ON THE FORWARD MOTION OF BOSTICK PLASMOIDS

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A theoretical interpretation is offered of the experimental observation that the velocity of Bostick plasmoids in vacuo, directly away from a button source, depends only upon the energy of the source capacitors in the button gun circuit.

When plasmoids are created by a pulsed capacitor discharge across the electrodes of a button gun in vacuo, experimental observations have revealed that the forward velocity of the plasmoids depends only upon the electrical energy of the source capacitors [1]. It is the purpose of this letter to offer a simple interpretation of this experimental observation in terms of the traditional toroidal model of Bostick plasmoid.

Because of Bostick's observation that a plasmoid travels from the button source as a plasma loop with a magnetic flux trapped within it for many microseconds [2], it follows that the plasma conductivity must be so high that the energy lost in Joule heating of the plasmoid, while it is still being formed by the source capacitor discharge, is negligible compared with the magnetic energy stored within the plasma loop. Now, it is the asymmetric magnetic forces created by the surge current flow that causes the plasma loop to expand and thus be projected away from the button gun. The magnetic energy W, therefore, is converted into kinetic energy of motion and we have

$$W = \tfrac{1}{2}\int_0^T I^2 \frac{dL}{dt}\,dt + \tfrac{1}{2}\int_0^T L\frac{d}{dt}(I^2)\,dt$$

where L is the self inductance of the torus, I is the surge current and T is the duration of the surge current. By using the first theorem of mean value [3], and the boundary conditions: $I(T) = I(0) = 0$, it follows that we may define a constant current $\langle I\rangle$ such that $W = \frac{1}{2}\langle I\rangle^2 L(T)$ for any particular discharge. In other words, we have replaced the true time dependent current I by a rectangular pulse of duration T. Now, the self inductance L of a torus is given by: [4]

$$L = 1.257\, r(\ln 8\, r/r_0 - 2 + \mu\delta)\times 10^{-6}\ \text{henry}\,, \qquad (1)$$

where r is the major radius of the torus in meters, r_0 is the minor radius, μ is the permeability of the material of the torus and δ is found from tables [4]. For a constant current $\langle I\rangle$, we find $\delta = 0.25$ and $\mu = 1$ for a plasma. It follows, therefore, from eq. (1) that the energy imparted to the plasmoid is given by:

$$W = \tfrac{1}{2}\,\alpha R\langle I\rangle^2\,, \qquad (2)$$

where

$R = r(T)$ and $\alpha = 1.257\,(0.32 + \ln(R/R_0)\times 10^{-6}$

and $\quad R_0 = r_0(T)$.

Now, if ρ is the mass per unit length (line density) of the plasma loop, then we may use the first theorem of mean value to write

$$W = \pi\langle\rho\rangle R\dot{R}^2 \qquad (3)$$

where $\dot{R} = (\partial r/\partial t)_{t=T}$. Hence, from eqs. (2) and (3), the forward velocity $\dot{R}$ of the plasmoid is given by

$$\dot{R} = \langle I\rangle\sqrt{\alpha/2\pi\langle\rho\rangle} \qquad (4)$$

Now, if the undirectional current pulse can be approximated by the first half cycle of a sine wave, with peak value I_0, then $\langle I\rangle \approx \sqrt{E/2\,L}$ [5] where E is the capacitor energy, L is the inductance of the circuit and we have interpreted $\langle I\rangle$ as being the root mean square current. Hence, because the plasmoid velocity has been shown experimentally to be a function of the energy alone, it follows from eq. (4) that $\sqrt{\alpha/\langle\rho\rangle}$ is a constant for a given discharge circuit. We deduce, from the formula for L therefore, that $\langle\rho\rangle$ and hence ρ, is a very slowly varying parameter in the source voltage range of interest (5kV - 15kV) [1,2]. If, now, we examine published high speed photographs of plasmoids [2] we may estimate $R/R_0 \approx 2$ so that $\alpha \approx 1.5\times 10^{-6}$ henry/m. Using this value of α and some published order of magnitude values of I_0 and $\dot{R}$ - say $I_0 = 5kA$ and $\dot{R} = 5 \times 10^4$ m/sec [2] - eq. (4) shows that $\langle\rho\rangle \approx 2.5 \times 10^{-8}$ kg/m. It follows, that the mass of any plasmoid is $2\pi\langle\rho\rangle R$ and if we estimate $R = \dot{R}T$ where T is the duration of the discharge ($\sim 0.3\mu$ sec) then the radius of the torus $R \approx 2.5$ cm and hence the plasmoid mass is abut 2 μg. If now the plasmoid is composed mainly of the ionized atoms of the electrode material (tungsten) [1,2], then these figures imply that there are approximately 0.7×10^{16} ions in such a plasmoid. This number correlates well well with Bostick's original estimate of 10^{15} to 10^{18} ions in a plasmoid [1].

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ON THE MAGNETIC STRUCTURE OF $MnAu_2$ *

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The ambiguity associated with the powder neutron diffraction analysis implies that a helical magnetic-spin alignment reported for $MnAu_2$, for example, is not the unique structure.

A number of ordered magnetic structures have been analyzed by means of neutron diffraction using powered crystals. However, because the individual powder reflection is frequently of multiplet, the unique structure is not always obtainable from the powder data. It appears that this ambiguity has not been fully realized particularly in the sinusoidally and incommensurably modulated moment structure. Here, the well-established diffraction equation is reformulated and the $MnAu_2$ structure [1] is examined as a typical example.

When the incident neutron beam is unpolarized, the observed intensity of the elastically and coherently scattered neutron by an ordered magnetic spin aggregate in a crystal is proportional to

$$|\vec{\mathcal{F}}(\boldsymbol{b})|^2 = \left(\frac{\epsilon^2\gamma}{2mc^2}\right)^2 \sum_{LL'} \sum_{jj'} Q_j^L Q_{j'}^{L'} f_j f_{j'}^* \exp\{2\pi i\, \boldsymbol{b}(\boldsymbol{R}_j - \boldsymbol{R}_{j'})\} , \tag{1}$$

where besides the customary notations, $\boldsymbol{b}$ is the scattering vector and $|\boldsymbol{b}| = 2\sin\theta/\lambda$; f_j is the amplitude form factor of a magnetic electron of the j-th atom at $\boldsymbol{R}_j = \boldsymbol{R}_L + \boldsymbol{r}_j$, $\boldsymbol{R}_L$ being the position vector of the L-th unit cell; $Q_j^L = \boldsymbol{e}(\boldsymbol{e}\, S_j^L) - S_j^L$, $\boldsymbol{e}$ being the unit vector of $\boldsymbol{b}$ and S_j^L representing the ordered moment of the j-th atom in the L-th unit cell. Now, a single-term Fourier expression is employed for S_j^L as follows:

$$S_j^L = \mu_1 \boldsymbol{u}_1 \cos 2\pi(\tau_1 \boldsymbol{p} \boldsymbol{R}_L + \varphi_1) + \mu_2 \boldsymbol{u}_2 \cos 2\pi(\tau_2 \boldsymbol{p} \boldsymbol{R}_L + \varphi_2) + \mu_3 \boldsymbol{u}_3 \cos 2\pi(\tau_3 \boldsymbol{p} \boldsymbol{R}_L + \varphi_3), \tag{2}$$

where the unit vectors, $\boldsymbol{u}_1$, $\boldsymbol{u}_2$ and $\boldsymbol{u}_3$, are chosen to be parallel to the unit-cell vectors, $\boldsymbol{a}_1$, $\boldsymbol{a}_2$ and $\boldsymbol{a}_3$, respectively; $\boldsymbol{p}$ is the propagation unit vector; φ's are the phase terms. We therefore obtain

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