Gravitational (Dynamic) Time Dilation According to Absolute Space-Time Theory

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Proceeding from our absolute space-time conceptions, we obtain the formula for the gravitational frequency shift in an extremely simple way. Using our "burst" model for photons, we show that the different rates of clocks placed in spatial regions with different gravitational potentials appear as a direct result of the gravitational frequency shift and the axiomatic assumption that at any space point the time unit is to be defined by light clocks with equal "arms," i.e., that at any space point the light velocity (in moving frames the "there-and-back" velocity) has the same numerical value c. Considering the principle of equivalence, we come to the logical conclusion that the kinematic (Einstein-Lorentz) time dilation is an absolute phenomenon.

1. THE GRAVITATIONAL FREQUENCY SHIFT

According to our absolute space-time theory,⁽¹⁾ the gravitational energy of two masses as registered in a frame of reference where the mass M is at rest is given by the formula

$$U = -\frac{k^2 mM}{r} \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \tag{1}$$

where k^2 is the gravitational constant, r is the distance between both masses, \mathbf{v} is the velocity of mass m, and c is the velocity of light.

The corresponding Lagrange equation of motion is

$$m \, d\mathbf{v}_0/dt = -\nabla U \tag{2}$$

where $\mathbf{v}_0 = \mathbf{v}(1 - v^2/c^2)^{-1/2}$ is the proper velocity of mass m.

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Forming the scalar product of both sides of (2) with $\mathbf{v} = d\mathbf{r}/dt$, we obtain

$$\frac{d}{dt} \left[mc^2 \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \right] = \frac{d}{dt} \left[\frac{k^2 mM}{r} \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \right]$$
(3)

or

$$\frac{de_0}{dt} = -\frac{d}{dt} \left(-\frac{k^2 e_0 E_0}{c^4 r} \right) \tag{3'}$$

where e_0 and E_0 are the *proper energies* of the masses m and M. This equation shows that the change in time of the proper energy of mass m interacting with mass M is equal to the negative change in time of its gravitational energy.

If we start with Eq. (3'), then we need as a basis only the energy conservation law and the form of the gravitational energy of mass m given in formula (3').

In addition, we must use the axiomatic assertion of our absolute spacetime theory that the energy (or rest energy) $e = mc^2$ of any material point with mass m can be expressed by its rest frequency v_r according to the relation $e = hv_r$, where h is the Planck constant. The proper energy $e_0 = e(1 - v^2/c^2)^{-1/2}$ is to be expressed by the frequency $v = v_r(1 - v^2/c^2)^{-1/2}$ of mass m according to the relation $e_0 = hv$. If the material point is a photon, then m = 0, v = c. The rest frequency of any photon is equal to zero; thus all photons have the same rest frequency, and only the frequencies of photons transferring different quantities of energy are different.

Substituting into (3') the proper energy e_0 expressed through the frequency ν of the material point considered, we obtain

$$\nu_2 - \nu_1 = \frac{k^2 M}{c^2} \left(\frac{\nu_2}{r_2} - \frac{\nu_1}{r_1} \right) = -\frac{1}{c^2} \left(\nu_2 \Phi_2 - \nu_1 \Phi_1 \right) \tag{4}$$

or

$$\nu_1 \left(1 + \frac{\Phi_1}{c^2} \right) = \nu_2 \left(1 + \frac{\Phi_2}{c^2} \right)$$
 (5)

where Φ_1 and Φ_2 are the gravitational potentials caused by mass M at the points of emission and reception, respectively, and ν_1 and ν_2 are the emission and reception frequencies of the photon.

For the case where the difference between the emitted and received frequencies is not large, we can write, on the right side of (4), $\nu_1 \cong \nu_2 \cong \nu_m$; and so we obtain

$$\nu_2 - \nu_1 = (\nu_m/c^2)(\Phi_1 - \Phi_2) \tag{6}$$

We call this effect (commonly known as the *red shift*) the gravitational (or *dynamic*) frequency shift. The frequency change of a photon emitted from

a body which moves with respect to the receiver is called the Doppler (or *kinetic*) frequency shift.

2. THE RATE OF TIME IN A GRAVITATIONAL FIELD

Now we shall show that the gravitational light frequency shift considered together with the tenth axiom of our absolute space-time theory⁽²⁾ leads to the conclusion that at points with different gravitational potentials time passes at different rates.

Here we shall briefly mention that the ten axioms, which represent the axiomatic basis of the classical (nonquantum) part of our theory, are: (1) Axiom for space. (2) Axiom for time. (3) Axiom for energy. (4) Axiom for gravitational (i.e., first type of space) energy. (5) Axiom for electric (i.e., second type of space) energy. (6) Axiom for kinetic (i.e., time) energy. (7) Axiom for magnetic (i.e., first type of space-time) energy. (8) Axiom for magnetic (i.e., second type of space-time) energy. (9) Axiom for conservation of energy. (10) Relativistic (i.e., high-velocity) axiom.

The tenth axiom reads: The material points called photons move with velocity c along all directions in absolute space and their velocity does not depend on their history. Light clocks with equal "arms" have the same rate in any frame, independent of the orientation of their "arms." At any point of any frame the time unit is to be defined by the period of light clocks with equal "arms," independent of the velocity of the frame and the local concentration of matter.

The crucial difference between the absolute space-time theory and relativity theory is the following: Our theory assumes that only the "there-and-back" velocity of light has the same numerical value at any point of any inertial frame, while the Einsteinian theory assumes that the "there" and "back" velocities separately have the same numerical value. As is shown in Ref. 1, in our theory for the unidirectional velocity of light we obtain the formula (4.30) of that paper, which was proved correct by our recent "coupled-mirrors" experiment. (3),3

- ² In the third axiom we introduce axiomatically the relations $e = mc^2$ and $e = hv_r$. Thus in our theory the quantities mass and frequency are derivative. Only three physical quantities are undefined (i.e., axiomatically introduced): space (three-dimensional quantity), time, and energy.
- ³ We note that now we are working on a repetition of the "coupled-mirrors" experiment in its so-called *interferometric* variant. According to preliminary estimations, we shall be able to measure the absolute Earth velocity with a measuring error less than 10 km/sec. In the old, so-called *deviative* variant of the "coupled-mirrors" experiment, ⁽³⁾ the fluctuation error alone was about 100 km/sec. The apparatus should be placed on a turntable and the whole measurement should last a couple of seconds, while in the first realization it lasted 24 hr. See Note Added in Proof on page 581.

Now we shall prove the assertion that at points with different gravitational potentials, time passes at different rates. Since now we are not interested in the *kinematic* aspect of the time dilation (i.e., the slowing of clocks moving with respect to absolute space), considered in Ref. 1, but only in the *dynamic* aspect of the time dilation (i.e., the slowing of clocks placed near local concentrations of matter), we shall work in a frame which is at rest in absolute space. Such a frame and the clocks attached to it are called *absolute* (better, *kinematically absolute*). The frames moving in absolute space and the clocks attached to such frames are called *proper* (better, *kinematically proper*). The clocks placed far enough from local concentrations of matter are called *dynamically absolute clocks*, while those placed near local concentrations of matter are called *dynamically proper clocks*. The regions between the galaxies (respectively, between the stars or between the planets) can be considered as situated far enough from local concentrations of matter.

Let us consider two points with gravitational potentials Φ_1 and Φ_2 , supposing that a photon is emitted from the first point and received at the second. Since according to the tenth axiom the time units at these two points are defined by light clocks with equal "arms," the velocity of light at both points will have the same numerical value c, if measured by the help of two proper clocks placed respectively at the points of emission and reception. Thus we have

$$\nu_1 \lambda_1 = c$$
 and $\nu_2 \lambda_2 = c$ (7)

where ν_1 and ν_2 are the emitted and received frequencies, and λ_1 and λ_2 the corresponding wavelengths, of the photon.

Substituting (7) into (5), we obtain

$$\frac{\lambda_1}{1 + (\Phi_1/c^2)} = \frac{\lambda_2}{1 + (\Phi_2/c^2)} \tag{8}$$

Hence the wavelength of a photon becomes larger when it passes from a region with a stronger gravitational potential to a region with a weaker gravitational potential (i.e., when $|\Phi_1| > |\Phi_2|$). As follows from formula (5), for such a case, the frequency of the photon becomes lower.

Let us now assume that the velocities of the photon at the first and second points are v_1 and v_2 , respectively, if measured with the help of an absolute clock. Then we shall examine the relation between λ_1 , λ_2 and v_1 , v_2 , using our "burst" model⁽⁴⁾ for photons.

For this purpose let us suppose that the gravitational potential changes from the emission to the reception point in a stepped form. The potential "steps" can be infinitely close to each other, but, for clarity, we shall assume the distances between them to be larger than the photon wavelength. Now, obviously, the rear bullet of the "burst," when passing the *i*th potential

"step," will change its velocity from v_i to v_{i+1} always with a time delay $\Delta t_i = \lambda_i / v_i$ after the leading bullet, λ_i being the wavelength of the photon in the *i*th region. Thus the wavelength of the photon after crossing the *i*th potential "step" will be

$$\lambda_{i+1} = v_{i+1} \, \Delta t_i = (\lambda_i / v_i) \, v_{i+1} \tag{9}$$

If from the emission to the reception points there are n "steps," we have

$$\lambda_n = (\lambda_{n-1}/v_{n-1}) \ v_n = (\lambda_1/v_1) \ v_n \tag{10}$$

This formula shows that λ_n can be different from λ_1 only if v_n is different from v_1 . Thus, from (10) and (8), we obtain

$$\frac{v_1}{1 + (\Phi_1/c^2)} = \frac{v_2}{1 + (\Phi_2/c^2)} \tag{11}$$

Let us emphasize that, under the assumption of the "burst" model for photons, one cannot explain the wavelength change prescribed by formula (8) [thus also the frequency change prescribed by formula (5)] if one assumes that the velocity of light in regions with different gravitational potentials is equal when measured on a unique clock.

Since the absolute times of emission and reception of the "burst" (i.e., the absolute periods of the emitted and received photons) are, respectively, $T_1^{ab} = \lambda_1/v_1$ and $T_2^{ab} = \lambda_2/v_2$, we obtain, from (10),

$$T_1^{ab} = T_2^{ab} (12)$$

The proper periods of the emitted and received photons are

$$T_1 = \lambda_{\rm I}/c$$
 and $T_2 = \lambda_2/c$ (13)

From (13) and (8), taking into account (12), we come to the conclusion that if a time t_1 , read on clock placed in a region with gravitational potential Φ_1 , and a time t_2 , read on a clock placed in a region with gravitational potential Φ_2 , correspond to the same absolute time interval t^{ab} , then the relation between t_1 and t_2 is

$$\frac{t_1}{1 + (\Phi_1/c^2)} = \frac{t_2}{1 + (\Phi_2/c^2)} \tag{14}$$

Remark. Formula (6) and the analogous approximate formulas which can be deduced from (11) and (14) are obtained also in the theory of general relativity. However, our exact formulas (5), (11), and (14) differ from those found in general relativity (see, for example, Landau and Lifshitz⁽⁵⁾).

Thus our theory, as well as the theory of general relativity, leads to the following assertions:

- 1. The velocity of any photon measured at any space point by the help of a clock placed there (imagine for clarity a light clock) has the same numerical value c. An experimental confirmation of this local constancy of light velocity is furnished by the equal aberration constant for all celestial objects. Indeed, light coming from different celestial objects is emitted from space points with different gravitational potentials, and if the corresponding gravitational shifts in the frequencies are due to the different velocities with which the corresponding photons pass the observer, then the large "red shifts" observed for certain celestial objects (say, quasars) will lead to different aberration constants for these objects. Here we do not want to discuss the problem of absorption and reemission of light in the interstellar medium nor the problem of whether the large "red shifts" for quasars and distant galaxies have a Doppler character (as the majority of astronomers assert) or a gravitational character (as our absolute space-time theory asserts⁽²⁾).
- 2. The velocities v_1 and v_2 of photons that traverse two regions with different gravitational potentials are different if measured on a unique clock, the relation being given by formula (11). An experimental confirmation of this relative changeability of light velocity is furnished by the experiment of Shapiro. (6) Shapiro measured the time it took an electromagnetic signal, traveling back and forth, to cover definite well-known distances between the Earth an Venus, first when the Sun is far from the line connecting both planets and then when the Sun is near this line. In the second case the gravitational potential in the region crossed by the electromagnetic signal was stronger (i.e., its absolute value greater), and the light velocity (measured in both cases with the help of a clock placed at the same gravitational potential of the Earth) was lower.
- 3. The rates of two clocks placed at points with different gravitational potentials are different if measured by the help of a unique clock, the relation being given by formula (14). An experimental confirmation of this gravitational time dilation is furnished by the experiment of Hafele and Keating. (7) They flew atomic clocks in jet planes at different heights, where the gravitational potentials are different, and compared the readings before and after the flights with a stationary atomic clock left in Washington. The differences registered in the readings can be explained well enough by the kinematic and dynamic time dilations that appeared during the flights.

We shall not dwell on the experimental confirmations of formula (5), which for many years has offered an important tool for the observational exploration of the universe.

Let us consider a mass m ($m \neq 0$) which, having velocity v_1 in the region with gravitational potential Φ_1 , acquires the velocity v_2 in the region with gravitational potential Φ_2 only as a result of the gravitational interaction between this mass and the masses producing the field. Proceeding from formula (3), we find

$$\frac{1 + (\Phi_1/c^2)}{[1 - (v_1^2/c^2)]^{1/2}} = \frac{1 + (\Phi_2/c^2)}{[1 - (v_2^2/c^2)]^{1/2}}$$
(15)

This is the energy conservation law for a point mass in a gravitational field according to our absolute space-time theory.

On the other hand, if two clocks move with velocities v_1 and v_2 with respect to absolute space, then, according to our theory, (2.8) the relation between their time rates is

$$\frac{t_1}{[1 - (v_1^2/c^2)]^{1/2}} = \frac{t_2}{[1 - (v_2^2/c^2)]^{1/2}}$$
(16)

Comparing formulas (14)–(16), we come to the following very important conclusion: If we want to change the rate of a given clock to a certain degree, we have to change either its velocity or its gravitational potential. In both cases we have to expend the same quantity of work. Here we must mention that we have to expend the same quantity of work in absolute value, since from (15) we obtain, within an accuracy of second order in 1/c,

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = m\Phi_1 - m\Phi_2 \tag{17}$$

and the gravitational energy (together with the gravitational potential) is negative, while the kinetic energy is positive. This can be established also with the following reasoning: If we want to slow the rate of a given clock "kinematically," we have to enhance its absolute velocity, and thus to do *positive* work, while if we want to achieve this "dynamically," we have to transfer the clock from a point with a weaker gravitational potential to a point with a stronger gravitational potential, and thus to perform *negative* work.

This represents an interesting manifestation of the so-called principle of equivalence between inertial and gravitational masses.

On the other hand, since the different time rates of clocks placed at points with different gravitational potentials is, obviously, an absolute phenomenon, the foregoing conclusion gives us enough certitude to maintain that the different time rates of clocks moving with different velocities also represent an absolute phenomenon. This is one of our basic physical conceptions, which contradicts the special relativity conception about the relative character of the kinematic time dilation.

3. THE PRINCIPLE OF EQUIVALENCE

Since the masses of the material points are a measure of their kinetic energy as well as of the gravitational energy to which they contribute, the so-called *principle of equivalence* can be formulated, which asserts the following: Any gravitational field in a small region around a given space point can be replaced by a suitable noninertial frame of reference (and vice versa), so that the behavior of material points in an inertial frame of reference in the presence of a gravitational field would be indistinguishable from their behavior in a suitable noninertial frame without the gravitational field.

The principle of equivalence can be applied to solve many kinematic problems with dynamic methods. For this purpose we have to consider the noninertial kinematic problem in an inertial frame of reference under the introduction of the so-called pseudo-gravitational potentials. The pseudo-gravitational potential is a fictitious gravitational potential ascribed to the space occupied by a noninertially moving frame of reference in order to transform away this noninertial motion. The pseudo-gravitational intensity corresponding to this pseudo-gravitational potential equals the real acceleration of the noninertially moving masses in the part of space under consideration.

Thus, if the pseudo-gravitational potential at a given space point is Φ , then the pseudo-gravitational intensity will be

$$\mathbf{G} = -\nabla \Phi = \mathbf{u} \tag{18}$$

where \mathbf{u} is the real acceleration of the masses, in a small region with radius vector \mathbf{r} , resulting from their noninertial motion with respect to absolute space.

Now we shall make a very interesting analysis of the kinematic time dilation with the aid of the principle of equivalence.

Any kinematic acceleration of a given material system (considered as a material point) which moves in a given inertial frame of reference can be decomposed into a tangential and a normal acceleration. First we shall consider the influence of the *normal acceleration* on the time dilation from a "pseudo-gravitational" point of view.

When the tangential acceleration is equal to zero and only the normal acceleration u exists, the system revolves with speed v in a circle with radius R such that

$$u = v^2/R = \Omega^2 R \tag{19}$$

where Ω is the angular velocity of the rotational motion.

Corresponding to the time t' read on a first clock which is at rest in absolute space (an absolute clock), a second clock moving with velocity v (a proper clock) will show a time t''; according to formula (16), for the difference $\Delta t = t'' - t'$ we obtain, within an accuracy of second order in v/c,

$$\Delta t/t = -\frac{1}{2}v^2/c^2 \tag{20}$$

where we have written $t \cong t' \cong t''$.

Let us take into account that during the time t the second clock has undergone the acceleration u. The difference between the pseudo-gravitational potentials at the space points along the circular trajectory of the second clock and the center of the circle, where the first clock rests, can be obtained from the following formula:

$$\Delta \Phi = \Phi'' - \Phi' = -\int_0^R \mathbf{G} \, d\mathbf{r} = -\Omega^2 R^2/2 = -v^2/2$$
 (21)

where we have substituted G on the grounds of (18) and (19).

Substituting (21) into formula (14), we obtain a result identical with (20).

Thus the time dilation of a given clock due to its inertial motion at velocity v with respect to absolute space is exactly the same as the time dilation due to its noninertial motion in a circle with radius R and acceleration (19). But the inertial motion of a material system, when its velocity keeps a constant magnitude and direction, can be considered as a rotation in a circle with radius tending to infinity and with a normal acceleration tending to zero. So we arrive at the extremely important conclusion of our absolute space-time theory: The time dilation obtained if one proceeds, on the one hand, from the "inertial" formula (16) and, on the other hand, from the "noninertial" formula (14), with the help of the artificially introduced pseudo-gravitational potentials, is one and the same. Hence if we calculate the time dilation from an "inertial point of view," taking into account the different velocities of the systems, we must ignore their different normal accelerations; and if we calculate the time dilation from a "noninertial point of view," taking into account the different normal accelerations of the systems and making use of the pseudo-gravitational potentials, we must leave aside their different velocities.

Let us now study the influence of the *tangential acceleration* on the time dilation from a "pseudo-gravitational" point of view.

Consider a clock (called the second clock) moving with an arbitrary acceleration *u* collinear with its velocity. How does the time read on this proper clock compare with the time registered on an absolute clock (called the first) which rests in absolute space?

First we use the "inertial" approach. Consider a small time interval t

so short that over it the acceleration can be considered as constant. Then divide this time interval into small intervals δt_1 , δt_2 ,..., δt_n , in any of which we may consider the velocities as constant, equal to v_1 , v_2 ,..., v_n , where v_1 is the velocity at the beginning of the time interval δt_1 , v_2 the velocity at the beginning of the time interval δt_2 , and so on.

Now, using formula (16), we find that the time elapsed on the second clock will be

$$t'' = \sum_{i=1}^{n} \delta t_i' \left(1 - \frac{v_i^2}{c^2} \right)^{1/2}$$
 (22)

and for the difference $\Delta t = t'' - t'$ we obtain, within an accuracy of second order in v/c, and writing $\delta t_i \cong \delta t_i' \cong \delta t_i''$,

$$\Delta t = -\frac{1}{2c^2} \sum_{i=1}^{n} \delta t_i \, v_i^2 \tag{23}$$

Let us now employ the "noninertial" approach, taking into account that during the time t the second clock has undergone the tangential acceleration u. The difference between the pseudo-gravitational potentials at the space points where the second clock was at the beginning and at the end of the time interval δt_i is

$$\delta \Phi_i = -u \, \delta r_i = -\frac{1}{2} u(v_i + v_{i+1}) \, \delta t_i
= \frac{1}{2} (v_i - v_{i+1})(v_i + v_{i+1}) = \frac{1}{2} (v_i^2 - v_{i+1}^2)$$
(24)

where δr_i is the distance covered in the time δt_i , and this formula is based on (18).

The result (23) for the difference between the readings of the moving and the stationary clocks is obtained under the assumption that, in the limit of small time intervals δt_i , the velocity of the moving clock changes instantaneously from v_i to v_{i+1} . Now we shall find the correction which is to be made when it is taken into account that during any of the time intervals δt_i the second clock has undergone the acceleration u. On the basis of formulas (14) and (24), assuming that δt_i are equal, i.e., $\delta t_i = \delta t$, we obtain, to within second order in 1/c,

$$\Delta t_{\text{corr}} = \sum_{i=1}^{n-1} \delta t_i \frac{\delta \Phi_i}{c^2} = \frac{\delta t}{2} \sum_{i=1}^{n-1} (v_i^2 - v_{i+1}^2) = \frac{\delta t}{2} (v_1^2 - v_n^2)$$
 (25)

Thus, when $\delta t \to 0$, we find $\Delta t_{\rm corr} \to 0$.

Consequently we establish that the tangential accelerations do not influence the time rates of the clocks when use is made of the pseudo-gravitational potentials.

The analysis performed in this section has shown that the calculations of the time dilation from an "inertial" point of view, with the help of formula (16), and that from a "noninertial" point of view, with the help of formula (14), where the pseudo-gravitational potentials artificially introduced and the principle of equivalence are used, lead exactly to the same results. We have also seen that in the calculation of the time dilation from a "noninertial" point of view only the normal accelerations of the moving system need to be taken into account because the tangential accelerations have no influence.

Here we have to point out that the velocities of the material systems are different in the different inertial frames of reference. However, the material systems have the same accelerations in all inertial frames of reference. This conclusion leads immediately to the fundamental conception of our absolute space—time theory concerning the absolute character of the kinematic time dilation.

NOTE ADDED IN PROOF

We have successfully carried out the interferometric "coupled-mirrors" experiment. The absolute velocity of the Sun registered by us has magnitude $v = 303 \pm 20 \text{ km/sec}^{-1}$ and the equatorial coordinates of its apex are $\delta = -22.5^{\circ} + 4^{\circ}$, $\alpha = 14^{\circ} 17' + 20'$.

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