAL FLUCTUATIONS IN THE MOON'S MEAN MOTION: COMPARISON OF THE NEW PHYSICAL THEORY WITH EIGHTY YEARS OF THE BEST MODERN OBSERVATIONS

From the nature of the new *Theoretical Fluctuations* now brought to light, and the physical cause on which they depend, as already fully explained, it appears that whilst the *periods* themselves may be calculated accurately from the recognized elements of the Moon's motion, it seems likely that values of the *coefficients* for the Fluctuations can only be obtained from the refined discussion of a long series of observations.

At present NEWCOMB's treatment of the subject is the only one available. We shall therefore derive the coefficients from the data there discussed, altering the empirical period of 275 years to the new theoretical period of 277.59 years, found from the elements of the Lunar motion, and adding two new fluctuations now brought to light in periods of 61.7006 and 18.0293 years respectively.

NEWCOMB concluded that the *Great Fluctuation* had a period which might lie anywhere between 250 and 300 years, but that the most probable value appeared to be 275 years, corresponding to the angular coefficient 1°.31 in his equation for the Great Fluctuation in longitude

$$\delta v = 12''.95 \sin \{1^\circ.31\ (t - 1800.0) + 100^\circ.6\} \tag{3}$$

(cf. NEWCOMB's *Researches on the Motion of the Moon*, 1912, p. 210).

Our new theoretical period, 277.590 years, differs from NEWCOMB's observational value, 275 years, only 2.6 years. In the *Monthly Notices* for January, 1909, NEWCOMB says that practically an angular coefficient 1°.32 will represent observations as well as 1°.31. Our new theoretical coefficient is 1°.29691, about 1°.30, and thus it evidently will represent observations practically as well as NEWCOMB's larger values 1°.32 and 1°.31.

By the discussion of a system of equations (p. 210) NEWCOMB concludes that the Moon's observed sidereal secular acceleration is expressed by the equation $7''.96\ T^2$, where T is counted in Julian centuries from the epoch 1800.0, Greenwich Mean Noon. This will not necessitate any change in virtue of the new theoretical period of 277.59 years, although it is possible that a slightly larger value, about 8''.08, would be an improvement, for the reasons indicated in my *Researches on the Evolution of the Stellar Systems*, Vol. II, 1910, pp. 290–291.

In regard to the *Minor Fluctuations* NEWCOMB seems to have reached no definite conclusion, either as to period or amplitude. In the *Monthly Notices* for January, 1909, which is an abstract of his whole Memoir of 1912, he says: "The minor deviations during the past 100 years may be empirically represented by a trigonometric series, the principal term of which would have a period of 60 years, more or less, and an amplitude of perhaps 3″. But we have no reason to believe that, how accurate soever the representation may be by such a series, it will represent the future course of the Moon." This remark clearly shows that NEWCOMB regarded a physical theory of the Moon's Fluctuations an urgent desideratum of Science.

BOTTLINGER (*Dissertation*, p. 21) examined the question of the short period fluctuation more closely than NEWCOMB had done; and although inferring that NEWCOMB's data supported the theory of an 18-year inequality, he sees nothing of a 60-year fluctuation, merely remarking, "Man kann vermuten, dasz die 18 Jährige Fluktuation NEWCOMBS ein wesentliches, periodisches Glied der Mond bewegung ist, wenn auch die wahre Ursache uns unbekannt ist." He points out that just as the Lunar eclipses are irregularly periodic, so also will be the action of the shadows in reducing the gravitation of the Sun at these times. Hence it is not at all necessary that the curve of the shadow effect should be very regular in appearance.

BOTTLINGER considered the agreement between his calculated shadow effects, 1830—1895, and NEWCOMB'S investigations so striking as to justify the conviction that it pointed to the true cause of the Moon's unexplained fluctuations.

In order to obtain a correct view of the total fluctuation in the Moon's mean motion, it is necessary to recognize clearly all the possible separate components of which it is made up. The chief component fluctuations, in order of duration, are as follows:

1. *The Great Fluctuation*, depending on the Perigee's relative shift through an angle of $4\pi + 11°.670$ with respect to a fixed node in a period of 17.9971 years. The movable conjunction line Node-Perigee shifts 11°.67 each time and revolves through 180° to the other fixed (descending) node in 15.422 such periods, thus completing the cycle of the long period disturbance in 277.590 years. The Great Fluctuation appears to have a coefficient of about 13″.0.

2. *The Large Fluctuation*, with period of 61.7006 years, and coefficient 3″.0, depending on the shifting of the three Node-Perigee *conjunction lines* which start at the fixed angles 360°, 240°, and 120° relatively to the Earth's orbit or fixed stars. At each complete revolution of the Moon's node, 18.6 years, the Perigee, by its more rapid motion, revolves forward relatively to these points through an angle of 36°.183; and after 3.31648 such periods, 3.31648×18.6 years $= 61.7006$ years, the Perigee shifts through 120° and thus meets the next original fixed

Node-Perigee point, when the cycle begins over again, exactly as at the outset.

3. The *Short Period Fluctuation*, depending on the *Saros* cycle of 223 Lunations = 6585.32 days = 18.0293 years, if the length of the sidereal year be taken at 365.2563582 days, as concluded by HANSEN and OLUFSEN. As this is the eclipse cycle discovered by the Babylonians, the length of the period requires no explanation; and we merely add that the coefficient of this fluctuation appears to be about $1''.0$.

4. *The Still Shorter Fluctuation, depending on one-third of the Saros*, 6.01465 years, in which period the Node-Perigee angles shift in opposite directions through very nearly the relative angle of 2π, so as to be near the *first conjunction line* at 240°, as explained in paragraph 2, above. This minor fluctuation would appear to have a coefficient not exceeding $0''.5$. As the comparison of theory with observations is not yet very refined, we have not attemped to treat this fluctuation in the present paper, nor in BULLETIN No. 1, but merely commend it to the attention of future investigators.

5. *The Minute Fluctuation, depending on the eclipse year of 346.62 days*, and too small to be certainly recognized in the present state of observation. The coefficient probably does not exceed $0''.2$; but exact calculations have not yet been made on this point, and the value is uncertain.

6. *The Infinitesimal Fluctuation depending on the Synodic month.* Here the coefficient may be evanescent, and perhaps vanishes entirely when the line of Nodes is nearly perpendicular to the direction of the radius vector Sun-Earth and the Moon's path is correspondingly remote from the axis of the Earth's shadow in space. At present these infinitesimal fluctuations have a purely theoretical interest, and thus the consideration of their practical importance must be left to the future.

The Great Fluctuation may be defined as in NEWCOMB's formula, but with the period 277.590 years, instead of 275 years, the annual coefficient being thus reduced from 1°.31 to 1°.29691. Thus the secular equation would be as in BULLETIN No. 1.

$$\Delta L_3 = 13''.0 \sin \{1^\circ.29691\,(t - 1800.0) + 100^\circ.6\} \tag{4}$$

In like manner the *Large Fluctuation*, with period of 61.7006 years, is represented by the secular equation

$$\Delta L_2 = 3''.0 \sin \{5^\circ.83597\,(t - 1800.0) + 126^\circ.35\} \tag{5}$$

And the *Short Period Fluctuation* is represented by the equation

$$\Delta L_1 = 1^\circ.0 \sin \{19^\circ.9675\,(t - 1800.0) + 239^\circ.42\} \tag{6}$$

Accordingly, it appears probable that these three equations will contain the whole theory of the Moon's sensible *Fluctuations;* and enable us to deduce the *theoretical fluctuations* at any period of time whatsoever. The table and diagram given in BULLETIN No. 1 show the agreement between theory and observation during the past 79 years, 1829.5 to 1908.3. With the above adjustment of periods and coefficients there remains outstanding only 15 residuals larger than $1''.0$ and most of these somewhat irregular, as if vitiated by systematic errors. On the whole, the indications, from the theory of probability, are that with the average residual of $0''.70$, there can be no outstanding fluctuation in the Moon's motion greater than $1''.0$.

A higher degree of refinement than this can scarcely be hoped for in the present state of Science. For owing to the mountains the Moon's limb is somewhat rough and uneven, so that the time of an occultation is uncertain by nearly $0^s.066$, or $1''.0$. Thus for the first time in history the Lunar Theory approaches completion, by the perfect conformity of observation with gravitational theory so modified as to take account of the reduction of the Sun's gravity in the transmission of the Electrodynamic Waves through the body of the Earth.

In more complete explanation of the diagram, given in BULLETIN No. 1, it suffices to note that the observed deviations from gravitational theory are represented by the small circles. The Great Fluctuation ΔL_3 with period of 277.59 years is first sketched; and the Large Fluctuation ΔL_2 with period of 61.7 years then added to the Great Fluctuation; finally the Short Period Fluctuation ΔL_1, with period of 18.0293 years, is added to $\Delta L_3 + \Delta L_2$, and the final result of all three fluctuations $\sum_{i=1}^{i=3} \Delta L_i = \Delta L_1 + \Delta L_2 + \Delta L_3$ is indicated by the larger circles. The series of larger circles are seen to follow the small circles with truly remarkable conformity.

As was justly remarked by NEWCOMB, the *Great Fluctuation* represents the observations roughly. The *Large Fluctuation* in 61.7 years brings a better accordance; and the *Short Period Fluctuation* in a *Saros* smoothes the irregularities still further, giving us an accordance between observation and theory which can scarcely be improved. The average residual is about $0''.70$, 27 minus, 29 plus, and 8 plus or minus. The minus residuals slightly exceed the plus residuals, but they are also slightly less numerous. Thus it is very doubtful if the present representation of the observations can be improved upon.

Accordingly, we appear to be confronted with the remarkable fact that by means of the new equations, based on physical theory, it is now possible to improve the accuracy of the Moon's mean motion at least a dozen fold. As the improvement rests on true causes it ought to be of value in the discussion of both ancient and modern observations.

If there be those who hesitate to acknowledge this obvious result they will find an uncertain attitude the more difficult, owing to this twelve-fold improvement in the accuracy of the Moon's theoretical mean longitudes.

The physical theory which represents the Moon's Motion so perfectly for the eighty years of most exact observations can scarcely fail us in coming ages. Thus at last we seem to have full assurance of reconciling the theory with the observations of the Moon — a problem which has presented difficulties far greater than that offered by any other heavenly body. This difficulty is due partly to the complicated character of the Lunar perturbations, and partly to the great proximity of our Moon and the great length of time over which it has been observed; so that small forces may at length become sensible* in the theory of its motion. In no other way could we have traced to its source the decrease of the Sun's Gravity near the shadow of the Earth. The Moon has thus introduced into Celestial Mechanics a new gravitational criterion of extreme delicacy.

V Expression for the Possible Absorption of Gravitational Energy Propagated Through a Spherical Earth: the Residual Character of the Moon's Great Fluctuation of 277.590 Years Foreseen by both Newcomb and Bottlinger, and now Confirmed by Calculation.

In §3 of Bottlinger's *Inaugural Dissertation* will be found a mathematical investigation of the loss of gravitation by absorption due to the rays passing through the spherical body of the Earth. It may not be necessary to consider this, in view of the refraction and dispersion of the energy concluded from the Electrodynamic Wave-Theory; but it seems best to overlook no cause which might contribute to the Lunar Fluctuations till the true physical cause of this phenomenon is clearly established, which is the object of the present paper.

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*It should be noted that the average residual 0″.70 is invisible in a six-inch telescope, such as a transit circle. At the Moon's distance 0″.1 corresponds to about 500 feet, 1″.0 to 5,000 feet, about a mile. It seems probable that no fluctuation remains in the Moon's mean motion greater than the average residual of 0″.70, which in absolute distance corresponds to about a kilometer. Accordingly we have the Moon's longitude so improved that the outstanding fluctuations are invisible in our best transit circles, and in absolute magnitude, at the Moon's orbit, do not exceed a kilometer!

BOTTLINGER justly remarks that if a screening of Gravitation exists, then even for bodies of finite extent, the Newtonian law is no longer valid. For when one heavenly body attracts another, the gravitational rays proceeding from the interior of the mass have to penetrate through the surrounding mass and are thereby slightly enfeebled. Hence arises a loss of gravitational energy.

He then considers the interaction between a mass-point and a sphere made up of homogeneous concentric layers. If μ be the mass of the mass-point, M the mass of the sphere, R its radius, and $\sigma = \sigma(r)$ its density; then the attraction on μ is the integral of the acceleration vector to the elements of mass of the sphere:

(7) $$z = \int B dm$$

where

(8) $$B = \frac{k^2}{\rho^2} e^{-\kappa \int_0^s \sigma d\rho}$$

k^2 being the Gaussian constant of attraction, κ the absorption constant, and ϕ the angle about the axis of symmetry, and the other quantities as illustrated in the figure.

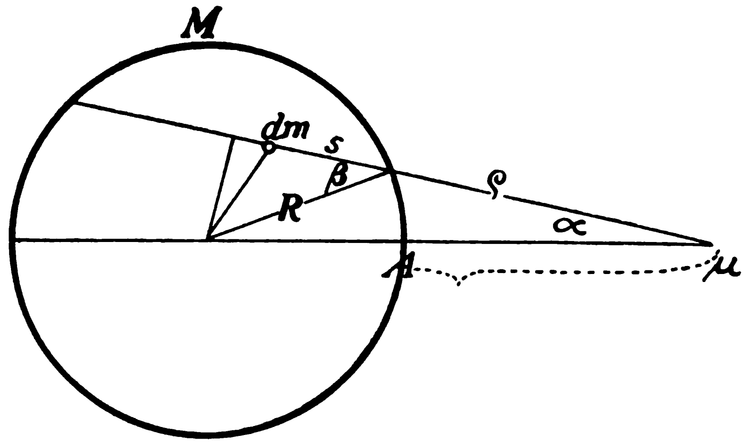

FIG. 3. BOTTLINGER'S Theory of a slight absorption of a ray of gravitation in traversing the Earth.

Accordingly, the attraction becomes

(9) $$z = \int B \cos \alpha \, dm = k^2 \int \int \int \sigma d\rho d\alpha d\varphi \sin \alpha \cos \alpha e^{-\kappa \int_0^s \sigma d\rho}$$

If we introduce the angle β, the integration being between o and $\frac{\pi}{2}$, we have

(10) $$\sin \alpha = \frac{R}{A} \sin \beta \;; \qquad \cos \alpha d\alpha = \frac{R}{A} \cos \beta d\beta$$

Then, the definite integral for the attraction becomes

$$z = \frac{k^2 R^2}{A^2} \int_0^{\frac{\pi}{2}} (\beta) \int_0^{2\pi} (\varphi) \int_0^{2r \cos \beta} (\rho) \sigma d\beta d\rho d\phi \sin \beta \cos \beta e^{-\kappa \int_0^s \sigma d\rho} \tag{11}$$

where

$$\sigma = \sigma(r) = \sigma \sqrt{R^2 + S^2 - 2RS \cos \beta} \tag{12}$$

Bottlinger points out that in this expression the integral is independent of A, the distance of the point μ from the sphere, and the attraction of the sphere of concentric layers reciprocally as the square of the distance, as in the Newtonian law. The constant of attraction alone remains indeterminate and a special function of the property of the sphere. He then proceeds to show that a factor should be introduced of the form

$$(1 - \tfrac{3}{4}\kappa\sigma R),$$

the fraction $\frac{3}{4}\kappa\sigma R$ representing the loss of gravitational energy due to absorption. He calls this factor h, and deduces for the Earth

$$h_{♁} = \frac{1}{160000}. \tag{13}$$

This outlines sufficiently the method of treatment followed, and we need only point out that the Electrodynamic Wave-Theory gives a much better physical basis for our reasoning than was available to Bottlinger. For we now see that there may be both absorption of gravitational energy and a refraction and dispersion of the waves propagated through a sphere made up of concentric shells. This is essentially the constitution of the Earth, and the outcome will be the production of the *Fluctuations* first recognized by Newcomb in the Moon's mean motion from the comparison of pure gravitational theory with observations since A.D. 1620.

It should be noted that the calculation already made of the period of the Long Inequality in the Moon's Mean Longitude rests wholly on the facts of observation, and is independent of any hypothesis as to the cause of the inequality. Yet the period being deduced from the facts, we have shown that there should be a period of 17.9971 years depending on the *forces* operating in the eclipse cycle, and leaving also a small residual effect for producing the secular equation extending over 277.590 years, owing to the lack of perfect compensation in the disturbing forces during the 18-year period.

If we represent by γ the *residual secular decrease* of gravity during an eclipse cycle of eighteen years, and these disturbing forces themselves by an unknown function Φ depending on the Moon's distance r, and the Sun's distance r'; the wave length λ of any chief element of the Electrodynamic Waves causing gravitation, the resistance ρ along the path of the ray through the terrestrial globe, thus being a direct function of the density σ, also depending on the moveable Node and Perigee; then the total secular effect for all angles of the Node-Perigee will take the form of the double integral:

$$\gamma = \sum_{i=1}^{i=i} h_i = \int_0^{\pi} \int_0^{2\pi} \left\{ \frac{\partial \Phi}{\partial \Omega} + \frac{\partial \Phi}{\partial \varpi} \right\} d\Omega d\varpi , \tag{14}$$

in which the limits of the integral are fixed by the considerations above indicated, and the unknown function has the form

$$\Phi\left(z, \frac{1}{r^2}, \frac{1}{r'^2}, \frac{1}{\lambda}, \frac{\sigma}{\rho}, \Omega, \varpi\right).$$

And in regard to the cycle representing periodic changes of gravity during the 18-year cycle, we shall have in like manner

$$\gamma' = \int_0^{2\pi - a} \int_0^{4\pi + a} \left\{ \frac{\partial \Phi}{\partial \Omega} + \frac{\partial \Phi}{\partial \varpi} \right\} d\Omega d\varpi \tag{15}$$

$$= \int_0^{\pi - \frac{a}{2}} \int_0^{2\pi + \frac{a}{2}} \left\{ \frac{\partial \Phi}{\partial \Omega} + \frac{\partial \Phi}{\partial \varpi} \right\} d\Omega d\varpi + \int_{\pi - \frac{a}{2}}^{2\pi - a} \int_{2\pi + \frac{a}{2}}^{4\pi + a} \left\{ \frac{\partial \Phi}{\partial \Omega} + \frac{\partial \Phi}{\partial \varpi} \right\} d\Omega d\varpi \tag{16}$$

On account of the *change* of the limits by $\pm \frac{a}{2}$, $\alpha = 11°.670$, there arises the small terms producing the *secular parts* in 277.590 years.

BOTTLINGER estimates the residual effect which gives rise to the *Great Fluctuation* in the Moon's motion at about two per cent. of the forces operating in the 18-year period. The angle of the Node-Perigee conjunction line shifts forward each time 11°.67; and as the whole circuit is 360°, this angular displacement is 3.24 per cent. of the path of the perturbing forces. As the *forces* at work are essentially proportional to the angular displacements occurring, it will be seen that our present theory agrees very well with that held by NEWCOMB, 1909, and BOTTLINGER, 1912.

In the *Monthly Notices*, January, 1909, NEWCOMB dwells on the fact that the shorter fluctuations imply the larger disturbing forces. "The fact is," he says, "that the variations of accelerating force necessary to produce the minor fluctuations are much greater than the forces necessary to produce the great one, the measure of this force being, not the actual deviation, but the degree of curvature at each point of the line representing the path."

From what we have now seen it is obvious that a residual effect of imperfect compensation, repeating itself in a cycle extending gradually over a long interval of time, explains all the phenomena. It was thus with the Great Inequality in the mean motion of *Jupiter* and *Saturn*. Before LAPLACE's discovery of the physical cause of this Great Inequality in 1785, it had proved so utterly bewildering to astronomers that some of them even began to doubt the rigor of the Newtonian law.

In the same way the most eminent mathematicians have recently admitted that the Newtonian law as heretofore understood will not explain the *Fluctuations* of the Moon's mean motion. NEWCOMB himself led the way by saying: "I regard these fluctuations as the most enigmatical phenomenon presented by the celestial motions, being so difficult to account for by the action of any known causes, that we cannot but suspect them to arise from some action in nature hitherto unknown."

It was from LAPLACE's procedure of 1785, in the case of the Great Inequality of *Jupiter* and *Saturn*, combined with the new insight into the nature of Gravitation, obtained from the Electrodynamic Wave-Theory developed since April, 1914, that I was enabled to detect the physical cause operating to produce the observed *Fluctuations* of the Moon's mean motion. In his *Inaugural Dissertation*, pp. 26-27, BOTTLINGER states that he had searched for an inequality of between 250 and 300 years in the Moon's mean motion, but hitherto had been unable to find such a Long Period Inequality.

VII OUTLINES OF THE PRINCIPAL PHENOMENA PRESENTED BY THE GREAT INEQUALITY IN THE MEAN MOTIONS OF JUPITER AND SATURN, FOR BRINGING OUT AN ANALOGY WITH THE LUNAR FLUCTUATIONS, WHICH REPRESENT A NEW TYPE OF LONG INEQUALITY IN THE CELESTIAL MOTIONS.

(*i*) *Brief description of the Great Inequality in the mean motions of Jupiter and Saturn.*

In his elementary work on *Gravitation*, pp. 143-155, §§163-177, Sir GEORGE AIRY gives a very instructive and very simple explanation of the physical cause of the great inequality in the mean motion of *Jupiter* and *Saturn*. He remarks (p. 155) that the long inequality of *Jupiter* and *Saturn* is the most imposing by its magnitude and the most celebrated for its history of any of the planetary inequalities, having been known to astronomers for more than a century before its cause at length was detected by LAPLACE in 1785. AIRY points out that before this great inequality was explained theoretically astronomers were completely bewildered by the strange irregularities in the motions of these planets, which even cast doubt on the rigor of the Newtonian law.

It is well known that in 1625 KEPLER found evidence that the motions of *Jupiter* and *Saturn* are not strictly uniform, but depart more and more from the positions resulting from the elements of PTOLEMY, as compared with the positions given by the elements deduced from the observations of TYCHO BRAHE (cf. KEPLER, *Opera omnia,* VI, pp. 617-18). In 1676 HALLEY perceived in the motions of *Jupiter* and *Saturn* some variations which appeared to have contrary signs, and which he imagined might run through 2,000 years, producing an equation of 3° 49′ in the mean longitude of *Jupiter* and 9° 15′ in that of *Saturn* (*Phil. Trans.*, 1683, p. 244). This result was therefore known to NEWTON at the time of the publication of the *Principia,* 1687, but no doubt put aside temporarily for more detailed investigation by NEWTON'S successors.

In 1773 the subject was examined somewhat carefully by LAMBERT, but in 1776 reinvestigated by LAGRANGE, who found that the inequality did not increase indefinitely; and moreover that when *Jupiter* was retarded *Saturn* was accelerated, and thus he concluded that there might be a long inequality depending in some unknown way upon the mutual actions of the two planets. Yet in vain did LAGRANGE labor to find in the mutual actions of *Jupiter* and *Saturn* the cause of the observed secular inequalities (cf. *Memoires de l'Acad. des Sciences,* Berlin, 1783, p. 223).

At length LAPLACE was more fortunate, for in 1785 he detected in his analysis certain terms with small divisors depending on the argument $2n - 5n'$ where n is the mean motion of *Jupiter* and n' the mean motion of *Saturn,* resulting from the near approach to commensurability in the mean motions, and acquiring large values by the double integration for the mean longitudes. Thus if R denote the disturbing function, and ϵ the epoch, or longitude of the planet at the assigned date, the term for the perturbation in the mean longitude

$$\delta L = k \int\int \frac{dR}{d\epsilon} dt dt \tag{17}$$

will by actual integration yield terms of the form (cf. LAPLACE's *Mécanique Céleste*, Lib. II, Chap. VII, §54, Chap. VIII, §65)

$$\delta L = \frac{kiP \sin \{(in - i'n')t + n - n' + Q\}}{(in - i'n')^2} \tag{18}$$

which becomes large when the divisor $in - i'n'$ is very small. In Dr. HILL's New Theory of *Jupiter* and *Saturn* $2n - 5n' = - 1467''.825$.

Applying this method of analysis to the planets LAPLACE found that the mean motion of *Saturn* was being retarded throughout the whole of the 17th century while that of *Jupiter* was being accelerated; but that during the 18th century the process began to reverse. From these circumstances, and the above considerations in the analysis of the mean motions, LAPLACE was enabled to discover the physical cause of the phenomenon, and to calculate the period and

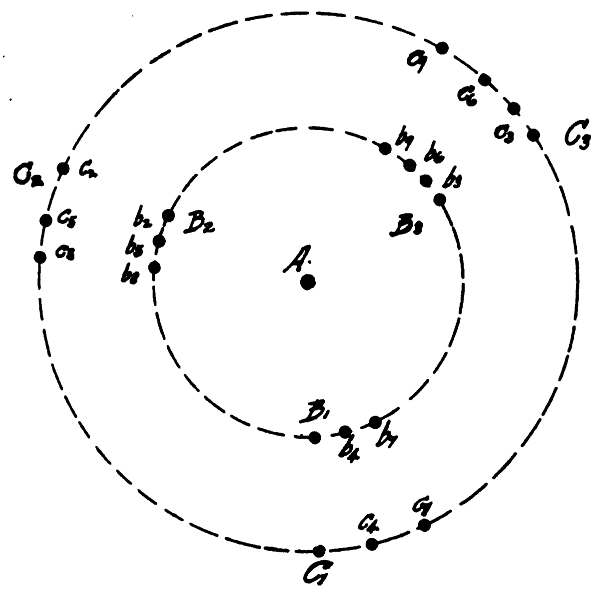

FIG. 4. Diagram of the Shifting Conjunctions of *Jupiter* and *Saturn*, for illustrating LAPLACE's discovery of the Cause of the Great Inequality, 1785.

amount of the mutual disturbance to the two planets, in a period of about 918 years.

Accordingly, from LAPLACE's theory it followed that during the first period of 459 years *Jupiter* is accelerated and *Saturn* retarded, while in the second period of the same length the reverse process goes on, *Jupiter* being retarded and *Saturn* accelerated. The conjunctions of the two planets occur near 360°, 240°, and 120°, and thus are situated in longitudes differing by about 120°. And as the mean motions are not quite commensurable, the line of the conjunctions shifts slowly around the orbits, as shown by the figure (AIRY, *Gravitation*, p. 145). The entire cycle is complete in about 918 years, when the mutual action of the planets is fully compensated, and the Great Inequality begins over again.

We have cited in some detail the history and circumstances of LAPLACE'S discovery of the cause of the celebrated long inequality in the mean motion of *Jupiter* and *Saturn*, because it is very instructive, and presents an

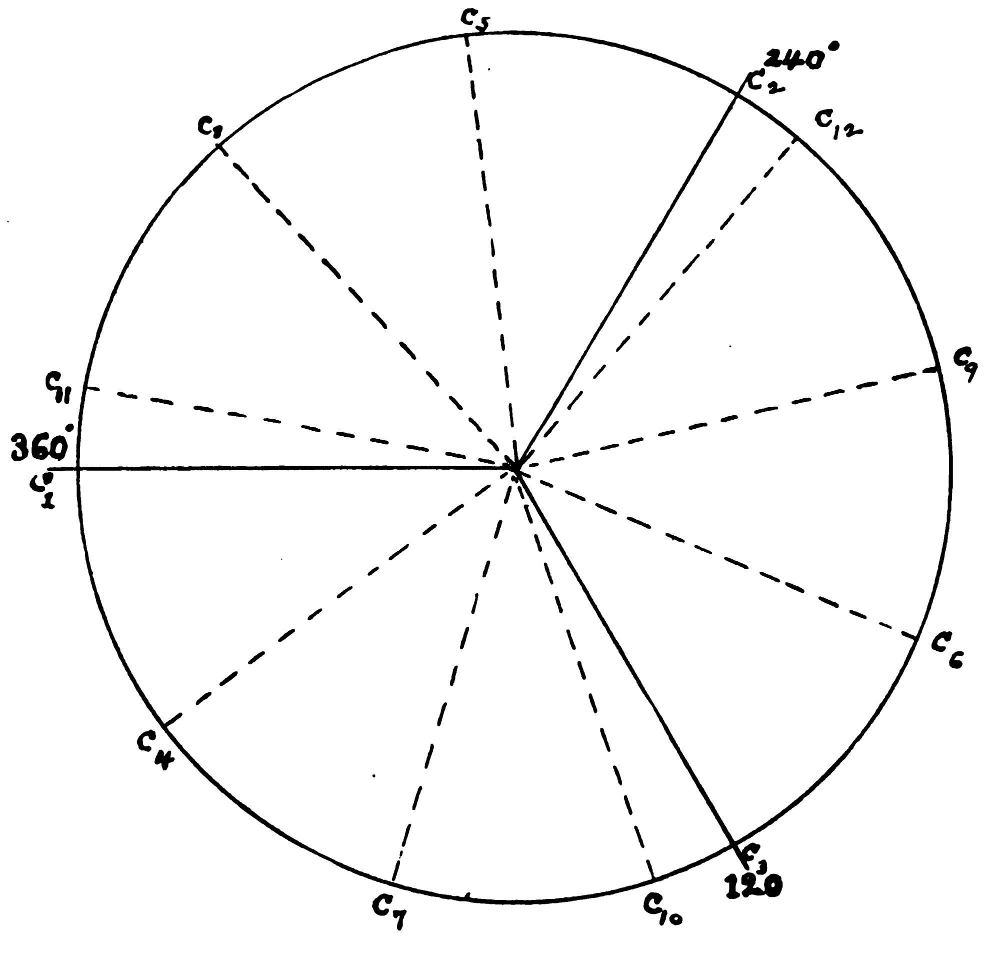

FIG. 5. Repetition of Fig. 1, for illustrating the closeness of the similarity of the Moon's 61.7-year Fluctuations to the Great Inequality of *Jupiter* and *Saturn*.

analogy which finally proved effective in disclosing the cause of the Moon's *Fluctuations*.

After what is shown in the foregoing development it will be obvious that the *Fluctuations* of the Moon's mean motion represents a new type of Long Inequality in the Celestial Motions. The excellent agreement of the new physical theory with the best observations of the eighty years immediately preceding 1908 appears to be a clear indication that the new theory rests on true laws of Nature. We have already shown how the different periods arise, and in BULLETIN No. 1 given the equations for calculating the Moon's position at any past or future time. But it seems advisable to recall the progress in finding out the dissimilarity with the phenomena of *Jupiter* and *Saturn* as well as the close analogy which enabled the discovery to be made.

(*ii*) *Outline of Progress in dealing with the Moon's Fluctuations.*

As the processes of discovery are scarcely less interesting than discovery itself, it may not be out of place to record the fact that in some quite mysterious way the impression steadily gained strength in my own mind, almost ever since NEWCOMB announced the existence of the *Unexplained Fluctuations* in the Moon's mean motion, 1909, that these irregularities eventually would be found to be analogous to, but not exactly similar to the phenomena presented by the Great Inequality of *Jupiter* and *Saturn*. In the autumn of 1914 I sent a letter containing such a suggestion, regarding the 60-year fluctuation, to Professor E. W. BROWN, but nothing ever came of it.

Late in November and early in December, 1916, while extending the researches on the cause of Gravitation, I discovered that in traversing the terrestrial globe the Electrodynamic Waves producing the Sun's gravitational attraction on the Moon, would be diminished in and near the shadow of the Earth, by refraction, dispersion and perhaps by absorption. And as this theoretical decrease of gravity by the interposition of the globe of the Earth in the path of the Sun's gravitational attraction would give rise to the very periodic disturbances of the Moon, in about eighteen years, which NEWCOMB's data had shown from the observations, and BOTTLINGER inferred theoretically should exist, a persistent search for the cause of the long-period fluctuation running through 275 years was kept up. At length, after the inquiry had been repeatedly renewed, under the most diverse forms, the cause of the *Great Fluctuations* with period of 277.590 years was discovered. Owing to a slight undetected numerical error it was at first believed to be 246.75 years, and so announced in the cablegram of Dec. 10, 1916. (*A.N.*)

After bringing to light this remarkable result, I was surprised to find that Dr. BOTTLINGER, under the direction of Professor von SEELIGER, had given some thought to finding such a long inequality in the Moon's motion, depending on eclipse action; yet he adds that, although he had searched carefully, hitherto he had not been able to find such a long inequality. He points out that the forces producing the 275 year inequality amount to only about two per cent. of those more powerful forces operating to prôduce the 18-year inequality. NEWCOMB also had called attention to the comparative feebleness of the forces producing the 275 year inequality.

To bring out the difference between the Great Inequality of *Jupiter* and *Saturn* and the Fluctuations of the Moon's motion we notice that in the case of *Jupiter* and *Saturn* we have a mutual action of two planets with conjunctions near longitudes 360°, 240°, and 120°, but slowly progressing and at length completely compensated by the shifting of the lines of conjunction through angles of 120°.

In the case of the Moon's *Fluctuations* there is indeed a similar series of conjunctions of the Node-Perigee movements, in contrary directions, also occurring near longitudes 360°, 240°, and 120°; and these points also shift forward, somewhat as in the case of *Jupiter* and *Saturn*.

But this does not lead us at once to a similar result, because the Node and Perigee are only *lines;* and can exert no *forces* on the system. Yet true *physical action* on the Moon, owing to the Earth's interposition in the path of the Sun's electrodynamic waves, must depend on the relative shifting of the perigee in respect to the line of nodes; and therefore this indirect action occurs, and we have to find the period for its complete compensation, in a semi-circumference, as above explained.

The conjunction line Node-Perigee shifts 11°.670 in a period of 17.9971 years, and the cycle runs through 180°, corresponding to the position of the other node, in 277.590 years. Therefore, whilst there is an analogy with the phenomena presented by the celebrated Long Inequality of *Jupiter* and *Saturn*, there is also a fundamental difference in the nature of the actions involved in the two cases.

One case represents direct actions between *planets* revolving in the same direction, and coming into conjunction near certain shifting longitudes. The other represents a relative shift of the *Node-Perigee conjunction line*, so as to carry it through a Perigee cycle at the other node of the Moon's orbit, thereby disturbing the Moon's mean motion in a period of 277.590 years. This is owing to the variations of the Sun's action on the Moon near the shadow of the Earth, as the conjunction line Node-Perigee slowly shifts till it reaches the original position of the other node, and the phenomena once more begin to repeat themselves in another similar period.

If BOTTLINGER was justified in saying that should a period of about 275 years be found to result from the cycles of the eclipse periods and eclipse action in modifying the Sun's gravity near the shadow of the Earth, then all the riddles of the Lunar Theory would be solved (*Inaugural Dissertation*, p. 27), it would seem indeed that we may now hope to have achieved this very wonderful result, and thereby harmonized both the ancient and modern observations of the Moon, extending from our own time through the Middle Ages, and the epoch of the Arabians, to the period of the Alexandrian School, and from the records of Lunar Eclipses then preserved by PTOLEMY to the era of the Babylonians, 720 B.C., thus embracing a total duration of 2,636 years!

This long record rests mainly on Lunar Eclipses, and naturally calls to mind also that associated with the disaster to the Athenians at Syracuse, under Nicias, in 413 B.C., through the eclipse of the Moon, which was celebrated in verse by HAMILTON,* whose mathematical methods we have found so useful.

As the Electrodynamic Wave-Theory is shown to explain the Moon's *Fluctuations*, to which the methods of HAMILTON are immediately applicable, the question may properly be asked whether, in turn, the discovery of the Cause of the *Fluctuations* does not at the same time practically establish the Electrodynamic Wave-Theory of Gravitation, from observed phenomena?

It is to be remembered:

1. The Electrodynamic Wave-Theory does completely explain the Moon's *Fluctuations*, which are utterly irreconcilable with the Newtonian form of Gravitational Theory, developed before electrodynamic waves were known to exist, or shown to be propagated with the velocity of light, which has been established by modern electrical measurements.

2. Thus there are important gravitational phenomena which do not admit of explanation except on the theory of wave action propagated in time, according to WEBER'S Fundamental Law of 1846. And since we cannot ignore progress in experimental science, it would appear necessary to adapt the traditional Theory of Gravitation to electrodynamic action, experimentally established by AMPÈRE and his successors, and so entirely free from valid objections, that it perfectly harmonizes the results of our laboratories with the most varied phenomena of the heavens.

(*iii*) *The Outstanding Difficulties in the Lunar Theory finally overcome.*

The Moon's motion long presented two outstanding difficulties which defied theoretical explanation. The secular acceleration calculated by LAPLACE was $\Delta L = 10''.1816213\, T^2 + 0''.1853844\, T^3$; but in 1853 ADAMS showed that it was about 4″ too large, and, as verified by DELAUNAY, should be only about

*GRAVES' *Life of Hamilton*, 1882, Vol. I, pp. 145-6.

6″.08, according to recent corrections by NEWCOMB. The most ancient eclipses observed at Babylon and preserved by PTOLEMY indicated more nearly 12″, while the Arabian eclipses investigated by NEWCOMB in 1878 indicated a secular acceleration of about 8″.4. The recent discussions of ancient eclipses by COWELL, FOTHERINGHAM and others have apparently rendered it certain that there are no reliable indications of a secular acceleration in excess of about 8″, yet this value is established and harmonized with the eclipse records of the Arabians, 829 to 1004 A.D.

Thus there remained an outstanding difference of 2″ between theory and observation until the year 1909, when the present writer showed that the Moon is a *captured planet*, nearing the Earth, from an original distance nearly twice its present value, but approaching with such excessive slowness that the interval since the Capture of the Moon is to be reckoned in something like 500 million years. The difference of about 2″ between theory and observation is thus an indication to us of the origin of our Moon, by slow approach, under the secular increase of the masses of both Earth and Moon. (cf. *Researches on the Evolution of the Stellar Systems*, Vol. II, 1910).

This finally removed one of the long standing difficulties in the theory of the Moon's motion; yet in the same year, 1909, an even greater one arose with the definite confirmation by NEWCOMB of the existence of the *Unexplained Fluctuations*. In the foregoing discussion we have been able to show that the *Fluctuations* arise from loss of effective gravitational energy when the Sun's attraction on the Moon has to be exerted through the body of the Earth. The Electrodynamic Waves producing the Sun's gravity are both refracted, and dispersed and perhaps somewhat absorbed, and thus partially deflected to one side in passing through our globe; so that the Moon suffers a relative reaction when in or near the shadow of the Earth, and thus arise disturbances in the Moon's motion depending on the *Saros*, and other periodic cycles connected with the Node-Perigee movements.

The new physical theory of these gravitational disturbances conforms to the observed periods of the Moon's hitherto *Unexplained Fluctuations*, and thus represents the observations so perfectly as to give a *twelve-fold* increase in the accuracy of the predicted places of our satellite, in the eighty years of the best modern observations immediately preceding 1908. This impressive fact must be held to justify the expectation that the new physical theory will account for any outstanding fluctuations in the motion of the Moon which the lapse of time may hereafter bring to light.

As the Theory rests on secure physical grounds and is fully borne out by mathematical analysis of the geometrical relations involved, it must be held to assign the true cause of the phenomena which long proved so utterly bewildering to the most eminent authorities — HANSEN, ADAMS, DELAUNAY, NEWCOMB, HILL, DARWIN, TISSERAND, POINCARÉ, and E. W. BROWN.

These geometers have laid the foundations on which the present theory is built. Above every other investigator of our time Professor E. W. BROWN has perfected and confirmed the purely mathematical part of the Lunar Theory; and his work alone made it certain that NEWCOMB's conclusions as to the existence of the *Fluctuations* rest on secure premises. This is the first necessary condition of progress; for so long as the premises can be questioned, there can be no solid advance towards the causes which underly the observed phenomena.

The writer's researches on the cause of Gravitation are as yet published only in part, and the whole of the results may not be available to the reader for some time. Without knowledge of this Electrodynamic Wave-Theory I believe that no effective attack on the cause of the Moon's *Fluctuations* could have been made.

The comprehensive treatment of the Lunar Theory given in TISSERAND's *Mécanique Céleste*, Tome III, 1894, has been of service to many investigators. Like BOTTLINGER (*Dissertation*, p. 5) I too long ago noted his sagacious remarks, concluding Chapter XIX, "Sur L'État Actuel de la Théorie de la Lune:"

"La théorie de la Lune se trouve arrêtée par la difficulté que nous venons de développer; déjà, a l'époque de Clairaut, la gravitation paraissait impuissante a expliquer la mouvement du périgée. Elle triomphera encore du nouvel obstacle que se présente aujourd'hui; mais il reste a faire une belle découverte!"

Accordingly, if we have settled the problem of the outstanding difference of 2″ in the secular acceleration, 1909, by the introduction of *the new conception* of the Capture of the Moon; and of the *Unexplained Fluctuations*, 1916, by the extension of the unpublished researches on the Electrodynamic Wave-Theory of Universal Gravitation, thus bringing to light a *New Type of Long Inequality in the Celestial Motions* which may become sensible where gravitation acts through solid bodies and suffers refraction, dispersion and perhaps absorption of the Electrodynamic Waves on which the force depends, — then, indeed, it would seem that we have now made and established the *beautiful discovery* foretold by TISSERAND twenty-two years ago, but heretofore beyond the reach of the geometer and natural philosopher, and, as was the case with the Great Inequality of *Jupiter* and *Saturn* before it was theoretically explained by LAPLACE, in 1785, utterly bewildering to the astronomer.

Starlight on Loutre,
Montgomery City, Missouri, June 18, 1917.

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RESULTS OF RESEARCHES

ON THE

ELECTRODYNAMIC WAVE-THEORY OF PHYSICAL FORCES

BULLETIN NO. 6

DEFINITE CONFIRMATION OF THE DISCOVERY OF THE PHYSICAL CAUSE OF UNIVERSAL GRAVITATION

By T. J. J. SEE

I Introductory Remarks

In the opening paragraph to the *Mécanique Céleste,* Vol. I, 1799, Laplace announces that "The nature of that singular modification, by means of which a body is transported from one place to another, is now, and always will be, unknown." Great indeed was the mystery then attaching to the nature of *Force.* The origin of the *Forces* which govern the operations of the material universe was so securely hidden that a feeling of despair came over the most illustrious mathematicians. The result was that very few natural philosophers even attempted to penetrate the invisible processes of the Ætherial Medium. Yet, as we have seen, a gradual change of view has been in progress during the past century, chiefly through the influence of the *Experimental Researches* of Faraday, and the mathematical and physical ingenuity of Maxwell, who interpreted these *Researches* to their successors.

In Bulletin No. 2 we have shown that Magnetic Attraction is due to decrease of stress by systematic properly directed wave action. *A magnet has two poles, always exactly equal, owing to the very nature of the waves, and the way the waves are directed in parallel planes normal to the lines of force, and largely normal to the Magnetic Axis.* Naturally the atoms have two corresponding poles, as viewed from either side of their equators, and the action of the entire Magnet represents the integral effect of the waves from all the atoms.

As Airy points out in his *Treatise on Magnetism,* 1870, p. 10, this attraction to poles, or *Duality of Powers,* is characteristic of *Magnetism.* But since the matter of artificial magnets once existed without the power of Magnetism, and may lose that power or have it greatly reduced by heating, shock, chemical action, etc., we justly conclude that the constitution of a magnet depends on the molecular arrangement, or the directions of the waves emitted by the atoms.

Beyond doubt the atoms of a piece of steel devoid of magnetism emit waves just like those of a magnet of the same size and material, but in the former body the waves are not lined up in parallel planes, as in the latter. And thus in unmagnetized bodies the waves are so confused by haphazard orientation that the magnetic power is lost by this mutually destructive tendency, — whereas in magnetized bodies the well ordered arrangement of the atoms gives *magnetic power,* which necessarily is directed to two equal poles. *This arrangement accounts for the existence of poles and their physical properties.*

If this Electrodynamic Wave-Theory of Magnetism alone be admissible for explaining the power of magnetic attraction, as shown in BULLETIN No. 2, then it follows also that the atoms of all bodies should emit waves which may give some power of attraction. Now FARADAY proved experimentally that all bodies are magnetic — the power varying from one body to another simply in degree — and thus it is established by careful experiments that their atoms do actually emit waves which under experimental influence may be more or less lined up in parallel planes.

Accordingly, bodies essentially non-magnetic or but feebly magnetic, like the Sun, Moon and Planets, necessarily must be centers of wave action, — just as the self-luminous stars are known to be by their light. Yea, as proved in BULLETIN No. 3, the magnetic observations themselves have definitely shown that the Sun and Moon both exert magnetic influence. And this power obviously proceeds from those individual atoms which are ordered and arranged to give magnetic action.

Now the great preponderance of the non-magnetic atoms of the Sun and Moon, which are arranged haphazard, must also emit waves like those which are magnetic; yet with their present orientation these waves so destroy one another that there is no action in respect to two poles, *but only a residual action in respect to the center of each mass. This unsystematized wave action towards the centers of the Sun, Moon and Planets, is what we call gravitation.* It is a curious fact that in his Article on *Gravitation, Ency. Brit.,* 9th edition, Sir ROBERT BALL estimates the force of gravitation to be millions of times feebler than *Magnetism,* as if it were a *residual action,* due to deterioration, under haphazard arrangement of the atoms.

In other respects Gravitation is analogous to Magnetism, since, as AIRY points out, *Cosmical Magnetism* seems to be one of the most general phenomena, in Nature. Moreover, whatever is due to *the systematized action of a few atoms* must, by NEWTON'S Rules of Philosophy, be ascribed also to *the vast majority of* *atoms having a purely haphazard arrangement*, and thus giving a *central action*, but no action in respect to poles.

We conclude, therefore, that since Magnetism is one kind of attraction, and definitely shown to be due to Waves, Gravitation as another kind of attraction must be ascribed to the same cause, in accordance with NEWTON'S *Second Rule of Reasoning in Philosophy, that "to the same natural effects we must, as far as possible, assign the same causes."* (*Principia*, Lib. III).

Now since the proof of the nature of Magnetism appears to be incontestible, it is not easy to see how this argument, based on Newtonian principles, can be evaded. But in order to decrease the possibility of such evasion, we have carefully investigated the nature of the disturbances operative in the *Fluctuations* of the Moon's Mean Motion, partially described in BULLETIN No. 1, and since fully confirmed in BULLETINS No. 4 and No. 5. If these *most enigmatical gravitative disturbances* can be explained by the recognized laws of Wave Action and by that cause alone, — as we have shown, — then obviously we have at once the strongest mathematical and physical grounds for denying the possibility of any other explanation of the phenomena of Universal Gravitation.

The *Fluctuations* of the Moon's mean motion thus become an *experimentum crucis* as to the true nature of the attractive force of Gravitation.

From the mere fact that a magnet is surrounded by an infinite system of waves, — which by the observed attraction and repulsion are proved to penetrate the spheres of activity of all other magnets, — we infer that if Gravity also be due to wave action, a similar interpenetration of gravitative influence should occur, each body extending its attractive force of gravitation into the sphere of activity of every other body. Now the researches of Sir ISAAC NEWTON for determining the law of gravitation from the celestial motions established the reality of this interpenetration of gravitative influence through all the celestial systems then known. And NEWTON'S early conclusions have been amply confirmed by the much more refined researches of the past two centuries.

Each body therefore emits an infinite system of waves from its atoms, and the waves travel in all directions with the velocity of light. The waves from the Sun enter the spheres of influence of the several planets, and those from the planets and satellites likewise enter that of the Sun and of each other mutually. It is from this mutual interpenetration of waves, propagated *spherically from these several centers*, that the *physical forces* arise which govern the motions of the planets in their orbits.

For in Bulletin No. 2 we have shown that when the rotations in the waves from the secondary magnet undo the rotations of the wave elements in the waves from the *primary magnet, the stress in the intervening medium decreases, the medium contracts, and thus draws the bodies together. It is very much the same way with the waves from the heavenly bodies which produce Universal Gravitation. As the waves from each body encounter waves from all other bodies, they mutually interpenetrate, and the contraction of the medium is greatest* in direct line with the other bodies — thus generating the very forces required for governing the heavenly motions.*

These forces always act in right lines, across free space, and thus operate to curve the planetary and cometary motions at all points of their orbits: which immediately justifies and indeed requires the denial of the *mystical view* of EINSTEIN that Gravitation is "not a force," but "a property of space." Nothing could be more misleading than such an unauthorized inference. Indeed it is doubtful if a more deceptive physical doctrine ever became current among men! It is nothing less than amazing that such *indistinctness of ideas, on the chief force of Nature,* should be prevalent in the Royal Society, to which NEWTON's immortal discoveries were communicated.

II ELECTRODYNAMIC WAVES FROM OPPOSITE DIRECTIONS NECESSARILY OPERATE TO DECREASE THE STRESS IN THE MEDIUM, THUS PRODUCING TENSION BETWEEN TWO HEAVENLY BODIES SUCH AS THE SUN AND EARTH

1. From either body an unlimited number of electrodynamic waves will be propagated in the form of the longer vibrations of the atoms: and on the average as many of these waves will rotate right-handed as left-handed. For outside of the planes of their equators, the waves will not be *flat,* but *helicoidal.*

2. Right-handed waves from S will combine with apparent left-handed waves from E, and thus operate to reduce the stress in the medium between the two bodies. In like manner left-handed waves from S will combine with apparent right-handed waves from E, thus making the whole of the waves, in two antagonistic groups, more or less operative to decrease the stress.

3. Accordingly, whichever way the waves from either body may rotate, as seen receding from that body, they will encounter waves rotating in the opposite sense as seen coming from the other body. And thus they each tend to undo the stress in the medium due to the oppositely directed system of waves, with apparent contrary direction of rotation. The stress in the medium between the two bodies

*In their mutual interpenetration the waves have a *relative velocity* which here becomes *double the Velocity of Light,* so that there is *maximum decrease of stress* and *maximum tension along the lines joining the bodies,* as in the observed phenomenon of Universal Gravitation.

is thus reduced by the interpenetration of waves; and hence the medium tends to contract and draw the bodies together.

4. Yet it is important to notice that beyond the two bodies the direction of rotation of the waves from the other body changes sign, and the effect is to increase the stress in this external region of space. Here the effect is to cause the medium to expand; and this external *pressure* thus aids the internal *tension* in holding the masses together. The phenomenon of gravitation requires unbroken continuity, for the internal and external equilibrium of the medium, and this is how it is effected.

5. We readily see that just as both right-handed and left-handed rotating light waves may issue from either body, — as actually observed in the Theory of Light, — so also from either body oppositely directed or like directed electrodynamic waves may travel, without mutual interference, but with changes of stress. Oppositely rotating waves from the two bodies combine to reduce the *stress* between them, thus causing contraction and thereby developing pulling in what we call *forces, acting in right lines connecting the two masses.*

6. In Figure 1 (p. 132) we have an illustration of the electrodynamic waves propagated from the Sun and Earth, and by their mutual interpenetration undoing the stress of the medium between the masses, but increasing it beyond. This causes the medium to tend to contract between the masses, which develops *tension,* while external to the bodies there is simultaneous increase of *stress,* and thus increase of *pressure,* which supports the internal *tension* in holding the bodies together.

7. The interpenetration of the waves is also mechanically illustrated by the pair of interlocked corkscrews below. *In the mutual undoing of the stress, the medium tends to contract and pull, giving a mechanical effect on E and S similar to the pulling on the bases of the interlocked corkscrews when they are rotated, and their interlocked helices are put under tension.*

8. The helices of the interpenetrating waves thus interlock and the rotations of the elements antagonize everywhere between S and E. *The effect is like that of screwing the points of two corkscrews together. When thus interlocked the movement of either screw will draw the other end on; so that if the bases were fixed, the helices of the interlocked screws would become stretched to a high tension, like the whirling filaments of the æther constituting the interpenetrating helicoidal waves between the Sun and Earth.* The average tendency for each set of waves is to undo somewhat the stress due to the other. In interpenetrating, the two sets of oppositely directed waves thus *mutually reduce the stress, the medium tends to contract, and the result is a tension along the line SE.*

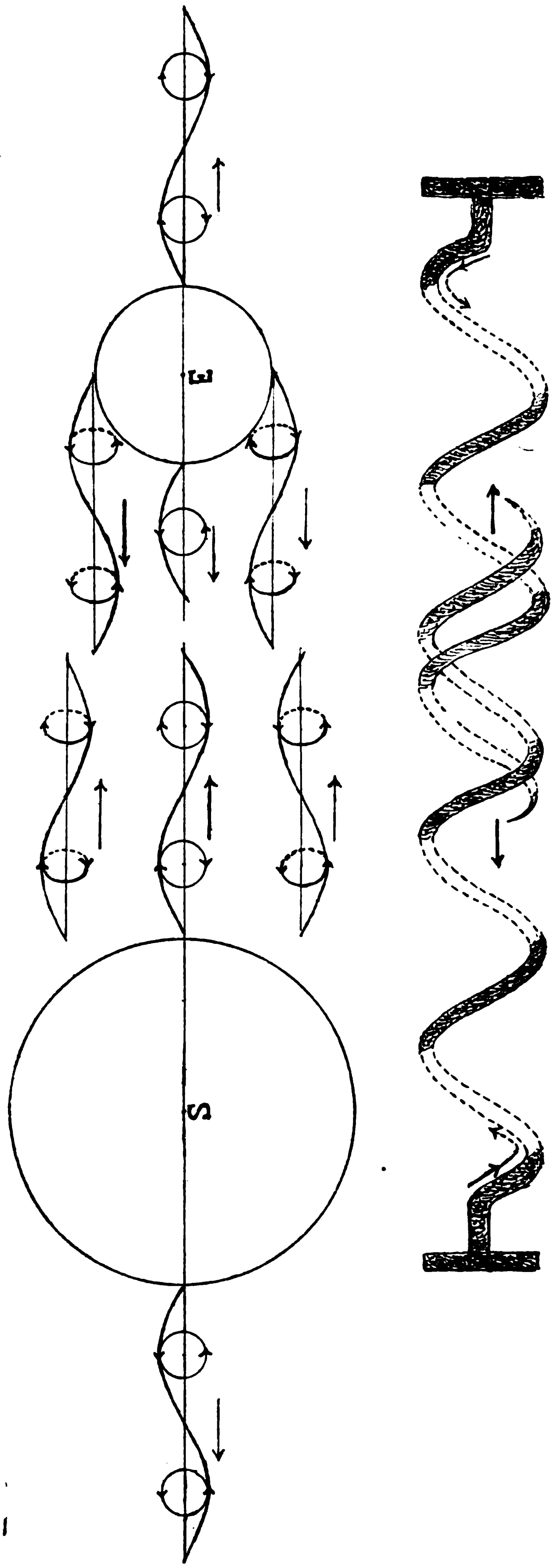

FIG. 1. Illustration of the decrease of stress and development of *tension* between the Sun and Earth; and of the increase of stress and the increase of *pressure* in the medium beyond these bodies.

From the figure we easily see how this *tension* arises in the case of *flat waves;* and now we find that the same tendency will hold for the helices, but that the power will be reduced owing to the factor depending on the cosine of an angle which may approach but never is equal to 90°.

9. Outside of the masses the waves have a common direction, and the two effects are cumulative as shown in Section III below. There is therefore a change of sign in the added wave action when the waves from either body pass beyond the other, and the two sets of waves begin to travel in the same direction.

10. *Our final conclusions in regard to the decrease of stress with development of tension between two masses, and the increase of stress with the development of pressure outside of the masses, where the two sets of waves act in the same direction, may be summarized as follows:*

11. (a) *The stress is decreased wherever similar and congruent waves travel in opposite directions — each thus undoing the other's stress — as between two heavenly bodies. Hence their attraction is due to tension between, excess of pressure beyond.*

(b) *The medium has the stress increased wherever similar and congruent gravitational waves travel in the same direction in space. Thus outside of two celestial bodies, the waves from either body support and add to the stress of the waves due to the other.*

III Exact Analysis of the Stresses Causing Universal Gravitation

The Æther is taken to be a corpuscular *moving medium,* the particles traveling 1.34 times faster than Light, in accordance with the views of Maxwell and S. Tolver Preston, *Phil. Mag.,* 1877, Vol. 4, pp. 206-213, 364-375; and having such high elasticity that all the atoms incessantly emit waves due to the corpuscular collisions, wave oscillations, or other transformation processes not yet fully understood. *All we know is that energy exists in the form of wave motion in the Æther: and these waves are incessantly transformed by contact with, and directed from, the atoms of matter.*

1. Since Magnetism is shown to be due to electrodynamic waves emitted by the atoms of iron and other magnetic substances, and these waves penetrate rapidly through all bodies, it is natural to assume that Universal Gravitation is also due to similar waves. And as the waves pervade all space, in the case of Magnetism, so will they also in the case of Gravitation. The force exerted varies

as the square of the amplitude, $a = \frac{k}{r}$, and therefore inversely as the square of the distance, $f = \frac{k^2}{r^2}$, as found by NEWTON in the *Principia*, 1687. Without an extensive system of all-pervading waves filling the universe, the resulting *forces* could scarcely have the *steadiness and continuity* they are observed to have, — which is indispensable to the preservation of the order of the world.

2. In entering upon an inquiry as to the way in which the waves act to stress the medium, and thus cause the attraction of Gravitation, we consider first a single spherical body, like the Sun. The atoms are arranged haphazard, with their equators distributed in all directions indifferently. The waves will be flat in the atomic equators, but at any point p the medium will be stressed by a preponderance of waves with rotating elements largely transverse to the line Sp, yet there will be some atoms with flat waves in or nearly coinciding with the line Sp. As the waves speed on there is thus a reaction towards the origin of the waves, which may be called the *central pressure* in the medium, substantially as conceived by NEWTON in the *Query 21* of the *Treatise on Opticks*, 1721, p. 325.

Reconciliation of the Electrodynamic Wave-Theory with NEWTON'S *View, 1721.*

NEWTON'S view that the *density* of the Æther decreases towards a central body like the Sun may be reconciled with that here developed by remembering that what he regarded as *hydrostatic pressure* we now designate by the more general term *stress*. And as the *wave amplitude* follows the law $\frac{k}{r}$, — the energy of the reaction in the medium giving the force $\frac{k^2}{r^2}$, as in Gravitation, — we see that NEWTON'S view, that a diminution of pressure inversely as the distance from the dense body would be required to explain Gravitation, can be met by using *wave amplitude* for what he calls *pressure*. When the *wave amplitude is large*, as near the attracting body, under the above law, the medium is so agitated by the large amplitudes of the passing waves as actually to have *small density*. This quite reconciles our present reasoning with that of NEWTON, 1721; and we perceive that the medium is denser inversely as the wave amplitude, and therefore directly as the distance from the center.

3. But so long as the central mass is the only source of waves, and it is spherical, with mass $m = \frac{4}{3}\pi\sigma r^3$, whether homogeneous, or made up of concentric spherical shells $dm_i = 4\pi\sigma_i r_i^2 = 4\pi\sigma_i(x_i^2 + y_i^2 + z_i^2)$ there is perfect symme-

try of stress about the origin, at the geometrical center of figure, and the stress is equal in all directions. By the rotations of the elements of the medium, as the waves pass the point p, there is a definite and steady central pressure in the agitated medium, the intensity of the pressure depending on the number of atoms in the mass emitting the waves.

4. This view of the stressing of the medium by waves and their reaction will perfectly explain the central attraction of gravitation in such a spherical body as the Sun. In 1875 MAXWELL said that hitherto he had not been able to imagine any cause for such a state of stress; but it is now traced to the reaction of the receding waves, without any hypothesis, except that of electrodynamic waves, of the type which we actually receive from the Sun. The records of "Magnetic Storms" show that they traverse the body of the globe with almost the velocity of light. Hence the electrodynamic waves are known facts, and it is shown that the larger disturbances come from the Sun, in *periodic fluctuations*, co-periodic with the Sun spot cycle.

5. To obtain a more general view of gravitation we now pass to a system of *two bodies* such as the Sun and Earth, and observe that the former perfect symmetry about S at once disappears, yet still exists in all directions about the line SE, which joins the two bodies. To investigate this new system more accurately, we place at the center of S the origin of co-ordinates of a system of rotating axes, with the axis of x coinciding with the line SE, which is taken in the plane of the Earth's orbit. Then it is evident that stresses will develop in the different regions of space as follows.

6. Whatever be the exact nature of the wave motion which gives rise to Gravitation, it is evident that beyond S, as at A, the waves from E will aid those from S. For they have a common direction of propagation, and in this direction the waves from the two centers will compound and work in concert. Accordingly, by the waves from E, *the stress in the medium throughout the infinite space beyond S is increased*. Therefore we have for the outward passing waves of both bodies giving combined inward reaction with *steady pressure* towards the center of S,

$$P' = P + p = \int\int \varpi_s dydz + \int\int \varpi_e dydz \tag{1}$$

Owing to this augmented stress, under the action of the two bodies, the medium tends to expand like compressed India rubber, in the unlimited region beyond S, and this pressure thus reacts to crowd S towards E.

7. For the same reasons, throughout the infinite space beyond E, as at B, A and B being on the line SE prolonged, we have also

$$P' = P + p = \int\int \varpi_s dydz + \int\int \varpi_e dydz \tag{2}$$

And here again the reaction of the stress in the infinite space beyond E tends by the expansion to crowd E towards S. The amount of this reaction in each case is proportional to the waves from E, and thus depends on the mass of the body E, the wave effects of which are added to those of S. *If the stress is thus proportional to the mass of either body, it necessarily is proportional to the mass of both, owing to the compounding of the waves for producing the attraction.*

8. Accordingly, we divide the whole of space into three parts: the two external parts A and B, beyond S and E; and the internal part C, between the two bodies. Let a plane be passed through the center of SE at right angles to this line, and then we may imagine the central space to extend along this plane to infinity in all directions. The supposed symmetry would hold rigorously for equal masses; but for very unequal ones, the gravitation of S so much exceeds that of E that the stress of S would everywhere prevail except in a small closed surface about E, first investigated by Dr. G. W. Hill in his celebrated *Researches in the Lunar Theory*, 1877, and thus called the Hill surface about the secondary body.

9. Temporarily neglecting, however, the limits of power of the two bodies over the interior space, we see that the waves from S, traveling towards E, will meet waves from E, traveling towards S. Each wave center gives a reaction in the medium towards that center; and in this interior region each set of waves tends to decrease somewhat the stress due to the other body. Hence for the interior region the stress becomes

$$P' = P - p = \int\int \varpi_s dydz - \int\int \varpi_e dydz \tag{3}$$

which agrees with the observed effects of gravitation.

10. It thus appears that the two bodies, acting from opposite centers, must necessarily operate to decrease the stress in the interior region, between S and E; while beyond these masses the stress is increased. The reaction of the medium in the regions A and B will yield increased central pressure, along the extended lines SA and SB, and on either side of the line of symmetry SE. On the other hand, in the region C, between S and E, there is a decrease of inward pres-

sure, as if centrifugal motion in the medium were at work about that line, giving a *tension* along it, not wholly unlike that acting along the FARADAY line of force, which causes the stress for the attraction of the opposite poles of two magnets.

IV TENSION BETWEEN THE MASSES ILLUSTRATED GRAPHICALLY BY THE FORMS OF THE EQUIPOTENTIAL SURFACES

1. The existence of this stress between the masses is made clear to the eye by the following figures of the surfaces of constant relative energy or equipotential surfaces about two equal stars, (cf. my *Researches on the Evolution of the Stellar Systems*, Vol. II, 1910, p. 169) and about the Sun and *Jove*, with the masses in the

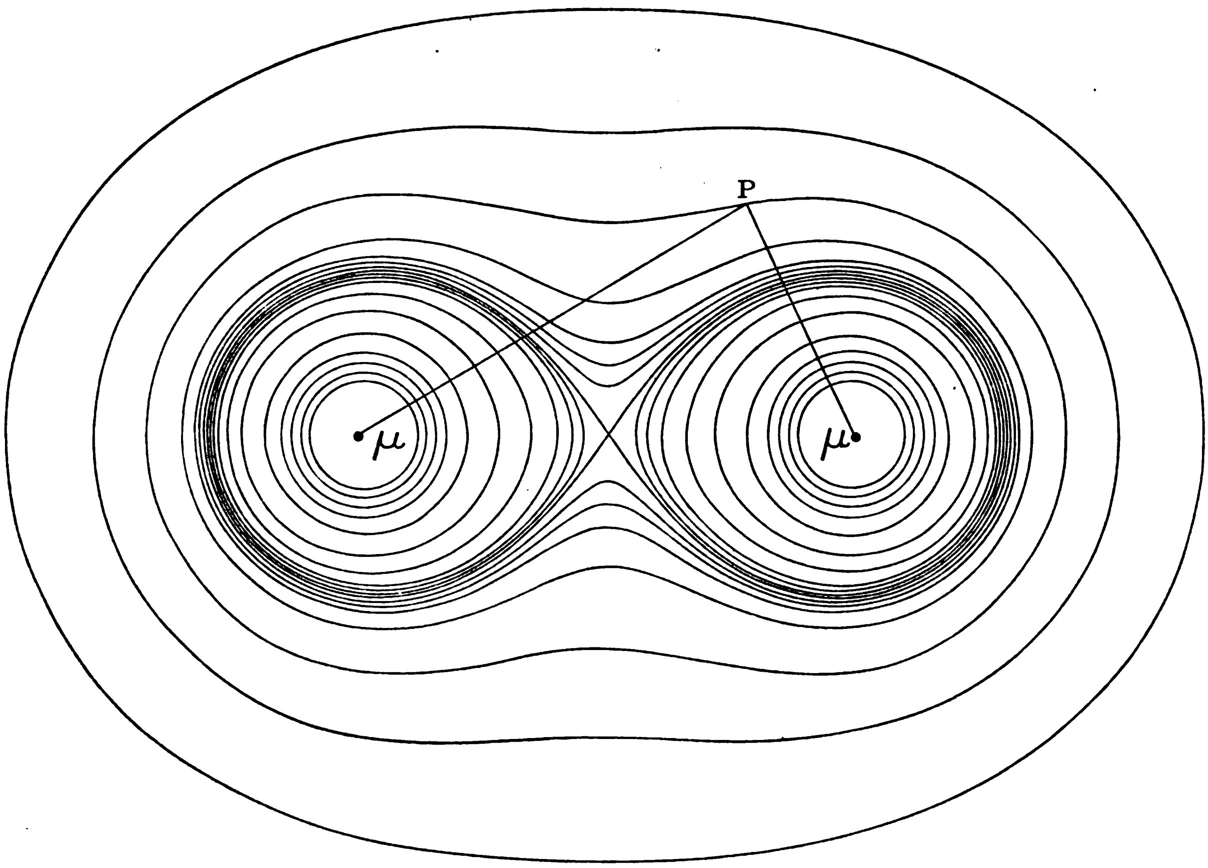

FIG. 2. Equipotential surfaces about two equal masses, such as we often find in a typical double star.

ratio of ten to one. The mathematical theory of these surfaces is fully explained in the *Researches*, Vol. II, pp. 169–171, and neither the formulæ nor the theory of the forms of the surfaces need be examined here.

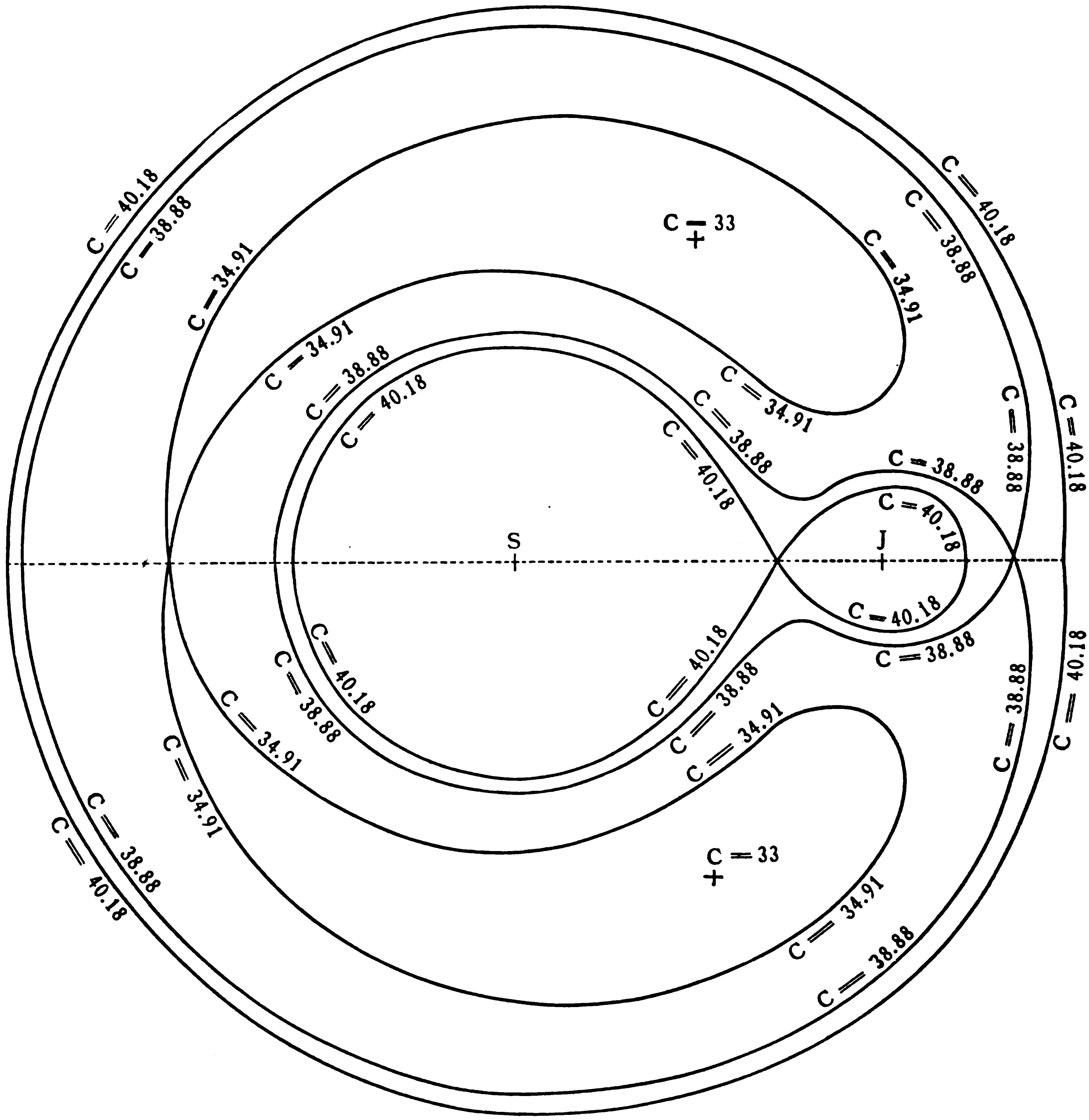

Fig. 3. Curves of zero relative velocity (Darwin)

This diagram illustrates the hour-glass shaped space through which the particle may move and drop down nearer the sun or planet, till it becomes captured by one of the larger bodies.

If the two masses are unequal, we have a special system of units which gives

$$M + m = 1 = (1 - \mu) + \mu \tag{4}$$

And the equation of the surface, referred to rotating axes, becomes (cf. *A. N.* 4341, p. 335):

$$x^2 + y^2 + \frac{2(1-\mu)}{\sqrt{(x - x_1)^2 + y^2 + z^2}} + \frac{2\mu}{\sqrt{(x - x_2) + y^2 + z^2}} = C \tag{5}$$

Under the action of a Resisting Medium the constant C of this Jacobian Integral increases with the time, the surface undergoes a secular contraction, and the satellites thus become captured, and can only move about S or J, but not about both, as the author first proved in 1909 (*A. N.* 4341). The late celebrated mathematician Poincaré verified this conclusion, (*Hypothèses Cosmogoniques*, 1911). It was also confirmed by Professor E. W. Brown in the *Monthly Notices* of the Royal Astronomical Society, March, 1911; and recently Mr. Harold Jeffreys, in the *Monthly Notices* for March, 1917, p. 448, by an independent process, reaches a similar result.

Now at every point of the surface of the hour-glass figure the total resultant force is *normal to the surface and directed inward.* There is thus a *process of constriction at work,* under the gravitational action of the two bodies, which is less powerful as the surfaces become more distant from the centers. The constant of Jacobi also correspondingly decreases, as the surfaces recede and are less constricted.

It is justly observed that the narrowing of the surface into an hour-glass form between the two bodies is a visible and obvious effect of the tension in the medium.

The pressure in the medium towards the separate centers, due to the waves propagated from those centers, thus gives *tension* between the bodies, tends to narrow the surfaces of constant relative energy, or equipotential surfaces, and gives them the forms here shown.

2. In his Article on *Attraction, Ency. Brit.*, 1875, Maxwell expressed the view that gravitational stress in the medium is a *central pressure* combined with numerically equal *tension* in all directions at right angles to the lines of gravitative force. Whilst this view of a *central pressure* towards each attractive center, due to the reaction of the receding gravitational waves, explains the attraction towards *the centers — occupied by matter* — it does not satisfactorily account for the attraction *between separate bodies,* such as the mutual gravitation between the Sun and planets. From what is shown above it is clear that planetary action is due to the interpenetration of waves from each mass. This stressing of the

medium everywhere is the corner stone of Celestial Mechanics, and involves a *tension* in the medium along the line joining the centers of the attracting bodies. Without a valid theory of the tension in the medium between the separate masses, due to the interpenetration of the waves, with decrease of stress and consequent contraction or tension in the medium, we cannot satisfactorily explain the phenomena of Universal Gravitation.

MAXWELL's statement that in the attraction of gravitation "there is a *pressure* in the direction of the lines of force combined with a tension in all directions at right angles to the lines of force," seems to imply that the *pressure* holds also between the Sun and a planet. It is therefore important to notice that there is *relative tension* in that line, but a *relative increase of pressure* in the medium beyond the two masses.

The planets are thus held in their orbits about the Sun by the *tension* of the medium between these bodies, and *the relative increase of pressure* in the infinite medium beyond. *The external pressure aids the internal tension in balancing the centrifugal force: and thus equilibrium is secured for the stable motions of the heavenly bodies.* Let it be noticed:

(a) As Magnetism undeniably is due to an infinite system of natural waves filling all space, and from any two magnets thus interpenetrating throughout the whole universe, it must be inferred that Universal Gravitation depends on similar waves, which thus likewise interpenetrate in all points of space.

(b) The *Moon's Fluctuations* prove that such waves do really exist in Universal Gravitation, and are sensibly refracted, dispersed and possibly absorbed in traversing the body of the Earth. No theory except that of electrodynamic waves will explain mathematically and physically the phenomena of the Lunar *Fluctuations*. As electrodynamic action is propagated with the velocity of Light, it follows therefore that Universal Gravitation also travels with the velocity of Light. It cannot possibly be instantaneous, as has been very generally assumed since the time of NEWTON; and corrections for the *time of propagation of gravitation* will be required in all future Astronomical Tables which aim at the highest accuracy.

V. THE MUTUAL INTERPENETRATION OF INFINITE SYSTEMS OF WAVES: CALCULATION OF THE SUM TOTAL OF GRAVITATIONAL WAVE-ENERGY IN THE UNIVERSE

1. About every pair of bodies we must therefore imagine a doubly infinite system of mutually interpenetrating waves. If the elements of mass be

$$dm_1 = \sigma_1 dx_1 dy_1 dz_1\,, \quad dm_2 = \sigma_2 dx_2 dy_2 dz_2\,,$$

and the wave energy due to these elements be forces of the type

$$\frac{k_1{}^2 a_1{}^2}{r_1{}^2}, \frac{k_2{}^2 a_2{}^2}{r_2{}^2},$$

where

$$\left.\begin{aligned} r_1{}^2 &= (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2\,, \\ r_2{}^2 &= (x' - x_2)^2 + (y' - y_2)^2 + (z' - z_2)^2\,; \end{aligned}\right\} \tag{6}$$

then the element of the stress in the medium will be

$$dP = \frac{\sigma_2 k_2{}^2 a_2{}^2}{r_2{}^2} \cos \varepsilon \frac{\sigma_1 k_1{}^2 a_1{}^2}{r_1{}^2}\, dx_2 dy_2 dz_2 dx_1 dy_1 dz_1, \tag{7}$$

where ε is the angle between the planes of the resulting rotating wave elements or total forces.

2. And the resulting attraction is therefore

$$A = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left(\frac{\sigma_2 \kappa_2{}^2 a_2{}^2}{r_2{}^2} \cos \varepsilon \frac{\sigma_1 \kappa_1{}^2 a_1{}^2}{r_1{}^2}\, dx_2\, dy_2\, dz_2\, dx_1\, dy_1\, dz_1\right) dx'\, dy'\, dz' dx\, dy\, dz \tag{8}$$

At any point of space the density of the waves is proportional to the density of the matter in the two elements of mass from which the waves proceed. Hence whilst the density applies to the masses, it can also be written in the formula for the stresses due to the waves, and we thus have a means of recognizing the bodies from which the waves proceed in developing the total forces at any point.

3. From the above considerations it appears that the bodies of the universe attract one another in pairs, owing to the stressing of the medium by a *mutual tension* between the masses and an increase of *pressure* in the regions beyond them.

If the function for the sum total of the wave action be Ω , and Δ_i be the distances above denoted by r_i, we may put for brevity

$$\begin{aligned} d\,\Omega = \frac{dm_0\, dm_1}{\Delta_0{}^2 \Delta_1{}^2} + \frac{dm_0\, dm_2}{\Delta_0{}^2 \Delta_2{}^2} + \frac{dm_0\, dm_3}{\Delta_0{}^2 \Delta_3{}^2} + \ldots\ldots + \frac{dm_0\, dm_n}{\Delta_0{}^2 \Delta_n{}^2} \\ + \frac{dm_1\, dm_2}{\Delta_1{}^2 \Delta_2{}^2} + \frac{dm_1\, dm_3}{\Delta_1{}^2 \Delta_3{}^2} + \ldots\ldots + \frac{dm_1\, dm_n}{\Delta_1{}^2 \Delta_n{}^2} \\ + \frac{dm_2\, dm_3}{\Delta_2{}^2 \Delta_3{}^2} + \ldots\ldots + \frac{dm_2\, dm_n}{\Delta_2{}^2 \Delta_n{}^2} \\ \vdots \qquad\qquad \vdots \\ + \frac{dm_{n-1} dm_n}{\Delta_{n-1}{}^2 \Delta_n{}^2}\,, \end{aligned} \tag{9}$$

in which the differentials of the masses show how they are combined in pairs.

Integrating sextuply for all space, and writing the densities σ_0 , σ_1 , σ_2 .... σ_n for the masses, we get for the sum total of the wave action of the Universe the result shown in the accompanying equation (10).

By using the double subscript (i, j), the resulting infinite expression may be written more compactly in the form:

$$(11) \quad \Omega = \sum_{i=0}^{i=i} \sum_{j=1}^{j=j} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left(\frac{\sigma_i k_i^2 a_i^2}{\Delta_i^2} \cdot \frac{\sigma_j k_j^2 a_j^2}{\Delta_j^2} \cos \varepsilon_{(i,\, j)}\, dx_i dy_i dz_i dx_j dy_j dz_j \right) dx'\, dy'\, dz'\, dx\, dy\, dz$$

It will be observed that this expression for the *Sum Total of the Gravitational Wave-Energy of the Universe,* viewed as made up of an unlimited number of bodies, thus becomes the side and diagonal of an Infinite Determinant. As remarked above, the bodies are grouped in pairs, and each element is a sextuple Integral between infinite limits.

It is difficult to imagine a more vivid picture of the intricacy of the mechanism of the Physical Universe than that here presented. The action is always propagated in right lines, across free space; and along FERMAT's minimum paths, if through solid masses, but in the free Æther the velocity of the wave action does not exceed that of Light, and through cosmical bodies the speed may be considerably less. *The Æther appears to have such high elasticity that when waves, as the vehicle of energy, are being transformed by contact with matter, they are renewed in the form of new waves appropriate to the structure of the particular atoms.*

Shorter waves than those which produce Gravitation also fill the universe, causing Heat, Light, Chemical Affinity and Molecular Forces, but they are not discussed in the present BULLETIN.

VI ASTRONOMICAL CONSEQUENCES OF ADOPTING WEBER'S ELECTRODYNAMIC LAW FOR THE SOLAR SYSTEM

Ever since the Law of Universal Gravitation was established by NEWTON, it has been the almost unbroken rule of mathematicians to assume that the actions of gravitational forces are instantaneous, at all distances, and the perihelia of the planets and satellites therefore fixed in space, as first concluded from approximate data by the illustrious author of the *Principia,* 1687.

But if at length we recognize WEBER's Electrodynamic Law as the true Law of Nature, it will become important to notice the Astronomical Consequences of substituting WEBER's Fundamental Law of 1846 for the approximate form of this law used by NEWTON.

The probability of the adoption of the Electrodynamic Law, in place of the approximate form found by NEWTON and since used by astronomers and mathematicians in the theories of the heavenly motions, has led to the suggestion that the results, in the form of the well known progression of the line of apsides, should be carefully calculated and tabulated for all the chief bodies of the Solar System. These results should include not only the major planets, but also the very different types of bodies represented by comets and satellites.

About the only appreciable changes which will be required in our Astronomical Tables are slight progressive motions of the perihelia. These consequences are shown in the accompanying carefully calculated tables. They were prepared in 1915, and with the following discussion communicated to Lord RAYLEIGH, July 10, 1915.

THEORETICAL PROGRESSION OF THE PERIHELIA, ACCORDING TO WEBER'S ELECTRODYNAMIC LAW

Planet	log. a	nt. in a Century	e	$\delta\varpi$ in a Century
Mercury	9.5878217–10	538101629.″25	0.20560478	14.″511
Venus	9.8593378–10	210664139.80	0.00684331	2.9125
The Earth	0.0000000	129597741.516	0.01677110	1.2964
Mars	0.1828971	68905080.14	0.09326113	0.45619
Jupiter	0.7162375	10925662.552	0.048255511	0.02104
Saturn	0.9794956	4399621.506	0.05606025	0.004613
Uranus	1.2831044	1542575.2	0.0469236	0.00080395
Neptune	1.4781414	786493.5	0.0084962	0.0002615

PERIODIC COMETS

Comet	a	Period	e	$\delta\varpi$
ENCKE	2.218375	3.304 yrs.	0.84600	0.″62198
TEMPEL	3.03245	5.281	0.54211	0.11461
BRORSEN	3.09907	5.456	0.81034	0.223072
TEMPEL–L. SWIFT	3.18275	5.678	0.63817	0.120988
WINNECKE	3.23857	5.828	0.71488	0.140366
DE VICO–E. SWIFT	3.49718	6.400	0.51566	0.078878
TEMPEL	3.49655	6.538	0.40194	0.067616
FINLAY	3.50279	6.556	0.72324	0.118332
D'ARREST	3.54920	6.686	0.62611	0.089622
BIELA (I)	3.55102	6.692	0.75242	0.12570

PERIODIC COMETS (*Continued*)

Comet	a	Period	e	$\delta\varpi$
WOLF	3.60506	6.845 yrs.	0.55534	0.″075939
HOLMES	3.61520	6.874	0.41135	0.062773
BROOKS	3.69455	7.101	0.46978	0.063389
FAYE	3.79385	7.390	0.56516	0.066373
TUTTLE	5.71621	13.667	0.82171	0.051076
PONS–BROOKS	17.23689	71.560	0.95500	0.011943
OLBERS	17.41125	72.650	0.93113	0.0077034
HALLEY	17.95545	76.080	0.96173	0.012637
NEWTON, *1680*	426.68	8813.8	0.9999854	0.0121184
DONATI, *1858 VI*	156.064	1949.7	0.9962933	0.0057557
1843 I	65.7108	533.0	0.9999157	0.219474
1882 II	84.069	772.0	0.9999078	0.1081733

SATELLITES OF THE PLANETS

Satellite	Period	a in Kilometers	e	$\delta\varpi$
The Moon	$27^d.32166$	384400	0.05489972	0.′00637
MARS:				
Phobos	$7^h.6542$	9377	0.02673	0.02651
Deimos	$30^h.2983$	23475	0.00591	0.011098
JUPITER:				
V	$11^h.9563$	180936	0.00308	4.233655
I	$1^d.7698605$	421632	0.001	1.821385
II	3.5540942	670859	0.001	1.14345
III	7.1663872	1070067	0.001335	0.715541
IV	16.7535524	1882150	0.007278	0.40508
VI	$250^d.618$	11456800	0.1550	0.068685
VII	265.0	11891000	0.0246	0.064658
VIII	930.73	27475000	0.44	0.034681
IX	773.6	24286000	0.27	0.034128

SATELLITES OF THE PLANETS (*Continued*)

Satellite	Period	a in Kilometers	e	$\delta\varpi$
SATURN:				
Mimas	$0^{d}.94242$	185465	0.0187	$1''.2403$
Enceladus	1.37022	237942	0.00455	0.966394
Tethys	1.887796	294555	0.00172	0.78066
Dione	2.736913	377258	0.00230	0.60811
Rhea	4.517500	526847	0.00085	0.43644
Titan	15.945417	1221340	0.02880	0.188423
Hyperion	21.277396	1479622	0.1242	0.157842
Japetus	79.329375	3559253	0.0290	0.102376
Phœbe	546.5	12886600	0.22	0.018751
URANUS:				
Ariel	2.520383	191312	0.001	0.18439
Umbriel	4.144181	266526	0.001	0.13235
Titania	8.705897	437174	0.00683	0.080504
Oberon	13.463269	584626	0.00837	0.060339
NEPTUNE:				
Satellite	5.87690	355518	0.010	0.185913

The formula developed in TISSERAND'S *Mécanique Céleste*, Tome IV, Chapter XXVIII, namely, $\delta\varpi = \left(\frac{V_0}{c}\right)^2 \frac{a_0}{a\,(1-e^2)}\,nt$, will apply to any body moving around the Sun, including the periodic comets, whatever be the eccentricities of their orbits.

No restrictions are imposed upon the eccentricity of the orbit of the body revolving about the Sun, in the derivation of this formula; and therefore it applies to the most eccentric ellipses described by the comets as rigorously as to the nearly circular orbits of the planets.

Yet when we come to extend the formula to bodies of the secondary systems of satellites revolving about small masses, it is necessary to modify it accordingly, to take account of the feeble central attraction of the planets.

This could be done by introducing ω, the ratio of the actual absolute velocity of the satellite to that of a fictitious body revolving in the same period about the Sun:

$$\delta\varpi' = \left(\frac{V_0}{c} \cdot \omega\right)^2 \frac{a_0}{a' (1 - e^2)} n't ; \tag{12}$$

where a' and n' refer to the semi-axis major and mean motion of the fictitious satellite revolving about the Sun, with the observed mean motion of the actual satellite.

But the calculation becomes somewhat simpler by the use of the following reductions. Neglecting the squares and higher powers of the eccentricity, the velocity V_1 for an intermediary planet, such as *Mercury*, and V for a satellite revolving about a planet of mass m becomes:

$$V_1^2 = \frac{k^2}{a_1} ; \quad V^2 = m \frac{k^2}{a} ; \tag{13}$$

where k^2 is the Gaussian constant, and the mass of the planet m is expressed as usual in units of the Sun's mass.

Accordingly, the ratio of the forces to the two centers of attraction becomes:

$$\frac{V^2}{V_1^2} = m\left(\frac{a_1}{a}\right) \tag{14}$$

Substituting this expression in the above formula (12), we get:

$$\delta\varpi' = \left(\frac{V_0}{c} \cdot \frac{V}{V_1}\right)^2 \frac{a_0}{a_1 (1 - e^2)} n_1 t = m\left(\frac{V_0}{c}\right)^2 \cdot \frac{a_1}{a} \cdot \frac{a_0}{a_1 (1 - e^2)} n_1 t ; \tag{15}$$

which takes the simple form

$$\delta\varpi' = m \left(\frac{V_0}{c}\right)^2 \frac{a_0}{a (1 - e^2)} n_1 t . \tag{16}$$

Here e is the eccentricity of the satellite orbit; $\frac{a_0}{a}$ the value of the Earth's mean distance, in units of the mean distance of the satellite; $n_1 t$ is the centennial mean motion of *Mercury;* m the mass of the planet about which the satellite revolves; and the ratio of the average velocity of the Earth's motion to the velocity of light $\frac{V_0}{c} = 1 : 10000$, log. $\left(\frac{V_0}{c}\right)^2 = 2.0000000 - 10$

The calculation of the centennial progression of the apsis lines of the satellite orbits is thus very simple, and the resulting values are included in the accompanying table.

As NEWTON points out in the *Principia*, the form of the law of attraction employed by him gives fixed *perihelia* and *periplaneta*. In WEBER'S law there is a slight progression, becoming larger with the orbital velocity or with slowness of the propagation of the attractive force across space. In our calculations we have always taken c = velocity of light, because in the Electrodynamic Theory all other velocities are excluded.

It will be seen that the perihelion of *Mercury* is the only outstanding motion of sensible magnitude. With a theoretical centennial progression of $\delta\varpi = 14''.51$, the oustanding difference of about 40″, between theory and observation first found by LEVERRIER in 1859 and since confirmed by NEWCOMB and others, is thus reduced to about 25″ per century. This is a decided improvement in one of the most difficult problems of Celestial Mechanics.

The fifth satellite of *Jupiter*, discovered by Professor BARNARD in 1892, has been found by observation to have a motion of its perijove of about 900° per annum, owing to the effect of the large oblateness of the figure of the planet. Thus, whilst the theoretical centennial progression of the perijove is now found to be 4″.23, it would take the finest observations over at least a hundred years before an effect so minute as 4″.23 in a century, with a total gravitational progression of 90000° = 324000000″, could become sensible, if indeed it ever could be recognized by observational test.

The motion of the Lunar Perigee is very accurately known, and at first sight one might suppose it would shed some light on the propagation of gravitation in time. Our eminent Lunar Theorist, Professor E. W. BROWN, assures me that the outstanding difference between observation and theory is less than 15″ per century, and most of this to be accounted for by slight admissible changes in the adopted oblatenesses of the Earth and Moon. On calculating the progression depending on WEBER'S law, I find it to be only $\delta\varpi = 0''.00637$ per century—a residual so very minute that it could not accumulate to sensible magnitude in less than 100,000 years!

Accordingly, it appears that neither the satellites nor the comets offer the slightest chance of an outstanding observational difference between the laws of WEBER and of NEWTON. For this purpose we must depend on the motion of the perihelion of *Mercury* alone. Yet as this outstanding difference tells decidedly in favor of WEBER'S law, as do also the periodic fluctuations of terrestrial magnetism — otherwise quite unintelligible — one is left with a clearly defined choice between the electrodynamic law and the traditional law without physical significance. To those who believe genuine physical causes underlie the operations of Nature, the decision necessarily will be in favor of the Electrodynamic Law.

VII Applications of Weber's Electrodynamic Law to the Motions of Binary Stars*

Actual calculation of the motions of the perihelia of the planets and comets, and of the periplaneta of the satellites of the solar system shows that there are no outstanding phenomena of our system which enable us to discriminate observationally between the effects of the ordinary law of gravitation, as given by Newton, and the Electrodynamic Law developed by Weber, except in the single case of the progression of the perihelion of *Mercury*, where Weber's law reduces the outstanding difference to about 25″ per century, instead of the 40″ per century found by observation from the Newtonian Law.

In order to take account also of every possible criterion offered by the sidereal universe for throwing light on the fundamental law of Nature, I have carefully examined the best determined physical systems among the Binary Stars, and found that no contradiction of Weber's Law may be anticipated from our studies in the sidereal heavens.

If Universal Gravitation be an Electrodynamic Phenomenon, as set forth in the memoir on this subject, dated Dec. 10, 1914, it is remarkable that the enormous velocity of propagation of this force, with the speed of light, enables the double stars to revolve in their orbits without disclosing motions of their periastra which could become sensible to observation in less than something like 100,000 years. This may be investigated as follows:

It is easy to demonstrate that for any Binary System of combined mass, $M + m$, the formula for the secular progression of the periastron becomes:

$$\delta\varpi = \left(\frac{V_0}{c}\right)\frac{M + m}{1 - e^2}\frac{a_0}{a}\,nt\,, \tag{17}$$

where e is the eccentricity of the orbit of the Binary, $\frac{a_0}{a}$ is the ratio of the Earth's mean distance to the mean distance of the components of the Binary, nt the mean motion of the Binary in a Julian century, and $\frac{V_0}{c}$ the same constant which was used in the solar system, $\text{Log.}\left(\frac{V_0}{c}\right)^2 = 2.000\,0000 - 10\,.$ (18)

* Extract from a letter to Lord Rayleigh, dated March 25, 1916.

The accompanying table gives the necessary elements for a dozen stellar systems. The accuracy of the data is unequal in different cases, owing to the minuteness of the annual parallaxes, and the difficulty inherent in measuring such small angular displacements. Yet in general the parallaxes are dependable, in giving us approximate absolute dimensions of the Binary orbits and the combined masses of their components.

In a few cases, such as 40 o_2 *Eridani* and *Castor*, the geometrical elements of the orbits are not yet defined with great accuracy; and in the case of *Algol* and *Capella* I have taken the eccentricity as 0.10, in conformity with the general

TABLE OF DATA ON BINARY STARS

Name of Star	$M+m$	a (Sun's Distance = 1)	e (Eccentricity)	P Period	Log nt (Julian Cent.)	$\delta\varpi$ (Cent. Motion Periastron)	π (Parallax Used)
η *Cassiopeiæ*.	4.0	53.3	0.51	196 yrs.	5.8203413	0″.00066902	0″.154
β *Persei*.....	0.67	0.034731 / 3230000 miles	0.10	2.8 days	10.2280296	3291.927	0.035
40 o_2 *Eridani* ...	1.64	38.	0.134	180 yrs.	5.8573249	0.000316413	0.166
α *Aurigæ*....	15.38	1.0763 / 100100000 miles	0.10	104 days	8.6581543	65.7045	0.08
α *Canis Maj.*	3.473	21.136	0.62	50 yrs.	6.4136274	0.0069185	0.37
α *Geminorum.*	12.66	115.	0.4409	346.82 yrs.	5.5724933	0.00051013	0.05
α *Canis Min.*	2.6	16.	0.45	40.0 yrs.	6.5105374	0.0066018	0.325
γ *Virginis*...	3.3	54.	0.8974	194.0 yrs.	5.8246957	0.00208893	0.074
α *Centauri*...	2.0	23.6	0.528	81.1 yrs.	6.2035765	0.0018777	0.75
70 *Ophiuchi* ..	2.83	28.07	0.50	88.4 yrs.	6.1661451	0.001968103	0.162
δ *Equulei* ...	1.9	4.	0.54	5.7 yrs.*	7.3567225	0.1524535	0.07
85 *Pegasi*	11.3	16.6	0.388	24.0 yrs.	6.7323862	0.0432729	0.054

*HUSSEY

law of small eccentricities for Spectroscopic Binaries found by observation (cf. my *Researches*, Vol. II, 1910, p. 571.) The orbit plane of *Capella* has also been taken to coincide nearly with the visual ray, as in the case of Algol, and the well known double star, 42 *Comæ Berenices*.

The column $\delta\varpi$ gives the motion of the periastron, in a Julian century, as deduced from WEBER'S Electrodynamic Law, in which the force of attraction between the masses is transmitted with the velocity of light. Thus for such well-determined systems as *Sirius*, *Alpha Centauri* and 70 *Ophiuchi*, we find the centennial motions of the periastra to be 0″.0069, 0″.00188, and 0″.00197 respectively. The orbits of these systems are comparable to that of the planet *Uranus*, but the masses are larger than that of the Sun and *Uranus*, and the eccentricities much higher than those of our major planets.

The larger Binary masses and their higher eccentricities both contribute to the augmentation of the motion of the periastra; yet it will be found that the theoretical motions here given are comparable to the theoretical motions of the perihelia of our major planets. In the outer parts of our solar system and among most of the Binary Stars, these motions are so very minute as to lie hopelessly beyond the reach of observation.

In the case of close Binaries, like *Algol* and *Capella*, the motions of the periastra are indeed large,—namely 3292″ and 66″ per Julian century respectively,—yet the components are so close together that the orbits are wholly invisible in our most powerful telescopes, and thus considerable uncertainty attaches to their geometrical elements.

Accordingly, it appears that the orbits of wide systems like *Sirius* and *Alpha Centauri* are well defined, but the motions of the periastra are slow. In such wide systems the attractive forces are comparatively feeble. Close systems, like *Algol* and *Capella*, and δ *Equulei*, revolve rapidly under enormous central forces and have motions of the periastra which are comparatively rapid; yet the orbits are either wholly invisible in our telescopes or so poorly defined, owing to the difficulty of separating the components telescopically, that an uncertainty of several whole degrees usually attaches to the position of the periastron.

The difficulty of fixing the geometrical elements of the orbits of Binaries with great accuracy results inherently from the great distances of the fixed stars from the Earth; and it is not easy to see how it can be overcome. It is true that *Algol*, *Z Herculis*, and some other variables disclose a slow shift of their eclipse paths, but as we cannot see how many unseen bodies are perturbing the systems or what oblatenesses of figure the component stars may have for producing analogous effects, it is clear that such systems offer very little chance for an observational criterion which will enable us to detect a progressive motion of the periastron depending on WEBER's Electrodynamic Law.

Accordingly, it is probable that the well defined excess in the motion of *Mercury's* perihelion will long remain our most useful observational criterion. It is doubtful if any corresponding criterion in the stellar universe could be established inside of thousands of years. Thus the periastron of 85 *Pegasi* would have a progressive motion of 43″ in 100,000 years, and the closer binary δ *Equulei* would require about thirty-eight thousand years to develop an equal progression of the line of apsides. But double star orbits are so minute compared to the large and well defined orbit of *Mercury* that it seems well-nigh hopeless to search for a criterion of WEBER's Electrodynamic Law among the fixed stars.

It is believed that these calculations may be useful, in that they fully authorize us to abandon unconditionally the traditional view of fixed perihelia, which has come down from the days of NEWTON. It appears to be incontestible that universal gravitation is an Electrodynamic Phenomenon and thus propagated across space with the velocity of light. The resulting progression of the perihelia and periastra is so small that only very slight corrections to the Newtonian law are required to enable it to represent all the phenomena of the heavens. Yet, as the cause of universal gravitation long remained unknown and proved utterly bewildering to all investigators, I venture to think that the Electrodynamic Theory may justly deserve the attention of natural philosophers.

VIII THE ELECTRODYNAMIC WAVE-THEORY OF UNIVERSAL GRAVITATION THE ONLY THEORY ADMISSIBLE.

1. In order to judge of the possibility of other causes than wave action we should first recall the Cause of Magnetism which is so general a phenomenon in the physical world, and of which the cause is definitely made out.

(a) In BULLETIN No. 2 the property of *Magnetism* is traced to the action of flat waves proceeding from the atoms, and all known magnetic phenomena are harmonized and co-ordinated.

(b) In BULLETIN No. 3, *Cosmical Magnetism* is traced to similar waves proceeding from the atoms, which are therefore of sufficient length to traverse the celestial bodies, but apparently with considerable loss of efficiency.

(c) It may be that the waves operative in Gravitation are the longest of the electrodynamic waves, but as those operative in Magnetism can be shown experimentally to penetrate all physical masses, it would not seem that there could be much difference in the wave length, if any.

(d) It will not do to deny that some gravitational waves may be so short as to be largely cut down in traversing the Earth; for the Moon's *Fluctuations* show that the decrease of the Sun's effective energy is experienced near the Earth's shadow, when the Sun's gravitational action is exerted through the body of the globe. There may be a great deal more gravitational wave energy rendered ineffective in masses than we heretofore have been aware of. As the figures of the heavenly bodies are spheroidal this decrease would be largely symmetrical in all directions, and would not be recognizable from the observations heretofore available.

2. *If Magnetism is universal in Matter,* — *as* FARADAY'S *experiments showed,* — *and its cause definitely traceable to waves, as shown in* BULLETIN No. 2 — *then it will follow that Gravitation also is due to similar electrodynamic waves.* For as waves will explain all the observed phenomena, we are not authorized by NEWTON'S Second Rule of Philosophy to ascribe *similar effects* in the way of attraction to *different causes.*

3. According to Modern Physics there are two distinct entities in the Universe: *Matter and Energy,* both indestructible, yet both transmutable into various forms. MAXWELL points out that in wave motion half the energy is potential, half kinetic, and thus the energy which pervades the medium when waves are in propagation is evenly balanced. It is only when wave energy undergoes transformation by contact with *matter* that inequality in the two types of energy develops. It seems to be matter alone that has the power to *transform the energy in waves,* receiving some wave energies and emitting others.

We do not yet understand the entire mechanism involved in this transformation. But since *Light* and *Heat* long ago were shown to be due to waves in the Æther, and now in BULLETIN No. 2 *Magnetism* is traced to the same agency, by perfectly definite proof, it is natural to hold that *Universal Gravitation* also is due to the same physical cause. This follows from *general principles of continuity,* and besides is supported by very specific phenomena in the Lunar *Fluctuations* which appear to admit of but one interpretation.

5. The phenomena of Cosmical Magnetism which show "Magnetic Storms" to occur simultaneously throughout our globe and to be traceable to prominence outbreaks and other disturbances in the Sun, furnish direct and obvious proof that such electrodynamic waves come from the Sun incessantly. Owing to the interruption of the flow by motion, the waves are thrown into a violent agitation when solar mass-movements are in progress. Any denial of this physical relationship, made obvious through the passage of electrodynamic waves between the Sun and Earth, is wholly untenable.

6. Accordingly, since the *Cosmical Magnetism* of the Sun and Moon admit of explanation only on the hypothesis of interpenetrating waves traveling with the Velocity of Light, and conforming to WEBER'S Law, we cannot admit that

NEWTON'S law is other than a close approximation. The old view that Gravitation is not propagated in time is no longer to be entertained.

7. It is obvious from the physical cause of *Magnetism*, and of *Electrodynamic Action*, established in BULLETIN No. 2, and such cosmical phenomena as the *Magnetic Tide depending on the Moon*, discussed in BULLETIN No. 3, that WEBER'S Electrodynamic Law governs the material universe. The celebrated equation of LAPLACE, for external space,

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0\,, \tag{19}$$

which holds rigorously true for the Newtonian Law, is only a close approximation under the Law of Nature where $\nabla^2 V = \alpha$, (20)
a series differing from zero. WHITTAKER'S general solution of the equation of LAPLACE (*Monthly Notices*, 1902, p. 617) obviously has corresponding limitations.

8. In like manner the more general equation of POISSON for the interior space of bodies

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -4\pi\sigma\,, \tag{21}$$

also ceases to hold rigorously true, and becomes

$$\nabla^2 V = -4\pi\sigma\beta \tag{22}$$

where β is a series differing from unity.

The modifications of the equations of LAPLACE and POISSON required by the use of WEBER'S Law depend on the second and third terms of the formula for the force:

$$F = \frac{mm'}{r^2}\left\{1 - \frac{1}{c^2}\left(\frac{dr}{dt}\right)^2 + \frac{2r}{c^2}\frac{d^2r}{dt^2}\right\}, \tag{23}$$

which represent the *Induction* and the *change of the Induction* respectively. Owing to an effect like that of DÖPPLER'S Principle, relative motion must always disturb the otherwise steady wave-field about every body.

9. When a body such as a comet undergoes a very rapid motion towards the Sun, as in approaching perihelion, this Inductive effect may become very large and probably will explain much of the observed sudden increase in brightness. In the same way Binary Stars revolving in very eccentric orbits may frequently develop sufficient luminosity from this changing Inductive Action to become notably *variable*.

In the course of time it will be an interesting question to find out what proportion of our *Variable Stars* may be explained in this way: at present we only know that the body of the phenomena of the light changes indicate dependence on orbital motion.

10. The eruptions of solar prominences frequently are sudden enough, and involve such large masses of matter in *relative motion*, as respects the Earth, that the observed "Magnetic Storms," Auroræ, "Earth Currents," etc., should occasion no surprise to the investigator. Thus whilst to the penetrating mind of GAUSS the Aurora was a *räthselhafter Erscheinung,—puzzling appearance*—it is to those who correctly interpret WEBER'S Law a necessary consequence of the Fundamental Electrodynamic Law which governs the physical universe.

It may be useful to emphasize certain results of these researches by the following analysis of the Cause of Gravitation.

11. The *Lunar Fluctuations* are explicable only by the refraction, dispersion and perhaps absorption of gravitational wave-energy. This physical action gives a new *distribution of the waves in space,* and the attractive force exerted is correspondingly modified, owing to the interposition of the Earth in the path of the Sun's gravitational waves.

12. If this argument be valid it follows that gravitation is explicable only by the Electrodynamic Wave-Theory. Thus the *Lunar Fluctuations* become an *experimentum crucis* for establishing the undulatory nature of gravitation.

13. We might experimentally determine if the waves of a magnet suffer a similar refraction, dispersion, etc., when they are made to pass through an artificial globe made up of layers becoming denser towards the center—for example, rock on the outside, and metals such as zinc, iron, copper, lead, within.

14. Such an experiment would be delicate, no doubt, but it easily could be made in our laboratories, and owing to the great interest attaching to it, ought to be tried very carefully.

15. In his *Opticks*, 1721, *Query 21*, p. 326, NEWTON *says that small magnets are stronger in proportion to their bulk than large ones: which shows by high authority —as is also recognized by modern writers—that some of the wave-energy is rendered ineffective when the transmission of the magnetic force is through a dense mass.*

16. This remark of NEWTON conforms to our present views entirely, since we know by the Moon's actual *Fluctuations* that the Sun's gravitation is decreased in efficiency by transmission through the solid body of the Earth.

17. This acknowledged *decrease of effective magnetism* by transmission through solid magnetizable bodies, and the *decrease of the Sun's effective gravitation* by transmission through the body of the Earth, — shown by the observed *Fluctuations* of the Moon's mean motion, — would seem to point to a general decrease in the efficiency of Gravitation when it has to be exerted through other matter. *Thus there would be more matter in the interior of all the heavenly bodies than is indicated by their power of attraction on external masses, such as planets, comets, satellites.*

18. Now the masses of the Sun, planets and satellites, are calculated from their observed actions on neighboring bodies; and if a screening of Gravitation occurs, the effect of such decrease in the force of attraction will be the same upon all external masses, and we cannot discover it by observation. Thus, up to the present time the researches of astronomers throw but little light on the amount of matter within the heavenly bodies. They have simply calculated the amount of matter within these masses which may make itself effective by external attraction; and the amount of matter actually there may be considerably larger than we have heretofore believed.

19. The remark of NEWTON near the close of the *General Scholium* to the *Principia* (1713), that "gravitation must proceed from a cause that penetrates to the very centers of the Sun and planets, without suffering the least diminution of its force," should therefore be accepted with great reserve. For at last we seem to have proof, drawn from the Lunar *Fluctuations*, that the efficiency of gravitation is decreased by transmission through solid masses, just as in the case of Magnetism mentioned by NEWTON in the *Opticks*, 1721, p. 326.

20. In the same paragraph of the *General Scholium*, NEWTON says that gravity "operates not according to the quantity of the surfaces of the particles upon which it acts (as mechanical causes use to do), but according to the quantity of the solid matter which they contain, and propagates its virtue on all sides to immense distances, decreasing always in the duplicate ratio of the distances. Gravitation towards the Sun is made up out of the gravitation towards the several particles of which the body of the Sun is composed; and in receding from the Sun decreases accurately in the duplicate proportion of the distances as far as the

orb of *Saturn*, as evidently appears from the quiescence of the aphelia of the planets; nay, and even to the remotest aphelia of the comets, if these aphelia are also quiescent. But hitherto I have not been able to discover the cause of these properties of gravity from phenomena, and I frame no hypotheses."

21. From the above considerations it appears that a decrease of gravitation, when it is transmitted through solid masses, is consistent with the reasoning of NEWTON; for this does not modify the law of distances, in surrounding space, on which the calculations of the motions of the planets and comets depend. It is only when a satellite passes near the shadow of a planet, that a sensible difference can be observed; and hence the high importance attached to the discovery of the cause of the *Fluctuations* of the Moon's mean motion.

22. Up to the present time no other known celestial phenomenon has been available for throwing a clear light on the profound mystery so long attaching to the Cause of Universal Gravitation. In the case of the Moon's *Fluctuations* this illumination is possible only by virtue of the great accuracy of the modern Lunar Theory, as the result of two centuries of laborious research by the most eminent mathematicians. For these reasons the present researches on the Cause of Universal Gravitation deserve the earnest consideration of astronomers and natural philosophers. It seems that at least a beginning has been made in unfolding some of the deepest mysteries of the Universe! Are we not about to obtain a distinct vision of things never before contemplated by mortal eye since the creation of the globe?

It is well known that GAUSS chose as his motto: "Thou, Nature, art my goddess; to thy laws my services are bound." (*Shakespeare*, King Lear, Act I, Scene II.)

If others seek to follow GAUSS in the sagacious suggestion herein contained, and thus trace the observed phenomena of Nature to their underlying Electrodynamic Law, they will be able to see more clearly than heretofore has been possible the connection between such purely physical phenomena as Magnetism and the stupendous Gravitational Forces which control the motions of the planets in their orbits.

CONCLUSION

Sir ISAAC NEWTON left the problem of the Cause of Universal Gravitation unsolved, and up to within a few years the most conflicting views of the nature of Gravitation have prevailed. On the one hand certain eminent authorities, either from habit or the force of venerable tradition, continue to adhere to the Mediæval conception of *Action at a Distance.* The French astronomers TISSERAND and POINCARÉ, on the other hand, have followed NEWTON, FARADAY and MAXWELL, and plainly recognize that there can be no action across space without a medium for communicating the action. Yet they admit that nothing is known as to how gravitation is propagated, or by what process physical forces are generated. This is about the view taken by HELMHOLTZ, KELVIN, DARWIN, LARMOR, NEWCOMB and HILL; and so recently as 1915 the English mathematicians JEANS and EDDINGTON have expressed the view that nothing is known as to the mechanism involved in electrical action through the æther, or the Cause of Gravitation.

Now it is evident that some *active influence* emanates from bodies to cause their *Mutual Gravitation.* In the fourth letter to BENTLEY, Feb. 25, 1692-3, Sir ISAAC NEWTON says we cannot imagine such an active influence to proceed from "inanimate brute matter," and to act at a distance through a void. This Mediæval idea likewise fails to explain the phenomena of Magnetism and of Cosmical Magnetism, such as the Moon's *Magnetic Tide,* and the "*Magnetic Storms,*" occurring simultaneously throughout the world.

Since Magnetism plainly is due to Electrodynamic Waves, it is obvious that the only way we can explain Gravitation is through *similar waves in the æther,* which appears to be so highly elastic a medium that *all matter* has the power to transform the waves now existing into others appropriate to the kind of matter acting. In our present ignorance of the atomic structures we cannot explain the full process by which this transformation occurs, or how the generation of the new waves arises.

But to us it is enough that such Electrodynamic Waves really exist, are shown to arise from the atoms, and to be propagated from them according to the laws which we have explained, and abundantly serve to account for the physical properties of Magnetism and of Universal Gravitation. Thus they indicate the correlation of all natural forces and tend to harmonize and co-ordinate the chief phenomena of Nature.

It is not our present purpose to enter into any discussion of the history of the Wave-Theory. But we think special attention should be directed to the earliest ideas of Dr. ROBERT HOOKE on this subject, 1671, (*Posthumous Works* of ROBERT HOOKE, edited by R. WALLER, London, 1705, pp. 181–190).

In view of what is now shown of the relationship between Electrodynamics and Gravitation, we must regard every Sun, Planet, and Satellite as a center of Electrodynamic Waves. Owing to this agitation, the *density of the æther* increases outwardly inversely as the *wave amplitude,* and thus directly as the distance. The waves stress the medium and thus give the *central pressure* required by NEWTON in 1721 for explaining Universal Gravitation.

Accordingly, it appears that after the lapse of nearly two centuries the early ideas of NEWTON have triumphed! Yet there will still remain many problems requiring further elucidation. Above all it is necessary to diffuse a knowledge of the Electrodynamic Wave-Theory of Gravitation among investigators, in order that it may receive adequate attention from men of science working from different points of view. Investigators may thus advantageously co-operate in extending the discoveries of KEPLER and NEWTON; and by connecting the Planetary Forces with those of Magnetism and Electrodynamics, develop a Modern Theory of the Physical Causes underlying the Laws of Nature. Such an extension of the existing Physical Sciences is an urgent *desideratum* of our time.

If the way to this exalted goal does not yet seem entirely clear and inviting, let us remember that the ascent of a range of mountains whose summits seem to near the stars has often appeared difficult and forbidding; yet also recall that when a path is once opened, the explorer who dares to persist will finally attain the dizzy heights above the clouds, and be rewarded with a wider *View of Nature*, which is always an ultimate object of research in the sublimer portions of Human Knowledge.

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