Stefan Marinov

CLASSICAL PHYSICS

Part III
HIGH-VELOCITY MECHANICS

EST-OVEST
Editrice Internazionale

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Published in Austria
by
International Publishers »East-West«

First edition, 1981

Addresses of the International Publishers »East-West« Affiliates:

ITALY — via Puggia 47, 16131 Genova. Tel. (010) 31.59.78.
BULGARIA — ul. Elin Pelin 22, 1421 Sofia. Tel. (01) 66.73.78.

DRUCK: RM-DRUCK GRAZ, AUSTRIA

PART III

HIGH-VELOCITY MECHANICS

§1. INTRODUCTION

High-velocity mechanics can be constructed on the same axiomatical grounds as low-velocity mechanics but the tenth axiom must also be taken into account.

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According to our conceptions, space and time are absolute categories. They have to be perceived in the way as the ordinary man does, and as Newton introduced them in the simplest and clearest way (see IIA, §2).

In our century, however, on the grounds of many experiments with light beams and particles moving at high speeds, the situation arose that the old conceptions for space and time were considered as corresponding only approximately to reality, i.e., when the velocities of the particles are low enough with respect to light velocity. A new theory, whose fathers are Einstein and Minkowski, allowed the explanation of all high-velocity experiments, but this theory led to tragic results: the sound absolute Newtonian conceptions for space and time have been replaced by new ones, generally called "relative", which contradict the healthy human mind drastically.

During the whole last century the scientists all over the world fought desperately to save the simple Newtonian conceptions but the struggle was in vain - all experiments supported Einstein's theory, so that since many years such physicists who continue to believe in Newton are treated as simple-minded people who cannot understand certain things which other people have already perfectly understood.

In the present part III of our book (and in parts IV and V) we show that a non-contradictory ABSOLUTE SPACE-TIME THEORY can be constructed, which explains all crucial experiments with light beams and fast moving particles performed up to now. We show that this is the old Newtonian theory, in which only some small corrections concerning the light kinematic effects of second order in v/c (see IIA, p. 20) have to be introduced, i.e., the NEWTON-AETHER MODEL for light propagation has to be substituted by the MARINOV-AETHER MODEL postulated in our tenth axiom. We show, however, that not only the photons have an "aether" character of propagation but also any massive particle (electron, neutron). As a matter of fact, the "aether" character of propagation of the particles is due to their attachment to the mass of the whole world. Thus,we think, it is inexpedient to speak about an "aether" and further on we shall speak simply about NEWTON and MARINOV MODELS OF LIGHT PROPAGATION.

The absolute space-time theory saves us not only from the tormenting necessity to accept Lorentz contractions, twin paradoxes, and fantastic velocity addition laws (let us emphasize that mankind has lost millions of working hours trying to "understand" these paradoxes), but it also explains experiments for which the theory of relativity gives wrong predictions. Nevertheless, the absolute space-time theory makes use of the 4-dimensional mathematical apparatus of Minkowski, and in the same way as Einstein wrote that the theory of relativity was unthinkable without Newton, we can say that the absolute space-time theory is unthinkable without Einstein. We have to add, however, that the birth and triumph of the relativity theory was not a historical necessity. If Michelson or other scientists at the beginning of our century have succeeded to measure the one-way light velocity with an accuracy higher than 0.1%, and thus to reveal the absolute motion of the Earth, Einstein's theory would have the same destiny as, say, the Ritz ballistic theory of light.

To stress the differences between our approach to the high-velocity problems and the Newton and Einstein approaches, we shall call the low-velocity mechanics NEWTON MECHANICS and the high-velocity mechanics which is adequate to physical reality (and which is presented in this book) MARINOV MECHANICS. We should like to point out in anticipation (the reader will see this when reading this book) that the differences between the Newton and Marinov mechanics are not so drastic as between the Marinov and Einstein mechanics. The fundamental differences between the Newton and Marinov mechanics are postulated:

a) in the tenth axiom where the Marinov model of light propagation is introduced differing from the Newton model within effects of second order in v/c,

b) in the fourth and seven axiom, according to which the gravitational charges of the particles are not their universal masses but their proper masses and that a gravitational energy analogue to the magnetic energy does exist,

c) in the third axiom, according to which one must operate not with the universal but with the proper quantities of the particles.

Let us note that when considering the time energy of the particles as their gravitational energy with the mass of the whole world (see §24), then postulate "c" follows directly from postulate "b".

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The fundamental axiomatical assumption on which the theory of relativity is based represents the so-called PRINCIPLE OF RELATIVITY which asserts the following: There is no possibility of establishing by a physical experiment, which of two material systems, moving at a constant velocity with respect to one another, is at rest and which in motion - each of them has absolutely the same right to be considered in motion or at rest. Hence, according to this principle, it is not possible to establish with the help of whatever physical experiment the constant velocity of propagation of a material system with respect to absolute space, and such an absolute space has to be treated as a pure fiction.

The principle of relativity (as the energy conservation law) is an axiom introduced in the theory on the basis of the human experience of centuries. We consider Galilei as the first person who introduced it explicitly in science, but formulations of the principle of relativity can be found in the writings of the Chinese philosophers before Christ. Poincaré and Lorentz were the first who considered the principle of relativity as an absolutely valid law, but it was Einstein who accepted it as a fundamental axiom and based his famous SPECIAL THEORY OF RELATIVITY on it.

The postive effect in our "coupled mirrors" experiment (see §51 and §52) showed that there is a possibility of measuring the absolute velocity of a laboratory. Thus, after the performance of our experiments, the principle of relativity must be rejected as not adequate to physical reality. The special theory of relativity must also be rejected as being in contradiction with physical reality.

Einstein introduced the isotropy of light velocity in any inertial frame of reference as another axiomatical assertion. This assertion is a corallary of the principle of relativity. Let us note, however, that according to Lorentz, who sustained the aether model of light propagation, the velocity of light is direction dependent in frames moving with respect to the aether, but there is no physical possibility of establishing this anisotropy. Our "coupled mirrors" experiments showed that the velocity of light is anisotropic in frames moving with respect to absolute space, and it was exactly this anisotropy which offered the experimental possibility of measuring the absolute velocity of the Earth and of giving an experimental refutation of the principle of relativity.

As another axiom Einstein introduced the assertion that the velocity of light does not depend on the velocity of the source. This assumption is true. It follows immediately from our tenth axiom which asserts (see IIA, p. 21): "The elementary particles called photons move with velocity c in absolute space and their velocity does not depend on their history."

If we call the principle of relativity "assertion A", the isotropy of light velocity in any inertial frame "assertion B", and the independence of light velocity on the velocity of the source "assertion C", then it is obvious that all these three assertions A, B, C (as accepted by Einstein in the theory of relativity) are incompatible. The logically compatible combinations are: A, B, non-C (as accepted by Ritz in the BALLISTIC THEORY of light propagation); this case is realized in low-velocity mechanics, for example, the shooting of a bullet by a riding gun-man (see, however, §44), or non-A, non-B, C (as accepted by Marinov in the absolute space-time theory).

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Since the masses of the material points are a measure of their kinetic energy as well as of the gravitational energy to which they contribute, the so-called PRINCIPLE OF EQUIVALENCE can be stated as follows: Any gravitational field in a small region around a given point in space can be replaced by a suitable non-inertial frame of reference (or vice versa), so that the behaviour of the particles in an inertial frame of reference in the presence of a gravitational field is indistinguishable from their behaviour in a suitable non-inertial frame without the gravitational field.

Einstein generalized and absolutized also the principle of equivalence (as he did with the principle of relativity), postulating that it is by no means possible to establish whether the acceleration exerted on particles in a laboratory has a kinematic (mechanic) character (thus being due to the accelerated motion of the laboratory, for example, by the thrust of a space ship) or a dynamic (gravitational) character (thus being generated by the action of nearby masses, for example, by the Earth's attraction). Einstein accepted the principle of equivalence as a fundamental axiom and based his famous GENERAL THEORY OF RELATIVITY on it. However, the results of our accelerated "coupled mirrors" experiment (see §53) have shown that there is a possibility of making a local laboratory distinction between a mechanic and a gravitational acceleration. Thus, after the performance of our experiment, the principle of equivalence must be rejected as not adequate to physical reality. The general theory of relativity must also be rejected as being in contradiction with physical reality.

Another axiomatical basis of Einstein's general theory of relativity is the assertion that there is no difference between "kinematic" (or "inertial") and "gravitating" mass. We show (see §24) that the notion "inertial" mass is superfluous. An "inertial" mass does not exist. The mass is only gravitational. Thus all experiments with the help of which one tries to find a difference between "inertial mass" and "gravitational mass" are a waste of efforst, time and money, and the theoretical speculations concerning their "equivalence", too.

We repeat, the fundamental conception of our absolute space-time theory represents the assertion (called generally MACH'S PRINCIPLE) that the gravitational energy of any particle with the mass of the whole world determines all its "kinematic" (or "mechanical") properties. This is true for massive particles (electrons, phonons) as well as for massless particles (photons). As the mass of the whole world determines a special frame of reference (this one in which its center of mass is at rest), the space attached to this frame must be called ABSOLUTE SPACE. The right description of the material systems can be done only when considering them in absolute space.

Chapter I

K I N E M A T I C S

§2. THE LIGHT CLOCK

As we said in IIA, p. 8, space intervals (lengths) can be measured by rigid rods and time intervals can be measured by light clocks. In fig. 2.1 the scheme of a light clock is represented. Let A be a point in space where time is to be measured and B a mirror placed at a distance d = 150,000 km. A short light pulse (i.e., a group of photons) is generated at a given initial moment. This pulse flies in half a second to mirror B, after another half a second it returns to A, where it puts a counter in motion showing the number "1" and generates (after amplification) a second pulse which flies to mirror B again; after the return of the second pulse the counter will show the number "2". So we have a perfect clock whose "wheels" move with the highest possible accuracy as the "Räder an der grossen Weltenuhr".

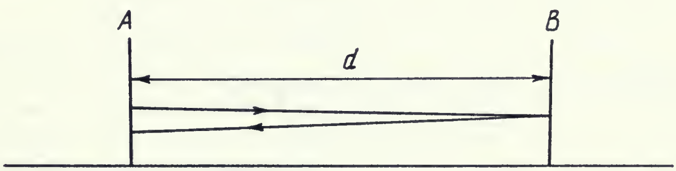

Fig. 2.1

The most important aspect of our tenth axiom is that the rate of any laboratory light clock does not depend on the orientation of its "arm" (see IIA, §5). This axiomatical assertion represents a crucial boundary between low-velocity and high-velocity mechanics which has concerned human mind for almost the whole 20th century. Indeed, if we try to find the rates of two identical light clocks proceeding with velocity v with respect to absolute space when their "arms" are, respectively, perpendicular and parallel to v, then the Newtonian conceptions should lead to the result that for $n_\perp$ periods of the "perpendicular" light clock there are $n_\parallel$ periods of the "parallel" light clock such that

$$n_\parallel = n_\perp(1 - v^2/c^2)^{1/2}, \qquad (2.1)$$

supposing (as we shall always do) that the "arms" of the clocks are equal.

Let us show this. If we denote by $T_\perp' = T_\perp'' = T_\perp/2$ the time in which light covers the "arm" d of the "perpendicular" clock "there" and "back", we shall have for these two cases

$$c^2T_\perp'^2 = d^2 + v^2T_\perp'^2, \qquad c^2T_\perp''^2 = d^2 + v^2T_\perp''^2, \qquad (2.2)$$

from where

$$T_\perp = T_\perp' + T_\perp'' = \frac{2d}{c(1 - v^2/c^2)^{1/2}}. \qquad (2.3)$$

On the other hand, if we denote by $T'_{\parallel}$ and $T''_{\parallel}$ the times in which light covers the "arm" of the "parallel" clock "there" and "back" (i.e., along the direction of propagation of the clock, and against this direction), we shall have for these two cases

$$cT'_{\parallel} = d + vT'_{\parallel}, \qquad cT''_{\parallel} = d - vT''_{\parallel}, \tag{2.4}$$

from where

$$T_{\parallel} = T'_{\parallel} + T''_{\parallel} = \frac{2d}{c(1 - v^2/c^2)}. \tag{2.5}$$

Hence it will be

$$T_{\perp} = T_{\parallel}(1 - v^2/c^2)^{1/2}. \tag{2.6}$$

Now if for a certain time t the "perpendicular" light clock makes $n_{\perp}$ ticks and the "parallel" $n_{\parallel}$ ticks, we shall have

$$t = n_{\perp}T_{\perp}, \qquad t = n_{\parallel}T_{\parallel}, \tag{2.7}$$

and from (6) and (7) we obtain (1).

Our tenth axiom asserts, however, that instead of (1) one must have

$$n_{\parallel} = n_{\perp}, \tag{2.8}$$

and thus the periods of the light clocks must be equal

$$T_{\parallel} = T_{\perp}. \tag{2.9}$$

This empirical fact was first proved by the Michelson-Morley experiment (see §49).

In the next sections of this chapter we shall see which conclusions are to be drawn from the assertion (9) of the tenth axiom and which must be the transformations of the space and time coordinates resulting from this axiom.

§3. TRANSFORMATION OF COORDINATES

A. The Galilean transformation

All transformations of the space and time coordinates which we consider in this section are between a frame K attached to absolute space and a frame K' moving with a constant velocity $\vec{V}$. To avoid trivial constants, we shall consider the so-called HOMOGENEOUS TRANSFORMATION (cf. I, p. 201), at which at the initial time (t = 0) the origins of both frames coincide (see fig. 3.1 wherefor the sake of simplicity a two-dimensional case is presented). The transformation shown in fig. 3.1 is called GENERAL TRANSFORMATION. If the axes of the moving frame K' are parallel to the axes of the rest frame K and if the velocity of K' is parallel to one of these axes (as a rule to the x-axis), this is called a SPECIAL TRANSFORMATION.

In fig. 3.2 a special transformation between frames K and K' is presented, where again a two-dimensional case is given. Let point P (see fig. 3.2) be at rest in the moving frame

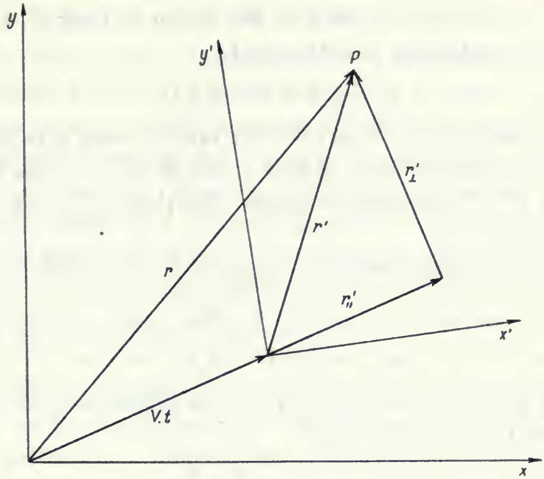

Fig. 3.1

K'. At the zero initial moment its absolute and relative radius vectors, $\vec{r}_0$ and $\vec{r}_0'$, will be identical. At an arbitrary moment t, they will be $\vec{r}$ and $\vec{r}' = \vec{r}_0'$. If a point covers the distance between the origin of frame K and point P during this time t, then the angle between the velocity of the moving point and the velocity $\vec{V}$ will be θ, if measured in frame K, and θ', if measured in frame K'.

Unless mentioned especially something other, we shall always consider a homogeneous general transformation.

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Thus the position of a point P may be represented (see IIB, §3) by the radius vector $\vec{r}$ (the absolute radius vector) in frame K and by the radius vector $\vec{r}'$ (the relative radius

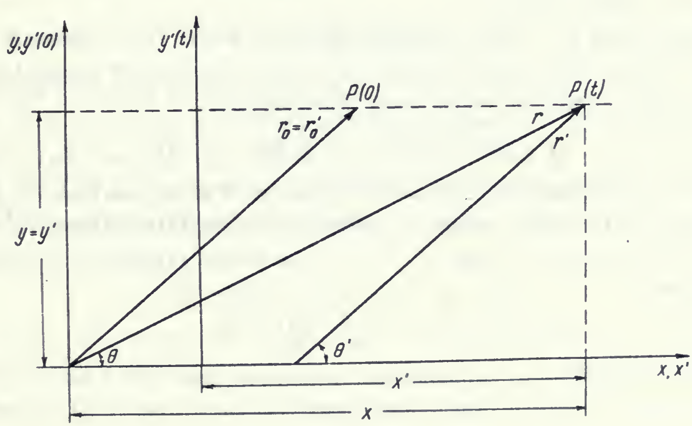

Fig. 3.2

vector) in frame K'. The radius vector $\vec{R}$ of the origin of frame K' in frame K (the transient radius vector) is given by (see fig. 3.1)

$$\vec{R} = \vec{V}t = \vec{V}_o t_o, \tag{3.1}$$

where t is the time read on a clock which is at rest in frame K (a universal clock) and $\vec{V}$ is the velocity of frame K' measured on this clock, while t_o is the time read on a clock which is at rest in K' (a laboratory or proper clock) and $\vec{V}_o$ is the velocity of frame K' measured on this clock.

According to the traditional Newtonian conceptions (see (IIA,3.8)), we have

$$\vec{r}' = \vec{r} - \vec{V}t, \tag{3.2}$$

$$\vec{r} = \vec{r}' + \vec{V}_o t_o. \tag{3.3}$$

Adding these two equations, we obtain (1). If we assume that the clocks attached to K and K' read the same time, we can write

$$t = t_o, \qquad \vec{V} = \vec{V}_o. \tag{3.4}$$

Therefore, in such a case, we can write the transformation formulas for the space and time coordinates in the form

$$\vec{r}' = \vec{r} - \vec{V}t, \qquad t_o = t, \tag{3.5}$$

$$\vec{r} = \vec{r}' + \vec{V}t, \qquad t = t_o. \tag{3.6}$$

Formulas (3.5) represent the direct, and formulas (3.6) the inverse GALILEAN TRANSFORMATION.

B. The Lorentz transformation

Now we shall seek a transformation of the space and time coordinates which will lead to the relation (2.9) between the periods of "perpendicular" and "parallel" light clocks, as required by our tenth axiom.

Let us decompose (see fig. 3.1) the radius vectors $\vec{r}$ and $\vec{r}'$ into components $\vec{r}_\perp$, $\vec{r}'_\perp$ and $\vec{r}_\parallel$, $\vec{r}'_\parallel$, respectively, perpendicular and parallel to the direction of propagation of K'. According to the traditional Newtonian conceptions, we have

$$\vec{r}' = \vec{r}'_\perp + \vec{r}'_\parallel = \vec{r}_\perp + (\vec{r}_\parallel - \vec{V}t) = \vec{r} - \vec{V}t. \tag{3.7}$$

We can meet the requirement (2.9) of our tenth axiom if we assume that the parallel component of the relative radius vector is contracted by the factor $(1 - V^2/c^2)^{1/2}$ when expressed by the coordinates in frame K', i.e., if we assume <u>axiomatically</u> instead of the Newtonian relations

$$\vec{r}_\perp = \vec{r}'_\perp, \qquad \vec{r}_\parallel - \vec{V}t = \vec{r}'_\parallel \tag{3.8}$$

the "Lorentzian" relations

$$\vec{r}_\perp = \vec{r}'_\perp, \qquad \vec{r}_\parallel - \vec{V}t = \vec{r}'_\parallel(1 - V^2/c^2)^{1/2}. \tag{3.9}$$

This "contraction" (when $\vec{r}_\parallel - \vec{V}t$ is expressed by $\vec{r}'_\parallel$) or "dilation" (when $\vec{r}'_\parallel$ is expressed by $\vec{r}_\parallel - \vec{V}t$) is neither a physical effect, as supposed by Lorentz, nor a result of measurement, as stipulated by Einstein. According to our conceptions, $\vec{r}'_\parallel$ and $\vec{r}_\parallel - \vec{V}t$ represent the same length (distance) between two material points which can be connected by a rigid rod or which can move with respect to one another, or between two non-material points, taken at a given moment (N.B. Lengths can only be considered at a given moment!) Thus $\vec{r}'_\parallel$ and $\vec{r}_\parallel - \vec{V}t$ are equal, and we write the second relation (9) only because the velocity of light has not an exact Newton-aether character. Making a transition from (8) to (9) we introduce a striking mathematical contradiction into the traditional Newtonian mathematical apparatus. This mathematical contradiction remains in all relevant formulas, and after years of intensive mathematical speculations we have not been able to find a way to get rid of it. We ask the reader to pay due attention to this statement and not to blame our theory for mathematical imperfection. This imperfection exists in Nature itself. One must realize once and for all that light has not an exact Newton-aether character of propagation since its "there-and-back" velocity (in a frame moving in absolute space) is isotropic, while according to the Newton conceptions it must be anisotropic. We repeat (see p. 3), this peculiarity in the propagation of light was called the Marinov-aether character of light propagation.

Thus, if we wish to meet the requirement (2.9), we have to write instead of relation (7) the following relation for the transformation of the radius vectors in frames K and K'

$$\vec{r}' = \vec{r}'_\perp + \vec{r}'_\parallel = \vec{r}_\perp + \frac{\vec{r}_\parallel - \vec{V}t}{(1 - V^2/c^2)^{1/2}}. \tag{3.10}$$

This formula, written in such a manner that only the absolute radius vector is represented but not its perpendicular and parallel components $\vec{r}_\perp$, $\vec{r}_\parallel$, has the form

$$\vec{r}' = \vec{r} + [\{\frac{1}{(1 - V^2/c^2)^{1/2}} - 1\}\frac{\vec{r}.\vec{V}}{V^2} - \frac{t}{(1 - V^2/c^2)^{1/2}}]\vec{V}. \tag{3.11}$$

We shall now show that at the transformation (11) of the radius vectors the requirement of the tenth axiom will be fulfilled. Let us assume that the extremities of the "arm" of our light clock (see fig. 2.1), which is at rest in frame K', have radius vectors $\vec{r}'_A$, $\vec{r}'_B$ in frame K' and $\vec{r}_A$, $\vec{r}_B$ in frame K. According to the assumption (9), the perpendicular and parallel components of the clock's "arm" d is frames K and K' will be connected by the relations

$$d_\perp \equiv r_{B\perp} - r_{A\perp} = r'_{B\perp} - r'_{A\perp} \equiv d'_\perp, \tag{3.12}$$

$$d_\parallel \equiv r_{B\parallel} - Vt - (r_{A\parallel} - Vt) = (r'_{B\parallel} - r'_{A\parallel})(1 - V^2/c^2)^{1/2} \equiv d'_\parallel(1 - V^2/c^2)^{1/2}. \tag{3.13}$$

According to the traditional Lorentzian interpretation, if the light clock has the length d when it is at rest, then its length will remain equal to d when the clock moves with a velocity V in absolute space in a direction perpendicular to its "arm", however, its length will become "contracted" to $d(1 - V^2/c^2)^{1/2}$ when the clock moves in a direction parallel to its "arm". As formulas (2.3) and (2.5) show, in this situation the periods of both a "perpendicular" and a "parallel" light clock will remain equal.

According to our conceptions, the non-Newtonian behaviour of light clocks is to be explained not by a "physical contraction" of the light clock's "arm" but is due to the peculiarity in the propagation of light. According to us, the "there-and-back" constancy of light velocity in any inertial frame is a quality inherent to light itself and has nothing in common with the behaviour of rigid rods when they move at high speeds. We repeat, the distance between two points does not depend on the frame in which this distance is measured. In §4C, when having obtained the formulas for the light velocity in a moving frame, we shall clearly see why the rate of a light clock is independent of the orientation of its "arm".

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Let us now find the formula for the inverse transformation, i.e., from $\vec{r}'$ to $\vec{r}$. Here we have two possibilities:

1. To assume that the principle of relativity is valid (the Einstein way).
2. To assume that the principle of relativity is not valid (the Marinov way).

The Einstein way leads to a transformation of the time coordinates where the radius vectors should appear, i.e., to relative time coordinates, while the Marinov way leads to a transformation of the time coordinates where the radius vectors should not appear, i.e., to absolute time coordinates.

The Einstein way leads to the Lorentz transformation formulas which are absolutely symmetric with respect to the coordinates in frames K and K' (as are the Galilean transformation formulas), i.e., one can assume frame K attached to absolute space and frame K' moving, as well as K' attached to absolute space and K moving.

It is generally accepted that the Lorentz transformation leads to the result that in any frame the velocity of light is isotropic. As we shall show in §4B, this is not true. Since the time coordinates in the Lorentz transformation are relative quantuties, this transformation leads to the following result: If frame K is considered as attached to absolute space, and thus in K the velocity of light is isotropic, this velocity will be anisotropic in any frame K' moving with respect to K. However, since the transformation is entirely symmetric, one can assume that K' is attached to absolute space, and then the velocity of light must be accepted isotropic in K' and anisotropic in K. From these contradictory conclusions Einstein drew the fatal paradoxical corollary that the velocity of light is isotropic in any inertial frame, and a terrible logical tempest troubled the quiet waters of Newtonian physics.

If one accepts that the principle of relativity is not valid, then one must arrive at the Marinov transformations which are not symmetric, i.e., if one assumes that frame K is attached to absolute space, then the velocity of light is isotropic in K only and it is anisotropic in any frame K' moving with respect to K.

Now we shall follow the first (Einstein) way and in §3C the second (Marinov) way.

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If we consider frame K' at rest and frame K moving, then, according to the principle of relativity, aiming at meeting the requirement (2.9), we must write instead of the relations (9) the following relations

$$\vec{r}'_\perp = \vec{r}_\perp, \qquad \vec{r}'_\parallel + \vec{V}t' = \vec{r}_\parallel(1 - V^2/c^2)^{1/2}. \tag{3.14}$$

Thus, instead of formula (10) we shall now have the following one

$$\vec{r} = \vec{r}_\perp + \vec{r}_\parallel = \vec{r}'_\perp + \frac{\vec{r}'_\parallel + \vec{V}t'}{(1 - V^2/c^2)^{1/2}}, \tag{3.15}$$

and instead of formula (11) the following one

$$\vec{r} = \vec{r}' + \left[\left\{\frac{1}{(1 - V^2/c^2)^{1/2}} - 1\right\}\frac{\vec{r}'.\vec{V}}{V^2} + \frac{t'}{(1 - V^2/c^2)^{1/2}}\right]\vec{V}. \tag{3.16}$$

Adding formulas (11) and (16), we obtain

$$-\left\{\frac{1}{(1 - V^2/c^2)^{1/2}} - 1\right\}\frac{\vec{r}.\vec{V}}{V^2} + \frac{t}{(1 - V^2/c^2)^{1/2}} = \left\{\frac{1}{(1 - V^2/c^2)^{1/2}} - 1\right\}\frac{\vec{r}'.\vec{V}}{V^2} + \frac{t'}{(1 - V^2/c^2)^{1/2}}. \tag{3.17}$$

If in this formula we substitute $\vec{r}'$ from (11), we shall obtain the transformation formula for time in which t' will be expressed through t and $\vec{r}$

$$t' = \frac{t - \vec{r}.\vec{V}/c^2}{(1 - V^2/c^2)^{1/2}}. \tag{3.18}$$

On the other hand, if in formula (17) we substitute $\vec{r}$ from (16), we shall obtain the transformation formula for time in which t will be expressed through t' and $\vec{r}'$

$$t = \frac{t' + \vec{r}'.\vec{V}/c^2}{(1 - V^2/c^2)^{1/2}}. \tag{3.19}$$

Formulas (11), (18) represent the direct, and formulas (16), (19) the inverse LORENTZ TRANSFORMATION. These formulas show that not only the radius vectors $\vec{r}$ and $\vec{r}'$ are two different quantities, but also that the time coordinates t and t' are two different quantities. If frame K is attached to absolute space, we shall call t UNIVERSAL TIME and t' LORENTZ (or RELATIVE) TIME. Lorentz time is a mathematical fiction. A clock attached to frame K' reads the laboratory (or proper) time t_0 which is an absolute category (see §3C), meanwhile the Lorentz time is a specific function of the universal time defined by formula (18). For the relative time t' Lorentz used the term "coordinate time".

In §4B we give the Lorentz transformation for the coordinates' differentials (see formulas (4.2) and (4.3)).

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Now we shall give the Lorentz transformation formulas in the case of a special transformation. Putting in formulas (11), (18), (16), and (19) $\vec{r}.\vec{V} = xV$ and $\vec{r}'.\vec{V} = x'V$, we obtain

$$x' = \frac{x - Vt}{(1 - V^2/c^2)^{1/2}}, \qquad y' = y, \qquad z' = z, \qquad t' = \frac{t - xV/c^2}{(1 - V^2/c^2)^{1/2}}, \tag{3.20}$$

$$x = \frac{x' + Vt}{(1 - V^2/c^2)^{1/2}}, \qquad y = y', \qquad z = z', \qquad t = \frac{t' + x'V/c^2}{(1 - V^2/c^2)^{1/2}}. \tag{3.21}$$

For short the following abbreviations may be introduced

$$\beta = V/c, \qquad \gamma = (1 - V^2/c^2)^{-1/2} = (1 - \beta^2)^{-1/2}. \tag{3.22}$$

With the help of these designations, formulas (21) will be written in the form

$$x = \gamma(x' + Vt'), \qquad y = y', \qquad z = z', \qquad t = \gamma(t' + \beta x'/c). \tag{3.23}$$

We label the transformations (20) and (21) by the name of Lorentz following the historical tradition, although they were first introduced by Voigt (1887) (divided by γ). The labelling of certain physical notions by the name of the person who has contributed most to the introduction of those notions into a theory cannot always be done entirely adequately. Very often a notion is not called after the name of the person who made the most important contribution. So Poincaré (1904) labelled the transformation (23) by the name of Lorentz, although Lorentz (1895) conceived it eight years after Voigt and, moreover, in an eroneous form.

C. The Marinov transformation

To deduce the Marinov transformation, we shall follow the second way mentioned on p. 12. We shall proceed from our tenth axiom, ignoring the influence of the gravitating masses on the rate of a light clock, a problem considered in IV, §65.

Thus, according to the tenth axiom:

a) Light clocks with equal "arms" have the same rate, independent of the orientation of their "arms".

b) In any frame the time unit is to be defined by the period of light clocks with equal "arms", independent of the velocity of the frame.

As we have shown in §2, the first assertion contradicts the traditional Newtonian conceptions. The second assertion does not represent such a drastic contradiction if in the frame of the traditional Newtonian conceptions one also defines the time unit in any inertial frame by the period of light clocks with equal "arms", provided the "arms" always form the same angle with the velocity of the used frame, e.g., their "arms" have to be perpendicular to that velocity. In this way the absolute time dilation phenomenon will also be introduced into the traditional Newtonian theory. Thus, at first glance, it seems that the second assertion has not such a "natural" character as the first one and represents rather a stipulation. However, not only the periods of light clocks become greater when the clocks move with greater velocity in absolute space (we repeat, a phenomenon which exists also in traditional Newtonian theory), but also the periods of other physical processes as the periods of spring clocks (see §55E), the periods of atomic clocks, the mean-lives of decaying elementary particles. So far there is no experimental evidence permitting one to assert that the period of any system (say, the pulse of a man, the period of ovulation) becomes greater with the increase of its absolute velocity; we think that the generalization must be made, although this problem needs additional theoretical and experimental investigation. At any rate the time dilation should be considered not as a stipulation but as a non-Newtonian phenomenon.

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Let us find first how the Galilean transformation formulas should be written if one assumes that in any inertial frame the time unit is defined by the period of light clocks with equal "arms", supposing for definiteness' sake that the "arms" of the clocks are always perpendicular to the absolute velocity of the frames.

The period of a universal light clock (see p. 10) whose "arm" is d will be

$$T = 2d/c. \tag{3.24}$$

A proper light clock with the same "arm" which moves with velocity V in absolute space with its "arm" perpendicular to V will have a period (see (2.3))

$$T_o = \frac{2d}{c(1 - V^2/c^2)^{1/2}} = \frac{T}{(1 - V^2/c^2)^{1/2}}. \tag{3.25}$$

If (for an appropriate choice of d) we choose T as a time unit in frame K (called UNIVERSAL SECOND) and T_o as a time unit in frame K' (called PROPER SECOND), then it is clear that if the time interval between two events is t universal seconds and t_o proper seconds, the relation between them will be

$$t_o/t = T/T_o = (1 - V^2/c^2)^{1/2}, \tag{3.26}$$

where T and T_o are measured in the <u>same</u> time units (universal or proper). Under this stipulation we shall obtain from (1) and (26)

$$\vec{V}_o = \frac{\vec{V}}{(1 - V^2/c^2)^{1/2}}, \qquad \vec{V} = \frac{\vec{V}_o}{(1 + V_o^2/c^2)^{1/2}}. \tag{3.27}$$

Thus the transformation formulas (2), (3), to which we attach the relation (26), are to be written in the follwoing form

$$\vec{r}\,' = \vec{r} - \vec{V}t, \qquad t_o = t(1 - V^2/c^2)^{1/2}, \tag{3.28}$$

$$\vec{r} = \vec{r}\,' + \vec{V}_o t_o, \qquad t = t_o(1 + V_o^2/c^2)^{1/2}. \tag{3.29}$$

We call formulas (28) the direct, and formulas (29) the inverse HIGH-VELOCITY GALILEAN

TRANSFORMATION.

In these formulas $\vec{V}$ is the velocity of frame K' with respect to absolute space measured in universal seconds (called UNIVERSAL TRANSIENT VELOCITY), $\vec{V}_0$ is the same velocity measured in proper seconds (called PROPER TRANSIENT VELOCITY), and c is the velocity of light along the "arm" of the universal clock measured in universal seconds, as well as along the "arm" of the proper clock measured in proper seconds.

The Galilean transformation, as well as the high-velocity Galilean transformation, lea to the result that a "parallel" and a "perpendicular" light clock have different rates. However, the historical Michelson-Morley experiment (see §49) showed that the rates of a "perpendicular" and a "parallel" light clock are equal. We have assumed this empirical fact in our tenth axiom. Thus the high-velocity Galilean transformation is not adequate to physical reality and we must look for another one which will satisfy all requirements of the tenth axiom.

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Now we shall find this transformation. We showed in §3B that to meet the requirement of our tenth axiom about the independence of the light clock's rate on the orientation o the clock's "arm", the transformation between the radius vectors is to be written in the form (10), i.e., in the form (11).

To obtain the inverse transformation, we proceed not from the symmetric formula (15) written on the grounds of the principle of relativity but again from the <u>same</u> formula (10) which we write in the form (see (1) and (27))

$$\vec{r} = \vec{r}_\perp + \vec{r}_\parallel = \vec{r}'_\perp + \vec{r}'_\parallel(1 - V^2/c^2)^{1/2} + \vec{V}t = \vec{r}'_\perp + \frac{\vec{r}'_\parallel}{(1 + V_0^2/c^2)^{1/2}} + \vec{V}_0 t_0. \qquad (3.30)$$

This formula, written in such a manner that only the relative radius vector $\vec{r}'$ is represented but not its perpendicular and parallel components $\vec{r}'_\perp$, $\vec{r}'_\parallel$, will have the form

$$\vec{r} = \vec{r}' + [\{\frac{1}{(1 + V_0^2/c^2)^{1/2}} - 1\}\frac{\vec{r}'.\vec{V}_0}{V_0^2} + t_0]\vec{V}_0. \qquad (3.31)$$

If we express here $\vec{V}_0$ through $\vec{V}$ according to the first formula (27), then from (11) and (31) we can obtain the second formula (28) in a manner similar to that used in §3B. On the other hand, if in (11) we express $\vec{V}$ through $\vec{V}_0$ according to the second formula (27), then from (11) and (31) we can obtain the second formula (29).

Let us combine formula (11) with the second formula (28) and formula (31) with the second formula (29)

$$\vec{r}' = \vec{r} + [\{\frac{1}{(1 - V^2/c^2)^{1/2}} - 1\}\frac{\vec{r}.\vec{V}}{V^2} - \frac{t}{(1 - V^2/c^2)^{1/2}}]\vec{V},$$

$$t_0 = t(1 - V^2/c^2)^{1/2}, \qquad (3.32$$

$$\vec{r} = \vec{r}' + [\{\frac{1}{(1 + V_o^2/c^2)^{1/2}} - 1\}\frac{\vec{r}'.\vec{V}_o}{V_o^2} + t_o]\vec{V}_o =$$

$$\vec{r}' + [\{(1 - V^2/c^2)^{1/2} - 1\}\frac{\vec{r}'.\vec{V}}{V^2} + t\,]\vec{V},$$

$$t = t_o(1 + V_o^2/c^2)^{1/2} = \frac{t_o}{(1 - V^2/c^2)^{1/2}}. \quad (3.33)$$

We call formulas (32) the direct, and formulas (33) the inverse MARINOV TRANSFORMATION. These formulas show that time, although differently measured in different inertial frames, is an absolute quantity. In §4C we shall show that the Marinov transformation satisfies the requirement of the tenth axiom.

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The special Marinov transformation has the form

$$x' = \frac{x - Vt}{(1 - V^2/c^2)^{1/2}}, \quad y' = y, \quad z' = z, \quad t_o = t(1 - V^2/c^2)^{1/2}, \quad (3.34)$$

$$x = x'(1 - V^2/c^2)^{1/2} + \frac{Vt_o}{(1 - V^2/c^2)^{1/2}}, \quad y = y', \quad z = z', \quad t = \frac{t_o}{(1 - V^2/c^2)^{1/2}}, \quad (3.35)$$

having written, in the inverse transformation (35), the transient velocity in absolute time.

The direct Marinov transformation was first considered by Tangherlini (1961). Tangherlini could not show the adequacy of this transformation to physical reality and remained a relativist, thaithful to the Einstein treatment of the Lorentz transformation.

D. Group properties of the Marinov transformation

After due examination of the Marinov transformations, it can easily be established that they form a group. For short we shall show this for the special Marinov transformations. As the y- and z-coordinates are subjected to an identical transformation, we shall ignore them.

Let us write the direct transformation (34) between the coordinates (x,t) in the absolute frame K and the coordinates (x_2,t_2) in a proper frame K_2 moving with velocity V_2 $V_2 \gtrless 0$) along the positive direction of the x-axis of the rest frame K

$$x_2 = \frac{x - V_2 t}{(1 - V_2^2/c^2)^{1/2}}, \qquad t_2 = t(1 - V_2^2/c^2)^{1/2}. \quad (3.36)$$

Let us then write the inverse transformation (35) between the coordinates (x_1,t_1) in a proper frame K_1 moving with velocity V_1 ($V_1 \gtrless 0$) along the positive direction of the x-axis of the rest frame K and the coordinates (x,t) in K

$$x = x_1(1 - V_1^2/c^2)^{1/2} + \frac{V_1 t_1}{(1 - V_1^2/c^2)^{1/2}}, \qquad t = \frac{t_1}{(1 - V_1^2/c^2)^{1/2}}, \tag{3.37}$$

where the velocity V_1 (as well as the velocity V_2) is measured in universal time.

Substituting formulas (37) into formulas (36), we can express the coordinates in frame K_2 through the coordinates in frame K_1

$$x_2 = x_1\left(\frac{1 - V_1^2/c^2}{1 - V_2^2/c^2}\right)^{1/2} + t_1 \frac{V_1 - V_2}{(1 - V_1^2/c^2)^{1/2}(1 - V_2^2/c^2)^{1/2}}, \quad t_2 = t_1\left(\frac{1 - V_2^2/c^2}{1 - V_1^2/c^2}\right)^{1/2}. \tag{3.38}$$

These formulas are absolutely symmetric with respect to the coordinates in both frames. Now we shall show that these transformations form a group.

A set of transformations, T_{12}, T_{23}, T_{34},....., forms a GROUP if it has the following properties:

1. TRANSITIVE PROPERTY: The product of two transformations of the set is equivalent to a member of the set, the product

$$T_{13} = T_{12}T_{23} \tag{3.39}$$

being defined as performing T_{12} and T_{23} successively.

If formulas (38) give a transformation T_{12}, a transformation T_{23} will have the same form in which the number 1 is replaced by 2 and the number 2 by 3. Substituting formulas (38) for the transformation T_{12} into the corresponding formulas for the transformation T_{23}, we obtain a transformation T_{13} which has the same form as (38) with the difference that the number 2 is replaced by 3.

Thus the transitive property is proved as correct. We shall like to emphasize that the transitive property for the Lorentz and Galilean transformations can only be verified if one takes into account the corresponding transformation for velocities. Meanwhile the transitive property for the Marinov transformation is established directly, i.e., without taking into account the transformation for velocities.

2. IDENTITY PROPERTY: The set includes one "identity" transformation, T_{ii}, whose product with any other member of the set leaves the latter unchanged. Thus

$$T_{12}T_{22} = T_{11}T_{12} = T_{12}. \tag{3.40}$$

The identity form of the transformation (38) occurs for $V_1 = V_2$.

3. RECIPROCAL PROPERTY: Every member of the set has a unique reciprocal (or inverse) which is also a member of the set. Thus the inverse of T_{12} is T_{21}, where T_{21} is a member of the set, and

$$T_{12}T_{21} = T_{11}. \tag{3.41}$$

The reciprocal of transformation (38) can be obtained by writing the number 2 instead of 1, and vice versa.

4. ASSOCIATIVE PROPERTY: If three successive transformations are performed, then

$$T_{12}(T_{23}T_{34}) = (T_{12}T_{23})T_{34}. \qquad (3.42)$$

The associative property can easily be verified.

Thus the Marinov transformations form a group.

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Making a comparison between the Lorentz and Marinov transformations, we establish that if the Lorentz transformation is considered from an absolute point of view (i.e., recognizing the existence of absolute space), then these two transformations are not two different mathematical constructions but merely represent two formally different interpretations of the same mathematical apparatus, i.e., that they are two faces of the same coin.

We emphasize that if one seeks to preserve the adequacy of the Lorentz transformation to physical reality, one must consider this transformation from an absolute point of view. We repeat, the seeming constancy of light velocity appearing in the Lorentz transformation is due to the introduction of space coordinates in the transformation formulas for time. At an appropriate calculation (see formula (4.19)), the correct value of light velocity in a frame moving in absolute space is obtained. In the Lorentz transformation the space as well as the time coordinates are relative quantities, while light velocity is considered as an absolute quantity. In the Marinov transformation both, space coordinates and light velocity are relative quantities, while time is an absolute quantity, as it is in NATURE.

In our book we shall work with both, the Lorentz and the Marinov transformations. However, as we shall see in chapter II, the Lorentz transformation permits the introduction of a 4-dimensional mathematical formalism which (as one will establish in many chapters in parts III and V) is very productive. Thus, as a mathematical tool, the Lorentz transformation is more useful but, for a conceptual clarification, the Marinov transformation is indispensable.

§4. TRANSFORMATION OF VELOCITIES

A. The Galilean transformation

The direct and inverse GALILEAN TRANSFORMATIONS FOR VELOCITIES are given by formula (IIB,4.12), or can be obtained by differentiation from formulas (3.5) and (3.6)

$$\vec{v}' = \vec{v} - \vec{V}, \qquad \vec{v} = \vec{v}' + \vec{V}, \qquad (4.1)$$

where $\vec{v}$ is the absolute velocity, $\vec{v}'$ the relative velocity, and $\vec{V}$ the transient velocity.

B. The Lorentz transformation

Differentiating formulas (3.11), (3.18) and formulas (3.16), (3.19), we obtain the direct and inverse Lorentz transformations for the differentials of the coordinates and time

$$d\vec{r}' = d\vec{r} + [\{\frac{1}{(1 - V^2/c^2)^{1/2}} - 1\}\frac{d\vec{r}.\vec{V}}{V^2} - \frac{dt}{(1 - V^2/c^2)^{1/2}}]\vec{V},$$

$$dt' = \frac{dt - d\vec{r}.\vec{V}/c^2}{(1 - V^2/c^2)^{1/2}} \qquad (4.2)$$

$$d\vec{r} = d\vec{r}' + [\{\frac{1}{(1 - V^2/c^2)^{1/2}} - 1\}\frac{d\vec{r}'.\vec{V}}{V^2} + \frac{dt'}{(1 - V^2/c^2)^{1/2}}]\vec{V},$$

$$dt = \frac{dt' + d\vec{r}'.\vec{V}/c^2}{(1 - V^2/c^2)^{1/2}}. \qquad (4.3)$$

The ABSOLUTE VELOCITY (i.e., the UNIVERSAL ABSOLUTE VELOCITY) of the considered particle is

$$\vec{v} = d\vec{r}/dt. \qquad (4.4)$$

For the relative velocity we can build two quantities, dividing the distance $d\vec{r}'$ covered in frame K', on one hand, by the universal time interval dt

$$\vec{v}' = d\vec{r}'/dt \qquad (4.5)$$

and, on the other hand, by the Lorentz (relative) time interval dt'

$$\vec{v}'_L = d\vec{r}'/dt', \qquad (4.6)$$

calling $\vec{v}'$ the UNIVERSAL RELATIVE VELOCITY and $\vec{v}'_L$ the LORENTZ RELATIVE VELOCITY.

The relationship between $\vec{v}'_L$ and $\vec{v}$, called direct and inverse LORENTZ (or EINSTEIN) TRANSFORMATIONS FOR VELOCITIES can be obtained dividing the first equations (2) and (3) by the second ones and introducing the notations (4) and (6)

$$\vec{v}'_L = \frac{(1 - V^2/c^2)^{1/2}\vec{v} + (\{1 - (1 - V^2/c^2)^{1/2}\}\vec{v}.\vec{V}/V^2 - 1)\vec{V}}{1 - \vec{v}.\vec{V}/c^2}, \qquad (4.7)$$

$$\vec{v} = \frac{(1 - V^2/c^2)^{1/2}\vec{v}'_L + (\{1 - (1 - V^2/c^2)^{1/2}\}\vec{v}'_L.\vec{V}/V^2 + 1)\vec{V}}{1 - \vec{v}'_L.\vec{V}/c^2}. \qquad (4.8)$$

The direct relation between $\vec{v}'$ and $\vec{v}'_L$ can be found substituting the second formula (3) into (5) and the inverse relation can be found substituting the second formula (2) into (6)

$$\vec{v}' = \frac{(1 - V^2/c^2)^{1/2}}{1 + \vec{v}'_L.\vec{V}/c^2}\vec{v}'_L = \frac{(1 - V^2/c^2)^{1/2}}{1 + v'_L V\cos\theta'/c^2}\vec{v}'_L, \qquad (4.9)$$

$$\vec{v}'_L = \frac{(1 - V^2/c^2)^{1/2}}{1 - \vec{v}.\vec{V}/c^2}\vec{v}' = \frac{(1 - V^2/c^2)^{1/2}}{1 - vV\cos\theta/c^2}\vec{v}', \qquad (4.10)$$

where θ' is the angle between $\vec{v}'_L$ and $\vec{V}$ (also between $\vec{v}'$ and $\vec{V}$, since $\vec{v}'_L$ and $\vec{v}'$ are collinear) and θ is the angle between $\vec{v}$ and $\vec{V}$. Note that in formula (10) the absolute velocity

$\vec{v}$ is present.

The direct relation between $\vec{v}'$ and $\vec{v}$ can be found by dividing both sides of the first equation (2) by dt and the inverse relation can be found by dividing both sides of the first equation (3) by dt

$$\vec{v}' = \vec{v} + [\{\frac{1}{(1 - V^2/c^2)^{1/2}} - 1\}\frac{\vec{v}.\vec{V}}{V^2} - \frac{1}{(1 - V^2/c^2)^{1/2}}]\vec{V}, \qquad (4.11)$$

$$\vec{v} = \vec{v}' + [\{\frac{1}{(1 - V^2/c^2)^{1/2}} - 1\}\frac{\vec{v}'.\vec{V}}{V^2} + \frac{1}{1 + \vec{v}'_L.\vec{V}/c^2}]\vec{V}. \qquad (4.12)$$

Note that in formula (12) the Lorentz relative velocity is present.

Squaring equation (11), we obtain the following transformation formula for the magnetudes of the velocities

$$v'^2 = \frac{v^2(1 - V^2/c^2) - 2\vec{v}.\vec{V} + (\vec{v}.\vec{V})^2/c^2 + V^2}{1 - V^2/c^2} = \frac{v^2(1 - V^2\sin^2\theta/c^2) - 2vV\cos\theta + V^2}{1 - V^2/c^2}. \qquad (4.13)$$

From formulas (7) and (8) the following two very important relations can be obtained

$$(1 - v_L'^2/c^2)^{1/2} = \frac{(1 - V^2/c^2)^{1/2}(1 - v^2/c^2)^{1/2}}{1 - \vec{v}.\vec{V}/c^2}, \qquad (4.14)$$

$$(1 - v^2/c^2)^{1/2} = \frac{(1 - V^2/c^2)^{1/2}(1 - v_L'^2/c^2)^{1/2}}{1 + \vec{v}'_L.\vec{V}/c^2}, \qquad (4.15)$$

We obtain (14) by squaring (7) and building the expression on the left side of (14). In the same way we obtain (15) from (8).

When $\vec{v}'_L$ is perpendicular to $\vec{V}$, i.e., for $\vec{v}'_L.\vec{V} = 0$, (8) is reduced to

$$\vec{v} = (1 - V^2/c^2)^{1/2}\vec{v}'_L + \vec{V}. \qquad (4.16)$$

A similar relation can be obtained between $\vec{v}'_L$ and $\vec{v}$ when $\vec{v}$ is perpendicular to $\vec{V}$, i.e., for $\vec{v}.\vec{V} = 0$.

When $\vec{v}'_L$ is parallel to $\vec{V}$, i.e., for $\vec{v}'_L.\vec{V} = v'_L V$, (8) is reduced to

$$v = \frac{v'_L + V}{1 + v'_L V/c^2}. \qquad (4.17)$$

A similar relation can be obtained between v and v'_L when $\vec{v}$ is parallel to $\vec{V}$, i.e., for $\vec{v}.\vec{V} = vV$.

If $v'_L = c$, we obtain from (17), supposing that $\vec{v}'_L$ and $\vec{V}$ are collinear,

$$v = \frac{c + V}{1 + V/c} = c. \qquad (4.18)$$

If $v'_L = v = c$, we obtain from formulas (13) and (9) for the UNIVERSAL RELATIVE LIGHT VELOCITY (write $v' = c'$)

$$c' = c\,\frac{1 - V\cos\theta/c}{(1 - V^2/c^2)^{1/2}} = c\,\frac{(1 - V^2/c^2)^{1/2}}{1 + V\cos\theta'/c}, \qquad (4.19)$$

since the LORENTZ RELATIVE LIGHT VELOCITY c'_L is equal to the UNIVERSAL ABSOLUTE LIGHT VELOCITY c.

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The direct and inverse special Lorentz transformations for velocities have the form

$$v'_{Lx} = \frac{v_x - V}{1 - v_xV/c^2}, \qquad v'_{Ly} = \frac{v_y(1 - V^2/c^2)^{1/2}}{1 - v_xV/c^2}, \qquad v'_{Lz} = \frac{v_z(1 - V^2/c^2)^{1/2}}{1 - v_xV/c^2}, \qquad (4.20)$$

$$v_x = \frac{v'_{Lx} + V}{1 + v'_{Lx}V/c^2}, \qquad v_y = \frac{v'_{Ly}(1 - V^2/c^2)^{1/2}}{1 + v'_{Lx}V/c^2}, \qquad v_z = \frac{v'_{Lz}(1 - V^2/c^2)^{1/2}}{1 + v'_{Lx}V/c^2}. \qquad (4.21)$$

C. The Marinov transformation

In the light of our conceptions for time, we introduce the following types of velocity:

1. The UNIVERSAL ABSOLUTE VELOCITY is the velocity $\vec{v}$ in the Loretzian approach defined by equality (4).

2. The UNIVERSAL RELATIVE VELOCITY is the velocity $\vec{v}'$ defined by equality (5).

3. The PROPER ABSOLUTE VELOCITY

$$\vec{v}_o = d\vec{r}/dt_o \qquad (4.22)$$

is the absolute velocity measured in proper time, i.e., on a clock attached to the moving frame.

4. The PROPER RELATIVE VELOCITY

$$\vec{v}'_o = d\vec{r}'/dt_o \qquad (4.23)$$

is the relative velocity measured in proper time.

Remark. Let us remember (see IIA, p. 21) that when the proper clock is attached to the laboratory which moves in absolute space, it is also called laboratory clock and measures laboratory time. The proper clock may be attached to the particle which moves in the moving frame K' considered as laboratory; then it measures the particle's proper time, and the clock attached to K' measures the laboratory's proper time.

Writing in the first formulas (3.32) and (3.33) $d\vec{r}$, dt, $d\vec{r}'$, dt_o instead of $\vec{r}$, t, $\vec{r}'$, t_o, dividung them by dt and introducing the notations (4) and (5), we obtain the direct and inverse MARINOV TRANSFORMATIONS FOR VELOCITIES

$$\vec{v}' = \vec{v} + \left[\left\{\frac{1}{(1 - V^2/c^2)^{1/2}} - 1\right\}\frac{\vec{v}.\vec{V}}{V^2} - \frac{1}{(1 - V^2/c^2)^{1/2}}\right]\vec{V}, \qquad (4.24)$$

$$\vec{v} = \vec{v}' + \left[\left\{(1 - V^2/c^2)^{1/2} - 1\right\}\frac{\vec{v}'.\vec{V}}{V^2} + 1\right]\vec{V}. \qquad (4.25)$$

These are the Marinov transformations for velocities written in universal time. If we divide not by dt but by dt_o, the Marinov transformations for velocities written in proper time should be obtained.

One may also write the transformation formulas for velocities in which the relative velocity is expressed in proper time and the absolute velocity in universal time. This would be the Marinov transformations for velocities written in mixed time.

The transformation formulas for the magnitudes of the velocities can be obtained from (24) and (25) by squaring

$$v'^2 = \frac{v^2(1 - V^2\sin^2\theta/c^2) - 2vV\cos\theta + V^2}{1 - V^2/c^2}, \tag{4.26}$$

$$v^2 = v'^2(1 - V^2\cos^2\theta'/c^2) + 2v'V\cos\theta'(1 - V^2/c^2)^{1/2} + V^2. \tag{4.27}$$

Evidently, formula (26) coincides with formula (13).

If we suppose that the moving particle is a photon, i.e., v = c, and we write v' = c', then these two equations (the second after a solution of a quadratic equation with respect to v') give formulas (19).

If we denote by c'_o the PROPER RELATIVE LIGHT VELOCITY, then its connection with the UNIVERSAL ABSOLUTE LIGHT VELOCITY c will be

$$c'_o = c\,\frac{1 - V\cos\theta/c}{1 - V^2/c^2} = \frac{c}{1 + V\cos\theta'/c}, \tag{4.28}$$

and its connection with the PROPER ABSOLUTE LIGHT VELOCITY

$$c_o = \frac{c}{(1 - V^2/c^2)^{1/2}} \tag{4.29}$$

will be the same as the one given by formulas (19) for the connection between c' and c.

Note that the velocities with respect to the moving frame K' are called relative, while the clocks attached to K' are called proper, and the velocities with respect to the rest frame K are called absolute, while the clocks attached to K are called universal.

We designate the relative quantities by primes and the proper quantities by zero subscripts. For this reason, in the Lorentz trasnformation (where time is relative), we designate the relative time coordinates by primes, and in the Marinov transformation (where time is absolute), we designate the proper time coordinates by zero subscripts.

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The special Marinov transformation for velocities has the form (we write only the inverse transformation)

$$v_x = v'_x(1 - V^2/c^2)^{1/2} + V, \quad v_y = v'_y, \quad v_z = v'_z. \tag{4.30}$$

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Now we shall show why the rates of a "perpendicular" and a "parallel" light clock are equal.

The angle between the velocity of light along the "arm" of a "peprendicular" light clock and the velocity $\vec{V}$ of the clock is $\theta' = \pi/2$, if measured in frame K', and $\theta = \pi/2 - V/c$, if measured in frame K, so that formula (28) gives

$$c'_o = c \tag{4.31}$$

for the proper relative light velocity, i.e., for the light velocity measured in frame K' on a clock attached to K'; thus for the period of the clock we obtain

$$T_\perp = 2d/c. \tag{4.32}$$

The angles between the velocity of light along the "arm" of a "parallel" light clock and the velocity $\vec{V}$ of the clock are $\theta' = \theta = 0$ when light propagates in the "direct" direction (i.e., along $\vec{V}$) and $\theta' = \theta = \pi$ when light propagates in the "opposite" direction (i.e., against $\vec{V}$), if measured both in K' and K, so that formula (28) gives for the proper relative light velocities

$$c_o'^{+} = \frac{c}{1 + V/c}, \qquad c_o'^{-} = \frac{c}{1 - V/c}, \tag{4.33}$$

and for the period we obtain

$$T_\parallel = d/c_o'^{+} + d/c_o'^{-} = 2d/c. \tag{4.34}$$

From (32) and (34) we obtain the requirement (2.9) of our tenth axiom.

§5. TRANSFORMATION OF ACCELERATIONS

A. The Galilean transformation

The GALILEAN TRANSFORMATIONS FOR ACCELERATIONS are given by formula (IIB,5.11), or can be obtained by differentiation of formulas (4.1)

$$\vec{u}' = \vec{u}, \qquad \vec{u} = \vec{u}'. \tag{5.1}$$

B. The Lorentz transformation

The general LORENTZ TRANSFORMATIONS FOR ACCELERATIONS can be found by differentiating formulas (4.7), (4.8), dividing them by the second formulas (4.2), (4.3), and introducing the UNIVERSAL ABSOLUTE ACCELERATION, $\vec{u} = d\vec{v}/dt$, and the LORENTZ RELATIVE ACCELERATION $\vec{u}'_L = d\vec{v}'_L/dt'$.

We shall only write the formulas for the inverse special Lorentz transformation of accelerations, proceeding from formulas (4.20) and the last formula (3.21),

$$u_x = \frac{u'_{Lx}(1 - V^2/c^2)^{1/2}}{(1 + v'_{Lx}V/c^2)^3},$$

$$u_y = \frac{u'_{Ly}(1 + v'_{Lx}V/c^2) - v'_{Ly}u'_{Lx}V/c^2}{(1 + v'_{Lx}V/c^2)^3}(1 - V^2/c^2),$$

$$u_z = \frac{u'_{Lz}(1 + v'_{Lx}V/c^2) - v'_{Lz}u'_{Lx}V/c^2}{(1 + v'_{Lx}V/c^2)^3}(1 - V^2/c^2). \qquad (5.2)$$

C. The Marinov transformation

The MARINOV TRANSFORMATIONS FOR ACCELERATIONS can be obtained by differentiating formulas (4.24), (4.25) and dividing them by the relevant time differentials (universal or proper).

We shall only write the formula for the inverse general Marinov transformation for accelerations written in universal time

$$\vec{u} = \vec{u}' + \{(1 - V^2/c^2)^{1/2} - 1\}\frac{\vec{u}'.\vec{V}}{V^2}\vec{V}, \qquad (5.3)$$

where $\vec{u}$ is the UNIVERSAL ABSOLUTE ACCELERATION and $\vec{u}'$ is the UNIVERSAL RELATIVE ACCELERATION.

More about the transformation of accelerations see in §25A.

§6. SPACE INTERVALS

A. Emission, reception, and middle distances

The deduction of the Lorentz and Marinov transformations contained a certain peculiar formalism connected with the phenomenon which has the general name LORENTZ LENGTH CONTRACTION. We repeat, according to our conception, the Lorentz contraction is not a physical phenomenon and must only be considered as a mathematical result of the Marinov character of light propagation. In the present section we shall consider the high-velocity aspects of space intervals following a more natural and clearer way. In this way one can perceive the need for the introduction of the Marinov character of light propagation and become familiar with the second order in v/c peculiarities of the space-time relations which, we repeat, are not Newtonian.

Consider fig. 6.1 where PQ is a rod proceeding with velocity $\vec{v}$ in absolute space so that its initial point crosses the origin of the rest frame of reference K at the zero initial moment. We are interested in how one can measure the length of the rod by using light signals, being stationary in the rest frame K. In parallel to the rod, we shall also consider a point q (it coincides with the final point of the rod) which proceeding with

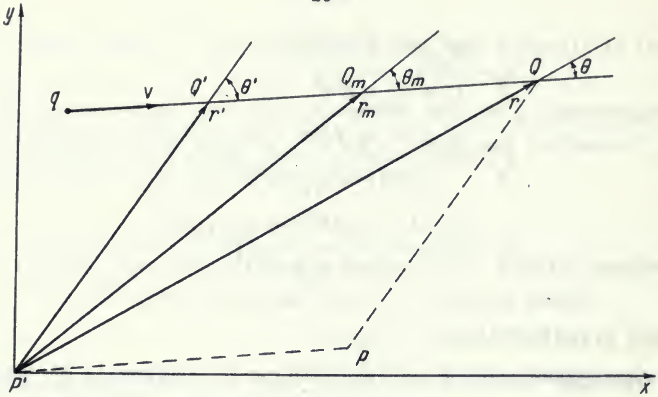

Fig. 6.1

the velocity $\vec{v}$ crosses point Q' at the zero initial moment, and we want to know its distance from the frame's origin at this very moment. Let us, for definiteness, analyse the case of a moving material point, and so to free our mind from the complicated notion "rigid rod".

If we have light signals at our disposal, there are two possibilities of measuring the distance between point q and the frame's origin at the moment when point q crosses the space point Q':

a) either to send a "runner" at the initial moment $t_{in} = 0$, called the EMISSION MOMENT which will cover the distance r' = Q'P' with velocity c to arrive at point P' at the moment

$$t' = r'/c, \tag{6.1}$$

called the RECEPTION MOMENT,

b) or to send a "runner" at the emission moment $t_{in} = 0$ which will cover the distance r = P'Q to arrive at point Q at the reception moment

$$t = r/c. \tag{6.2}$$

Using formulas (3.5) and (3.6), in which we write $\vec{V} = \vec{v}$, we obtain for the first and second case, respectively, the following relations between the distances r and r'

$$r = r'(1 + 2v\cos\theta'/c + v^2/c^2)^{1/2} = r'\{(1 - v^2\sin^2\theta/c^2)^{1/2} + v\cos\theta/c\}, \tag{6.3}$$

$$r = \frac{r'}{(1 - v^2\sin^2\theta'/c^2)^{1/2} - v\cos\theta'/c} = \frac{r'}{(1 - 2v\cos\theta/c + v^2/c^2)^{1/2}}, \tag{6.4}$$

where θ' is the angle between $\vec{v}$ and $\vec{r}'$, called the EMISSION ANGLE, while θ is the angle between $\vec{v}$ and $\vec{r}$, called the RECEPTION ANGLE. The distance r' can be called the EMISSION DISTANCE and the "track" distance r can be called the RECEPTION DISTANCE.

When the "photon-runner" covers distance r' between the emission and reception moments one has

$$\frac{r'}{c} = \frac{r\cos\theta - r'\cos\theta'}{v}, \tag{6.5}$$

from where the follwoing relation between r and r' can be obtained

$$r = r' \frac{\cos\theta' + v/c}{\cos\theta}, \tag{6.6}$$

while when the "photon-runner" covers distance r between the emission and reception moments, one has

$$\frac{r}{c} = \frac{r\cos\theta - r'\cos\theta'}{v}, \tag{6.7}$$

from where the following relation between r and r' can be obtained

$$r = r' \frac{\cos\theta'}{\cos\theta - v/c}. \tag{6.8}$$

Let us now find the relation between r and r' when the "photon-runner" covers the MIDDLE DISTANCE r_m, starting at the emission zero moment from Q_m (or from P') and arriving t P' (or at Q_m) at the reception moment

$$t_m = r_m/c. \tag{6.9}$$

In this case we shall have

$$\frac{r_m}{c} = \frac{r\cos\theta - r'\cos\theta'}{v}. \tag{6.10}$$

To find a relation between r', r, θ', θ, v, and c which would correspond to this condition, we have to define an expression for r_m through certain of the elements r', r, θ', θ, v, and c substantially different from (10), and, putting this expression fot r_m into (10), we will obtain a suitable relation between r', r, θ', θ, v, and c. We exerted many efforts to do this in a mathematically correct manner but we always obtained too complicated expressions (for example, write the law of cosines (I,54.36) for the triangles $P'Q'Q_m$ and $P'Q_mQ$ which involve the angles θ' and θ, and exclude then the quantity r_m). Thus we arrived to the conclusion that the simplest but mathematically incorrect way is the following:

Introduce the approximate relations

$$r' = r_m - \frac{r_m}{2c} v\cos\theta_m, \qquad r = r_m + \frac{r_m}{2c} v\cos\theta_m, \tag{6.11}$$

where θ_m is the angle between v and r_m, called the MIDDLE ANGLE, and write the equation (10) in the approximate forms

$$r = r' \frac{\cos\theta' + r_m v/r'c}{\cos\theta} = r' \frac{\cos\theta' + v/c + v^2\cos\theta_m/2c^2}{\cos\theta}, \tag{6.12}$$

$$r = \frac{r'\cos\theta'}{\cos\theta - r_m v/rc} = \frac{r'\cos\theta'}{\cos\theta - v/c + v^2\cos\theta_m/2c^2}. \tag{6.13}$$

Multiplying these two equations and taking the square root, we obtain approximately

$$r = r'\left\{\frac{\cos\theta'(\cos\theta' + v/c)}{\cos\theta(\cos\theta - v/c)}\right\}^{1/2}. \tag{6.14}$$

We assume that the relation (14) is the exact expression (with an exactitude higher than of second order in v/c) corresponding to condition (10). It is strange, however, that the expression (14) cannot be obtained following a mathematically correct way from the condition (10) and the geometry presented in fig. 6.1. This engenders an interesting mathematical problem to which we call the attention of the mathematicians. May be, the Marinov character of light propagation has not some "physical" ground but is a simple "mathematical" result, following from the impossibility to find a relation between r', r, θ' θ, v, and c which should correspond to condition (10).

From a physical point of view, all three cases (1), (2), and (9) of how to send the "photon-runner" are realizable. Nature, however, realizes only the case (9). Only at the realization of case (9) the Michelson-Morley experiment will give a null result and both a "perpendicular" and a "parallel" light clock will display the same rate. We must emphasize that all these conclusions have some strange character. Although having ruminated during many years over the Marinov character of light propagation, we cannot present a clear explanation why light has this peculiarity of motion. Let us add that this peculiarity concerns also the motion of any massive particle (see §44), and it enters into the whole body of gravimagretism and electromagnetism (see the Liènard-Wiechert potentials in IV, §3 and V, §3).

May be, in future days somebody will be able to give a clear and logic explanation of the Marinov character of light propagation. Let us hope. Fecit quod potui, feciant meliora potentes.

From fig. 6.1, according to the law of sines (I,54.54), we have

$$r'/r = \sin\theta/\sin\theta', \qquad (6.15)$$

and from (14) and (15) we obtain the following relations between the angles θ' and θ

$$\cos\theta = \frac{\cos\theta' + v/c}{1 + v\cos\theta'/c}, \qquad \cos\theta' = \frac{\cos\theta - v/c}{1 - v\cos\theta/c}. \qquad (6.16)$$

From formulas (15) and (16) we find

$$r = r'\,\frac{(1 - v^2/c^2)^{1/2}}{1 - v\cos\theta/c} = r'\,\frac{1 + v\cos\theta'/c}{(1 - v^2/c^2)^{1/2}}, \qquad (6.17)$$

from where

$$r = r'\left(\frac{1 + v\cos\theta'/c}{1 - v\cos\theta/c}\right)^{1/2}. \qquad (6.18)$$

From (17) we get

$$(1 + v\cos\theta'/c)(1 - v\cos\theta/c) = 1 - v^2/c^2. \qquad (6.19)$$

Since it is approximately

$$\cos\theta' = \cos\theta_m + a, \qquad \cos\theta = \cos\theta_m - a, \qquad (6.20)$$

where a is a positive or negative quantity, we can write (18) approximately in the form

$$r = r'\left(\frac{1 + v\cos\theta_m/c}{1 - v\cos\theta_m/c}\right)^{1/2}. \qquad (6.21)$$

xxxxx

The relations (17) can also be obtained if one proceeds from the Lorentz transformation. We shall show this for the special Lorentz transformation; the demonstration for the general transformation can be performed in the same manner.

Let us have a "rod" which has an arbitrary position in frame K' (see fig. 6.1) and let us find the track which a light signal covering the length of the "rod" will leave in the rest frame K. Let us suppose that the light signal is sent at the moment t_1 (t_1') and it arrives at the other extremity of the rod at the moment t_2 (t_2'). Thus, according to the Lorentzian conceptions we shall have

$$t_2 - t_1 = \frac{1}{c}\{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\}^{1/2} = \frac{r}{c}, \tag{6.22}$$

$$t_2' - t_1' = \frac{1}{c}\{(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2\}^{1/2} = \frac{r'}{c}, \tag{6.23}$$

where (x_1', x_1', x_1'), (x_2', y_2', z_2') are the coordinates of the extremities of the rod in frame K' and (x_1, y_1, z_1), (x_2, y_2, z_2) are the extremities of the track which the light signal has left in frame K.

Proceeding from formulas (3.20), we build the differences $x_2' - x_1'$. $y_2' - y_1'$, $z_2' - z_1'$. Squaring these differences, adding their left and right sides, respectively, taking the square root from the equation obtained, substituting in its right side (22), and using the relation (see fig. 3.2)

$$(x_2 - x_1)/r = \cos\theta, \tag{6.24}$$

we obtain the following relation

$$r' = r\,\frac{1 - V\cos\theta/c}{(1 - V^2/c^2)^{1/2}} = r\,\frac{1 - \vec{r}.\vec{V}/rc}{(1 - V^2/c^2)^{1/2}}, \tag{6.25}$$

where $\vec{r}/r$ is the unit vector along the track which the light signal has left in absolute space (in frame K). This formula coincides with the first formula (17), if we put $V = v$.

In the same way, proceeding from formulas (3.21), we build the differences $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$. Squaring these differences, adding their left and right sides, respectively, taking the square root from the equation obtained, substituting in its right side (23), and using the relation (see fig. 3.2)

$$(x_2' - x_1')/r' = \cos\theta', \tag{6.26}$$

we obtain the following relation

$$r = r'\,\frac{1 + V\cos\theta'/c}{(1 - V^2/c^2)^{1/2}} = r'\,\frac{1 + \vec{r}'.\vec{V}/r'c}{(1 - V^2/c^2)^{1/2}}, \tag{6.27}$$

where $\vec{r}'/r'$ is the unit vector along the rod. This formula coincides with the second formula (17), if we put $V = v$.

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Finally, let us show that the relations (17) follow immediately from the formulas

(4.19) for the relative light velocity.

Indeed, if an observer in frame K will measure the time in which light will cover the length of the moving rod r' sending a light pulse from the initial point P' to the final point Q' which will be "caught" at the space point Q (the sending is at the moment when P' crosses point P), he will register the universal time interval

$$t = r/c. \tag{6.28}$$

An observer attached to the moving rod, with respect to which the velocity of light measured in universal time is given by formulas (4.19), will measure the universal time interval

$$t = \frac{r'}{c'} = \frac{r'}{c}\,\frac{(1 - v^2/c^2)^{1/2}}{1 - v\cos\theta/c} = \frac{r'}{c}\,\frac{1 + v\cos\theta'/c}{(1 - v^2/c^2)^{1/2}}. \tag{6.29}$$

From the last two formulas we obtain the relations (17).

If the observer attached to the moving rod registers the time on a proper clock, he has to use the expressions (4.28) for the velocity of light and will measure the proper time interval

$$t_0 = \frac{r'}{c_0'} = \frac{r'}{c}\,\frac{1 - v^2/c^2}{1 - v\cos\theta/c} = \frac{r'}{c}(1 + v\cos\theta'/c), \tag{6.30}$$

which, according to the second formula (3.32) is equal to the universal time interval if divided by $(1 - v^2/c^2)^{1/2}$. Thus again from (28) and (30) we can obtain the relations (17).

B. Advanced, retarded, and observation distances

In this subsection we shall introduce certain generalizations of the notions introduce in §6A which are very useful when one analyses the propagation of electromagnetic waves.

Let us consider again a point q proceeding with an arbitrary velocity $\vec{v}$ in the rest frame of reference K (see fig. 6.2 and compare it with fig. 6.1). When q crosses the spac point Q', a light signal (a "photon-runner") is sent towards point P, which we shall call the REFERENCE POINT. This light signal, covering distance r', reaches point P at the moment t, called the OBSERVATION MOMENT, when q crosses point Q. At this very moment a ligh signal is sent from P which, covering distance r", catches q when it crosses point Q".

The moment

$$t' = t - r'/c, \tag{6.31}$$

at which a light signal is sent from q when it crosses point Q', is called the ADVANCED MOMENT.

The moment

$$t'' = t + r''/c, \tag{6.32}$$

at which a light signal sent from P reaches q when it crosses point Q", is called the RETARDED MOMENT.

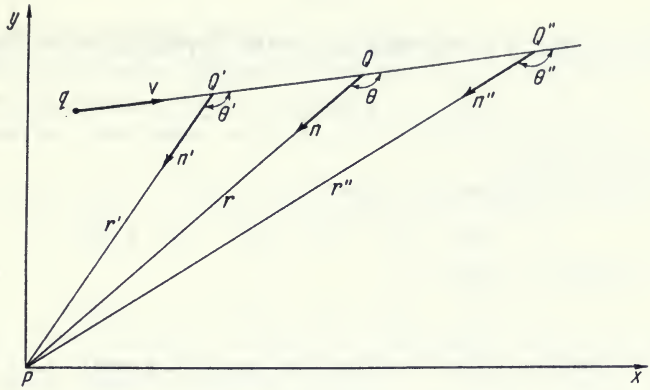

Fig. 6.2

Slightly different values for the advances and retarded moments should be obtained if in formulas (31) and (32) we write r instead of r' and r".

We call r', r", and r, respectively, the ADVANCED, RETARDED, and OBSERVATION DISTANCES.

When comparing fig. 6.2 with fig. 6.1, we must take into account that the triangle P'Q'Q in fig. 6.1 corresponds either to the triangle PQ'Q or to the triangle PQQ" in fig. 6.2. Take also into account that in fig. 6.1 the radius vectors $\vec{r}'$ and $\vec{r}$ (i.e., the unit vectors $\vec{r}'/r'$ and $\vec{r}/r$) point from the rest point P' to the moving point q, while in fig. 6.2 the unit vectors $\vec{n}'$, $\vec{n}$, and $\vec{n}''$ point from the moving point q to the rest point P. We see immediately that if the emission moment is the advanced moment, then the reception moment is the observation moment, while if the emission moment is the observation moment, then the reception moment is the retarded moment.

Thus, on the grounds of formulas (25) and (27), we can write

$$r = r'\,\frac{1 - \vec{n}'.\vec{v}/c}{(1 - v^2/c^2)^{1/2}} = r'\frac{1 - v\cos\theta'/c}{(1 - v^2/c^2)^{1/2}} = r'\,\frac{(1 - v^2/c^2)^{1/2}}{1 + \vec{n}.\vec{v}/c} = r'\,\frac{(1 - v^2/c^2)^{1/2}}{1 + v\cos\theta/c}, \quad (6.33)$$

$$r = r''\,\frac{1 + \vec{n}''.\vec{v}/c}{(1 - v^2/c^2)^{1/2}} = r''\frac{1 + v\cos\theta''/c}{(1 - v^2/c^2)^{1/2}} = r''\,\frac{(1 - v^2/c^2)^{1/2}}{1 - \vec{n}.\vec{v}/c} = r''\,\frac{(1 - v^2/c^2)^{1/2}}{1 - v\cos\theta/c}. \quad (6.34)$$

C. The proper distance

The Marinov character of light propagation leads to a certain contradiction between the formulas for space intervals. As a matter of fact, this contradiction was introduced during the deduction of the Lorentz (p. 11) and Marinov (p. 16) transformations for coordinates. We should like to add here a couple of words more.

In the previous two subsections we analysed the space relations in the rest and moving frames when one sends a light signal between two points in the moving frame (or between a moving point and a point at rest). Now we shall analyse these relations at a single moment.

Thus let the coordinates of a moving rod be (x_1', y_1', z_1') and (x_2'', y_2'', z_2'') in the moving frame K'. Let the coordinates of the rod's extremities registered in the frame K at the same moment

$$t_1 = t_2 \qquad (6.35)$$

be (x_1,y_1,z_1) and (x_2,y_2,z_2).

The length of the rod (registered in frame K') is

$$r' = \{(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2\}^{1/2}. \qquad (6.36)$$

This length registered in frame K is

$$r = \{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\}^{1/2}. \qquad (6.37)$$

According to our absolute conceptions these two lengths are equal, i.e.,

$$r = r', \qquad (6.38)$$

however, because of the Marinov character of light propagation, we shall obtain another relation between r and r', if we should use the Lorentz or Marinov transformation. Indeed, using in (36) the Lorentz transformation (3.20) or the Marinov transformation (3.34) under the condition (35), we obtain

$$r_o = \left[\frac{(x_2 - x_1)^2 + (1 - v^2/c^2)\{(y_2 - y_1)^2 + (z_2 - z_1)^2\}}{(1 - v^2/c^2)^{1/2}}\right]^{1/2}, \qquad (6.39)$$

where we have replaced the symbol r' by the symbol r_o to emphasize that the length of the rod which is at rest in K' is expressed not by the coordinates in K' but by its coordinates in K.

Using the relation (24), but emphasizing that now r is not the track of a light signal left in K which has covered the length of a moving rod but the momentary snap-shot of the moving rod in frame K, so that now $\theta = \theta'$, we can write (39) in the form

$$r_o = r\left(\frac{1 - v^2\sin^2\theta/c^2}{1 - v^2/c^2}\right)^{1/2}. \qquad (6.40)$$

For $\theta = \pi/2$ this formula leads to the relation

$$r_o = r, \qquad (6.41)$$

which coincides with the relation (38), if putting $r' = r_o$. However, for $\theta = 0$, we obtain

$$r_o = \frac{r}{(1 - v^2/c^2)^{1/2}}. \qquad (6.42)$$

When a certain distance (a length of a "rod") in the moving frame K' is expressed by the coordinates of its extremities registered in the rest frame K, we shall call this PROPER DISTANCE (or PROPER LENGTH) and denote it by r_o. Because of the Marinov character of light propagation the proper distance r_o is not equal to the distance r registered in frame K and the relation is given by formula (40). To this mathematical peculiarity contemporary physics gives the name LORENTZ CONTRACTION, considering it as a physical contraction of the moving bodies. We repeat, according to our conception, this is not true. The peculiarity presented by formula (40) when comparing it with formula (38) is only mathematical and results from the Marinov character of light propagation.

For mathematical convenience, we introduce a SECOND PROPER DISTANCE according to the relation

$$r_{oo} = r_o(1 - v^2/c^2)^{1/2} = [(x_2 - x_1)^2 + (1 - v^2/c^2)\{(y_2 - y_1)^2 + (z_2 - z_1)^2\}]^{1/2} \tag{6.43}$$

§7. TIME INTERVALS. KINEMATIC (LORENTZ) TIME DILATION

Let us find the relation between the readings of a universal and a proper clock, proceeding from the Lorentz transformation. Since the proper clock is at rest in frame K', its radius vector there will be a constant quantity and for any two moments t_1', t_2' we shall have

$$\vec{r}_1' = \vec{r}_2'. \tag{7.1}$$

The radius vectors of the clock in the frame K for the corresponding universal moments t_1, t_2 will be related by the formula

$$\vec{r}_2 - \vec{r}_1 = \vec{V}(t_2 - t_1). \tag{7.2}$$

Making use of any of the formulas (3.11), (3.18), (3.16), or (3.19), under the conditions (1) and (2), we obtain

$$t_2' - t_1' = (t_2 - t_1)(1 - V^2/c^2)^{1/2}. \tag{7.3}$$

Denoting $t_2' - t_1' = \Delta t_o$, $t_2 - t_1 = \Delta t$, $V = v$, we obtain

$$\Delta t_o = \Delta t(1 - v^2/c^2)^{1/2}, \tag{7.4}$$

which is formula (3.26). We call (4) the KINEMATIC (or LORENTZ) TIME DILATION. It was considered first by Larmor (1900) and then by Einstein (1905), but since it is a result of the Lorentz transformation, we attribute to it the name of Lorentz. While the Lorentz length contraction (see p. 32) is a mathematical fiction the Lorentz time dilation is a real physical phenomenon.

One cannot make the symmetric opposite assumption (as one does in the theory of relativity) that the clock at rest in frame K is moving with respect to frame K' (in such a case, according to the Lorentz transformation, one would obtain that the rate of the universal clock will be delated with respect to the rate of the proper clock). Indeed, the absolute frame K is defined as the one in which the center of mass of the whole world is at rest. Evidently, the proper clock moves with respect to this center; the proper clock cannot be such a one with respect to which the whole world moves.

In the Marinov transformation the time dilation is introduced explicitly.

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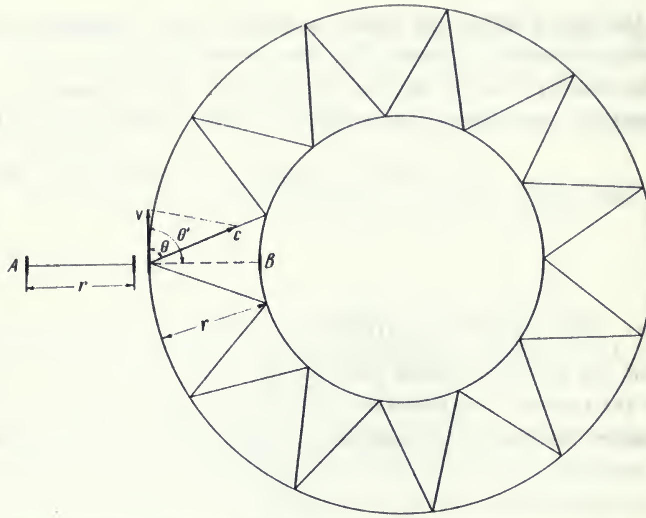

Fig. 7.1

Now we shall give certain simple examples which manifest clearly the absolute character of the time dilation.

Let us have (see fig. 7.1) two light clocks, one of which (clock A) is at rest in the reference frame attached to absolute space, and the other (clock B) performs a rotational motion in such a manner that its "arm" always remains perpendicular to the linear velocity of rotation. If clocks A and B have the same "arms", they will go exactly at the same rate when being at rest. However, if clock B performs the above-mentioned rotational motion, its light pulse will always arrive, with a specific time delay, later than the corresponding light pulse in clock A. As we have already established (the same results are obtained whether proceeding from relativistic or from non-relativistic conceptions), if the periods of both clocks are measured in the same time (universal or proper), the relation between the period T_0 of the proper clock and the period T of the universal clock will be given by formula (3.25). Thus, after having covered a closed path, the proper clock will be found to be late, the difference in the readings being (use formula (4) calculating within an accuracy of second order in v/c)

$$\delta t = \Delta t - \Delta t_0 = \frac{1}{2}\frac{v^2}{c^2}\Delta t. \tag{7.5}$$

In fig. 7.1 the motion of clock B is non-inertial during the whole time of separation from clock A. We shall now show that the same effect of time dilation will be obtained when the motion of clock B is inertial during the predominant part of the separation time.

Indeed, let us have (see fig. 7.2) a light clock A which is at rest in absolute space, and an identical light clock B which passes near A (at point b) with velocity v. Up to point b' clock B moves inertially with velocity v. From point b' to point b" its velocity diminishes to zero, and from point b" to point b' its velocity increases again to v, but oppositely directed. Clock B then begins to move inertially and, with the same velocity v, passes near clock A. Assuming that the time of non-inertial motion is insignificantly short with respect to the time of inertial motion, we obtain again relations (4) and (5).

In both cases presented in fig. 7.1 and fig. 7.2 we supposed that the "arm" of the moving light clock B is perpendicular to its velocity, so that the calculations could be performed proceeding from high-velocity as well as from low-velocity conceptions. Because of the Marinov character of light propagation, any light clock will have the same rate independently of the orientation of its "arm".

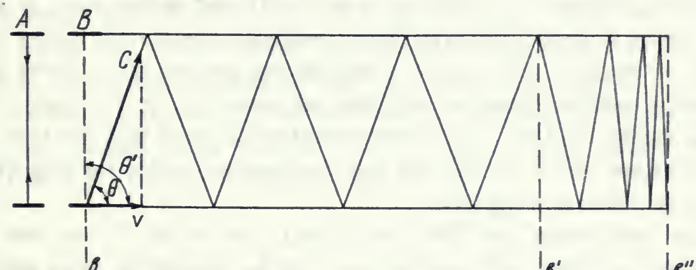

Fig. 7.2

§8. THE TWIN PARADOX

In the theory of relativity the rest frame K and the moving frame K' have exactly the same rights. According to the principle of relativity, one can consider frame K at rest and frame K' moving, as well as K' at rest and K moving. In this approach the time dilation must be not an absolute but a relative phenomenon, i.e., for an observer in frame K the clocks attached to frame K' go with a lower rate, while for an observer in frame K' the clocks attached to frame K go with a lower rate. From this Einsteinian paradigm the so-called twin paradox has arisen.

The TWIN (or CLOCK) PARADOX can be formulated as follows: If one of two twins remained fixed in an inertial frame, while the other undertakes a journey at great speed, then the twin-traveller would be found to have aged less upon returning than his stay-at-home brother (assume the twins were clocks).

We must emphasize that the twin paradox arises not from the fact that there is a physical time dilation in material systems moving with respect to each other (and thus with respect to absolute space), but from the relativistic dogma that this time dilation is absolutely symmetric and mutually opposite for both systems. Thus the insurmountable logical difficulty for the theory of special relativity consists in the following: to obtain an asymmetric result from two absolutely symmetric effects. Thousands and thousands of pages written by very clever men are dedicated to the logical explanation of this unresolvable paradox.

In this section we shall show that the twin paradox cannot be resolved within the framework of special relativity, and thus the efforts of the Einstein disciples to explain it with mathematical and logical speculations of different quality and quantity were a terrible waste of time for authors and readers.

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Let us consider the traditional twins who carry with themselves identical clocks, say, their hearts or fine and precise atomic clocks. This signifies that when the twins are at rest in an inertial frame of reference, the hands of their clocks always show the same readings on their clock-faces.

In any textbook on relativity, with the help of different motivations, as a rule, the following conclusion is drawn: the twin who has suffered acceleration during the time of separation will be found to have aged less than the one who has not suffered acceleration. Here the following question always arises: What has become of the twin (and of the clock carried by him) during the time of accelerated motion? To avoid this question, we shall consider two twins who suffer exactly the same acceleration during the time of separation. Such a case can be realized as follows:

At the initial zero moment both twins are at rest with respect to the used inertial frame of reference, and they switch on the thrust of two rockets in which both of them have taken a seat. When the velocities of their rockets become equal to v, both twins switch off the thrust and the rocket of the second twin flies further under its own momentum, while the first twin immediately switches on a backward thrust, reducing the velocity from v to -v. Arriving at the velocity -v, the backward thrust is switched off and again a forward thrust is switched on until the velocity of the rocket becomes equal to zero and the first twin lands at the start. There he awaits the home-coming of his brother.

According to special relativity, the twin paradox appears as a result of the free flight. Hence during the time when the twins have suffered acceleration (which time, obviously, is <u>equal for both twins</u>) they can stop if not their hearts, then at least their clocks, and compare only the readings for the free flight time of the twin-traveller and the rest time of his stay-at-home brother. If moreover we assume that the "free-of-acceleration" time intervals are much longer than the time intervals when the twins have suffered acceleration, the latter can be neglected with respect to the former.

Assume that the readings of the clocks of both twins at the initial moment are equal to zero. If after their meeting again the readings of the clocks are, respectively, t_1 and t_2, then, according to special relativity, one has

$$t_2 = t_1(1 - v^2/c^2)^{1/2}. \qquad (8.1)$$

This formula can be written on the grounds of the Lorentz transformation only by the first twin, if proceeding from the principle of relativity. The second twin has to write exactly the opposite relation. If both twins establish, after comparing the readings of their clocks, that relation (1) is the really existing one, then we have to conclude that this relation is written by an absolute hand, by someone who uses not only the Lorentz transformation formulas - an observer who knows that twin 1 has taken once a full portion of anti-acceleration pills, while twin 2 has taken three times anti-acceleration pills: the first and the third time 1/4th part of the full portion any time and the second time 1/2th portion. Correspondingly the clock of twin 1 has been stopped once, while the clock of twin 2 has been stopped thrice. Without these additional conditions no conclusions can be made.

Let us suppose that the second twin noted down in his log-book the reading t" of his clock at the moment when he switched on the backward thrust, taking the decision to end his forward journey and to return home. We pose the following question to special relativity: which is the relation between the reading t" and the reading $t' = t_1/2$? According to the relativistically treated Lorentz transformation formulas, from the point of view of twin 1 (i.e., working in his inertial frame of reference with respect to which the second twin flies with velocity v), the relation must be

$$t'' = t'(1 - v^2/c^2)^{1/2}, \tag{8.2}$$

while from the point of view of twin 2 (i.e., working in his inertial frame of reference with respect to which the first twin flies with velocity v) the relation must be

$$t' = t''(1 - v^2/c^2)^{1/2}. \tag{8.3}$$

The readings t' and t" of both clocks cannot be directly compared because between both twins the largest distance of their separation extends. With the aim of establishing which of relations (2) and (3) is the true one, let twin 1 send at the moment t' a third twin who will fly with velocity 2v (in the frame of twin 1) along the direction in which the second twin has flown, and let twin 2 return home with a daughter rocket, leaving a fourth twin to fly on with velocity v (in the frame of twin 1) with the mother rocket.

Twin 3 will "catch" twin 4 when the readings of their clocks will be, respectively, t_3 and t_4, and, according to special relativity, one has

$$t_3 = t_4(1 - v^2/c^2)^{1/2}, \tag{8.4}$$

because twin 3 flies (with respect to the inertial frame where twin 4 is at rest) with a velocity -v during the first half of his journey and with a velocity v during the second half of the journey.

Putting into formula (1) $t_1 = 2t'$ and $t_2 = 2t''$, and into formula (4) $t_3 = 2t'$ and $t_4 = 2t''$, we obtain formulas (2) and (3), respectively, which are mathematically contradicting one another.

Here the unique objection can be made: whether one can be sure that the relations $t_2 = 2t''$ and $t_3 = 2t'$ are true. However, the theory of special relativity cannot pose this objection, because for this theory absolute space does not exist and only the mutual velocity of the twins is of importance. Hence, for special relativity, if twin 2 has spent time t" for the "there" journey in the inertial frame of twin 1, then he must spend exactly the same time for the "back" journey. The same is to be said for the third twin who spends time t' for the "there" journey and exactly the same time for the "back" journey in the inertial frame of twin 4.

Thus the existence of time dilation within the framework of special relativity is a logically inconsistent assertion, and it is to be regretted that the father of this theory defended such an inwardly contradictory conclusion.

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Proceeding from the Lorentz transformation formulas, we can come to the following two logically consistent alternatives: either to reject the time dilation dogma, treating it as a seeming fictitious effect but not as a physical one (as it is done by certain relativists), or to accept this dogma together with the assumption of absolute space (as it is done in our absolute space-time theory).

According to our theory, the courses of two clocks do not depend on their mutual velocity but on their velocities with respect to absolute space. Hence in our theory we arrive at the following conclusions: If the first twin is at rest in absolute space, the relation between the readings t' and t" will be given by (2); if the fourth twin is at rest in absolute space, the relation will be given by (3). As it can easily be verified, in both cases the relation between t_1 and t_2 will be given by (1), and the relation between t_3 and t_4 will be given by (4).

Indeed, let us show this for the case where twin 1 rests in absolute space. Then relation (2) will be valid and relation (1) can immediately be obtained. Furthermore we shall have

$$t_4 = 2t'' = 2t'(1 - v^2/c^2)^{1/2} \cong 2t'(1 - v^2/2c^2) \qquad (8.5)$$

and

$$t_3 = t' + t'(1 - 4v^2/c^2)^{1/2} \cong 2t'(1 - v^2/c^2), \qquad (8.6)$$

so that from (5) and (6) we obtawn relation (4).

If twin 1 flies with a velocity v' with respect to absolute space and twin 4 (thus also twin 2 in the first half of his journey) flies with a velocity v" with respect to absolute space, the relation between t' and t" will be (see formula (3.38))

$$\frac{t'}{(1 - v'^2/c^2)^{1/2}} = \frac{t''}{(1 - v''^2/c^2)^{1/2}}, \qquad (8.7)$$

as it can immediately be established, taking into account a fifth twin who is at rest with respect to absolute space.

Thus in our theory we come to the following assertion: Absolute space does exist, and the course of time in material systems moving with respect to absolute space is different. These assertions do not lead to any paradoxical results.

According to us, the twin-traveller will age less than his stay-at-home brother, but from a mathematical point of view there is nothing paradoxical in this conclusion. A layman may consider as a strange and amazing result only the natural fact that time has different course in material systems moving at different velocities in absolute space. This phenomenon, however, is demonstrated quite simply by the help of a light clock, taking into account the Marinov character of light propagation. Thus the absolute time dilation contains no more paradoxical elements than, say, the conclusion to which Archimedus came, establishing that all bodies lose on their weight proportionally to their volumes when being put in a liquid.

§9. SLOW TRANSFER OF CLOCKS

Consider two spatially separated clocks in the moving frame K'. We say (see IIA, p. 20) that there is a NEWTONIAN TIME SYNCHRONIZATION between these two clocks if at any moment they show the same readings. As we said in part IIA, and we showed by our "coupled mirrors" experiment (see §51 and §52), a Newtonian time synchronization can be realized by the help of a rotating rigid shaft. An EINSTEINIAN TIME SYNCHRONIZATION between spatially separated clocks is realized by the help of light signals, assuming that the velocity of light is isotropic and equal to c in any inertial frame. Then the reading of the synchronized clock is set equal to the reading of the clock placed at the frame's origin (a "mother" clock) at the moment when the light signal has been sent plus r/c, where r is the distance between the clocks. Let us note that a Newtonian time synchronization can be made by the help of light signals, too, as we show in §56.

It is quite obvious that under an Einsteinian time synchronization only the clocks placed along a line perpendicular to the velocity of frame K' will display a Newtonian time synchronization, too. The clocks in the hemisphere with a pole taken at the apex of the frame's velocity will be slow, proportionally to the distance from the frame's origin and the cosine of the angle concluded between this distance and the absolute velocity, while the clocks placed in the hemisphere with a pole taken at the anti-apex of the frame's velocity will, in a similar manner, be fast.

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Certain authors assert that by a slow transfer of clocks one can realize a Newtonian time synchronization. This is not true. We shall show that by a slow transfer of clocks one realizes an Einsteinian time synchronization.

Indeed, let us transfer a clock with the low relative velocity $\vec{v}'$ from the frame's origin O' to a point in space P' having a relative radius vector $\vec{r}'$. If the transfer of the clock begins at the universal moment $t = 0$, corresponding to the proper moment $t_0 = 0$, and ends at the universal moment t, the reading of the transferred clock at the moment of arrival will be

$$t_0^{tr} = t\{1 - \frac{(\vec{V}+\vec{v}')^2}{c^2}\}^{1/2} = t - \frac{V^2}{2c^2}t - \frac{v'V}{c^2}t\cos\theta' - \frac{v'^2}{c^2}t, \qquad (9.1)$$

where $\vec{V}$ is the absolute velocity of frame K' and θ' is the angle between $\vec{v}'$ (thus $\vec{r}'$) and $\vec{V}$. Since it is $v't = r'$, we obtain from (1), for $v' \to 0$,

$$t_0^{tr} = t - \frac{V^2}{2c^2}t - \frac{r'V}{c^2}\cos\theta' - \frac{r'v'}{2c^2} \cong t - \frac{V^2}{2c^2}t - \frac{r'V}{c^2}\cos\theta'. \qquad (9.2)$$

In equation (1) we work within an accuracy of second order in "velocity/light velocity". If we should like to work with a higher accuracy, instead of the Galilean formula for velocity addition (4.1), we have to use the Marinov formula (4.27).

Let us now find the reading of a clock, placed at the point P', which is synchronized by the "mother" clock, placed at the frame's origin, according to the Einstein synchronization convention. If we assume that the synchronizing signal arrives at the moment when the transferred clock arrives at point P', we conclude that it had to be sent from O' at the proper moment

$$t_0^{se} = t(1 - V^2/c^2)^{1/2} - r'/c_0' = t - \frac{V^2}{2c^2}t - (\frac{r'}{c} + \frac{r'V}{c^2}\cos\theta'), \qquad (9.3)$$

where c_0' is the proper relative light velocity of the signal propagating from O' to P', and we have used formula (4.28).

According to the Einsteinian time synchronization convention, the clock placed at P' will, at the universal moment t, show the following proper time

$$t_0^{ar} = t_0^{se} + r'/c = t - \frac{V^2}{2c^2}t - \frac{r'V}{c^2}\cos\theta'. \qquad (9.4)$$

Comparing formulas (2) and (4), we conclude that the reading of the transferred clock will be equal to the reading of the clock placed at point P'. Thus a slow transfer of clocks leads not to a Newtonian but to an Einsteinian time synchronization.

Chapter II

K I N E M A T I C S I N T H E 4 - S P A C E

§10. 4-SPACE

Minkowski has shown that it is more convenient from a mathematical point of view if one considered the time axis as a fourth coordinate axis along which the time coordinate is taken in the form

$$\tau = i\,c\,t, \tag{10,1}$$

i.e., as an <u>imaginary position coordinate</u> (i is the imaginary unit), the absolute value of which is equal to the time coordinate, t, multiplied by the velocity of light, c. Now all coordinates, space and time, are measured in length units, and thus one can consider the "space-time" position of any material point in a hypothetical 4-DIMENSIONAL SPACE, in which all four coordinate axes are mutually perpendicular. Of course, the time axis preserves a special mathematically different character, since the time position coordinates are imaginary numbers, while the space position coordinates are real numbers. Any point in the 4-space is called a WORLD POINT. Any world point corresponds to a physical EVENT, i.e., to the occurence of something at a given space point at a given moment. Obviously, the notions "world point" and "event" are to be considered as synonyms.

We attach a 4-DIMENSIONAL FRAME OF REFERENCE to the 4-dimensional space. Thus the 4-dimensional frame is composed by an ordinary 3-dimensional space frame and by a time axis along which the coordinates are taken in the form (1). The coordinates of the origin O of the 4-dimensional frame are $x = 0$, $y = 0$, $z = 0$, $\tau = 0$ (i.e., $t = 0$). The coordinates of an arbitrary world point P are x, y, z, τ (i.e., t). The radius vector pointing from the origin of the 4-frame to the world point P is called the 4-RADIUS VECTOR and is denoted by $\overset{\mapsto}{r}$, i.e., putting above its letter symbol the sign $\mapsto$. The 4-radius vector consists of the ordinary 3-radius vector $\vec{r}$ and the time "radius vector" $\tau = i\bar{r}$; the magnitude of the imaginary (time) component of the 4-radius vector is denoted by the same letter r but above it the sign "-" is put. Thus the components of the 4-radius vector can be written in the following different forms

$$\overset{\mapsto}{r} = (r_1,r_2,r_3,r_4) = (r_x,r_y,r_z,r_\tau) = (r_x,r_y,r_z,ir_t) = (x,y,z,\tau) = (x,y,z,ict) = (\vec{r},i\bar{r}). \tag{10.2}$$

The unit vectors along the four coordinate axes of the 4-frame K are designated by $\hat{\vec{x}}$, $\hat{\vec{y}}$, $\hat{\vec{z}}$, $\hat{\vec{\tau}}$. Thus the 4-radius vector can be presented in the form

$$\overset{\mapsto}{r} = x\hat{\vec{x}} + y\hat{\vec{y}} + z\hat{\vec{z}} + ict\hat{\vec{\tau}}. \tag{10.3}$$

The absolute value of the 4-radius vector of the world point P represents the "length" of the line segment in the 4-dimensional space between the frame's origin and this world point, thus (cf. I, p. 192)

$$|\vec{r}| = \{(x - 0)^2 + (y - 0)^2 + z - 0)^2 + (ict - 0)^2\}^{1/2} = (x^2 + y^2 + z^2 - c^2t^2)^{1/2}. \quad (10.4)$$

This 4-dimensional distance is called 4-INTERVAL.

If there are two world points P_1 and P_2, the 4-interval between them will be the magnitude of the difference of their 4-radius vectors

$$|\vec{r}_2 - \vec{r}_1| = \{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - c^2(t_2 - t_1)^2\}^{1/2}. \quad (10.5)$$

§11. TRANSFORMATION OF COORDINATES IN THE 4-SPACE

Let us now consider another 4-frame K' in our 4-space. As we know (see IIA, p. 7), any two frames in the 3-space are of the first, second, or third class (i.e., translated, rotated, or reflected). Any two 4-frames in the 4-space may be also of the first, second, or third class. We shall consider 4-frames only of the first and second class.

The 4-radius vector of the world point P in the 4-frame K' will be (cf. (10.3))

$$\vec{r}' = x'\hat{x}' + y'\hat{y}' + z'\hat{z}' + ict'\hat{\tau}', \quad (11.1)$$

and the 4-interval between the world points P_1 and P_2 in the 4-frame K' will be (cf. (10.5))

$$|\vec{r}_2' - \vec{r}_1'| = \{(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2 - c^2(t_2' - t_1')^2\}^{1/2}. \quad (11.2)$$

The distance between two points considered in two frames of the first and second class is the same quantity, and thus we can write

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - c^2(t_2 - t_1)^2 = (x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2 - c^2(t_2' - t_1')^2. \quad (11.3)$$

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Let us find the kind of relation between the space and time coordinates of a world point considered in the 4-frames K and K', i.e., under a transformation from the 4-frame K to the 4-frame K'. A transformation of the first class, representing a shift of the reference frame parallel to itself (the unit vectors remain respectively parallel), is of no interest, since it only leads to a sfift of the origin of the space part and to a change of the time reference point (i.e., a shift of the origin of the time part). Thus we have to analyse a transformation of the second class representing a rotation of the 4-frame.

Every rotation in the 4-space can be dissolved into six simple rotations in the planes (xy), (xz), (yz), (xτ), (yτ), and (zτ), as every rotation in the 3-space can be dissolved into three rotations in the first three planes. The first three of these six simple rotations transform only the space coordinates; they correspond to the usual space rotations, and, thus, there is no motion of the space part of K' with respect to the space part of K.

Let us consider the rotation in the $x\tau$-plane. Under this rotation the y- and z-coordinates do not change, and thus this must be a special transformation (see p. 8). Let us designate the angle of rotation by ϕ. If we use the transformation formulas (IIB,3.13), writing τ intead of y, we shall have

$$x = x'\cos\phi - \tau'\sin\phi, \qquad \tau = x'\sin\phi + \tau'\cos\phi. \tag{11.4}$$

We shall now determine the angle ϕ which can only depend on the relative velocity V with which the space part of K' moves with respect to the space part of K. Let us consider the motion in frame K of the origin of the space part of K'. Then $x' = 0$, and formulas (4) take the form

$$x = -\tau'\sin\phi, \qquad \tau = \tau'\cos\phi. \tag{11.5}$$

Dividing the first of them by the second, we obtain

$$x/\tau = -\tan\phi. \tag{11.6}$$

Taking into account that at the zero initial moment the origins of the space parts of frames K and K' coincide, we shall have

$$x = Vt, \tag{11.7}$$

and making use of (10.1), we obtain from (6)

$$\tan\phi = iV/c. \tag{11.8}$$

We wrote equation (7), supposing that the velocity V is constant, i.e., that the space parts of the frames K and K' are inertial. If the velocity V is not constant, then a unique 4-dimensional frame cannot correspond to the non-inertial frame of reference K'.

Now substituting (8) into the mathematical relations (see (I,9.5))

$$\sin\phi = \frac{\tan\phi}{(1 + \tan^2\phi)^{1/2}}, \qquad \cos\phi = \frac{1}{(1 + \tan^2\phi)^{1/2}}, \tag{11.9}$$

we obtain

$$\sin\phi = \frac{iV/c}{(1 - V^2/c^2)^{1/2}}, \qquad \cos\phi = \frac{1}{(1 - V^2/c^2)^{1/2}}. \tag{11.10}$$

The substitution of (10) into (4) gives

$$x = \frac{x' - iV\tau'/c}{(1 - V^2/c^2)^{1/2}}, \qquad y = y', \qquad z = z', \qquad \tau = \frac{\tau' + iVx'/c}{(1 - V^2/c^2)^{1/2}}. \tag{11.11}$$

We write here also the transformation formulas for the y- and z-coordinates, which undergo an identical transformation, as we consider only a rotation in the $x\tau$-plane, i.e., when the motion of the space part of K' is along the x-axis of frame K.

If we use once again (10.1), formulas (11) yield the inverse special Lorentz transformation formulas (3.21).

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Our first and second axioms affirm that the 3-dimensional space is homogeneous and isotropic and time homogeneous. If one considers the 4-space mathematically in the same manner as the 3-space, one must accept that the 4-space is also homogeneous and isotropic. Only under this assumption, for a transformation of the space-time coordinates in the 4-space under a rotation, we can use the <u>same type</u> of transformation formulas which have been used in the 3-space. As our analysis in this section has shown, the transformation formulas in the 4-space, under the assumption of its isotropy, are the Lorentz transformation formulas.

Under the aspect of the Newton model of light propagation, there are two kinds of effects which lead to an anisotropy of the inertial frames (note that a moving frame is <u>not</u> a 3-dimensional frame; any 3-dimensional frame, in which space is isotropic according to our first axiom, is considered <u>at a given moment</u>, as if being at rest in absolute space):

a) the second-order in V/c effects which make the "there-and-back" light velocity direction dependent,

b) the first order in V/c effects which make the "there" and "back" light velocity direction dependent.

As the 4-space must be isotropic, these two anisotropy effects <u>are to be annihilated</u> in the transformation formulas of the space-time coordinates. The annihilation of the second order in V/c anisotropy is performed by introducing the Marinov character of light propagation into the Lorentz transformation, thus by the appearance of the factor $(1 - V^2/c^2)^{1/2}$, while the annihilation of the first order in V/c anisotropy is performed by making the time coordinates relative, thus depending on the space coordinates. As the reader has seen, these two "annihilations" of the anisotropy effects are introduced <u>automatically</u> following the logic of the mathematical speculations. This <u>mysterious power</u> of mathematics legitimately impresses human mind, if one takes into account that the second-order in V/c anisotropy is annihilated by Nature <u>physically</u>, while the first-order in V/c anisotropy exists physically in Nature, and in the Lorentz transformation the annihilation is only formal. The Lorentz transformation and the mathematical structure of 4-space which is a <u>mathematical image of our physical space and time</u> have been analysed and discussed by mankind for a whole century and will further be investigated, showing that human mind is still too far from understanding its proper wonderful and enigmatic creation called "mathematics".

In the 4-space we always work with the Lorentz transformation. The Marinov transformation cannot be used in the 4-space because, according to the Marinov transformation, the inertial frames are not isotropic. However, using the "isotropic" Lorentz transformation, <u>we have to treat it from an absolute point of view</u>, always attaching the rest frame K to absolute space, where the velocity of light is, indeed, isotropic.

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In the same manner as in §3D, one can show that the Lorentz transformations form a group. However, we repeat (see p. 18), the transitive property can only be verified if one takes into account the Lorentz transformation formulas for velocities (4.20) and (4.21). Analogically, the transitive property of the Galilean transformations, which are also in conformity with the principle of relativity, can only be verified if one takes into account the Galilean transformation formulas for velocities (4.1).

There is, however, an important difference between the Galilean and Lorentz transformations. The first have the general property of commutativity, i.e., the combined result of two successive transformations (with different velocities $\vec{V}_1$ and $\vec{V}_2$) does not depend on the order in which the transformations are performed. Meanwhile, the result of two successive Lorentz transformations generally does depend on their order. This is already apparent - purely mathematically - from the formal description of these transformations as rotations of the 4-dimensional frame: the result of two rotations (about different axes) depends on the order in which they are carried out. The sole exception is the case of transformations with parallel vectors $\vec{V}_1$ and $\vec{V}_2$(which are equivalent of two rotations of the 4-dimensional frame about the same axis).

§12. TRANSFORMATION OF VELOCITIES IN THE 4-SPACE

For simplicity's sake, we have only considered a special transformation of the coordinates in the 4-space. We shall also consider a special transformation of the velocities.

Let us have two successive rotations of the 4-dimensional frame in the plane $(x\tau)$, the first to an angle ϕ_1, i.e., corresponding to a velocity $V_1 = -\,ic\tan\phi_1$, and the second to an angle ϕ_2, i.e., corresponding to a velocity $V_2 = -\,ic\tan\phi_2$ (see (11.8)). The resultant angle $\phi = \phi_1 + \phi_2$ will correspond to a velocity V, for which we obtain (see (I,9.6))

$$V = -\,ic\tan\phi = -\,ic\,\frac{\tan\phi_1 + \tan\phi_2}{1 - \tan\phi_1\tan\phi_2} = \frac{V_1 + V_2}{1 + V_1V_2/c^2}, \qquad (12.1)$$

which represents formula (4.17).

§13. 4-VECTORS

The Euclidean transformations in the 3-space have been considered in I, §56 and can be called 3-DIMENSIONAL EUCLIDEAN TRANSFORMATIONS. The transformations in the 4-space considered in §11 can be called 4-DIMENSIONAL EUCLIDEAN TRANSFORMATIONS.

Two types of 4-dimensional Euclidean transformations can be considered:

1. Pseudo-Euclidean transformations.
2. Proper Euclidean transformations.

Under a PSEUDO-EUCLIDEAN TRANSFORMATION the fourth (time) axis is an imaginary one, i.e., the coordinates along the time axis are taken in the form (10.1).

Under a PROPER EUCLIDEAN TRANSFORMATION the fourth (time) axis is a real one, i.e., the coordinates along the time axis are taken in the form

$$\tau = ct. \tag{13.1}$$

According to our Newtonian conceptions about the absoluteness of space and time, we can only use the pseudo-Euclidean transformation because we assume time to be a quantity which is qualitatively absolutely different from space, and it is an extremely happy occasion that in mathematics there are two types of entirely different numbers which can by no means be mixed.

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Using the designations (10.2), we can write formulas (11.11) in the form

$$r_1 = \frac{r_1' - i\frac{V}{c}r_4'}{(1 - V^2/c^2)^{1/2}}, \qquad r_2 = r_2', \qquad r_3 = r_3', \qquad r_4 = \frac{r_4' + i\frac{V}{c}r_1'}{(1 - V^2/c^2)^{1/2}}. \tag{13.2}$$

Now we introduce the notion "4-vector" as follows (cf. I, p. 235):

4-DIMENSIONAL VECTOR (or 4-VECTOR) $\overset{\mapsto}{a}$ is any set of four quantities a_1, a_2, a_3, a_4 which must be transformed under a special Lorentz transformation according to formulas (2), i.e., according to the formulas

$$a_1 = \frac{a_1' - i\frac{V}{c}a_4'}{(1 - V^2/c^2)^{1/2}}, \qquad a_2 = a_2', \qquad a_3 = a_3', \qquad a_4 = \frac{a_4' + i\frac{V}{c}a_1'}{(1 - V^2/c^2)^{1/2}}. \tag{13.3}$$

By analogy with the 4-radius vector, we call the first three components of every 4-vector SPACE COMPONENTS and the fourth component - TIME COMPONENT. The space components a_1, a_2, a_3 are real quantities and the time component a_4 is an imaginary quantity.

The components of the 4-vector $\overset{\mapsto}{a}$ can be written in the following different forms

$$\overset{\mapsto}{a} = (a_1,a_2,a_3,a_4) = (a_x,a_y,a_z,a_\tau) = (a_x,a_y,a_z,ia_t) = \begin{matrix} (a_\alpha), & \alpha = 1,2,3,4, \\ (a_a,a_4), & a = 1,2,3. \end{matrix} \tag{13.4}$$

Remark. We assume the following convention which will be preserved throughout this book:

When the index of a 4-vector is written with a Greek letter, it signifies that this letter takes the values 1,2,3,4, i.e., with a Greek index we denote all components of the 4-vector.

When the index of a 4-vector is written with a Latin letter, it signifies that this letter takes the values 1,2,3, i.e., with a Latin index we denote only the space components of the 4-vector.

The summation convention (see I, p. 235) will be used for Latin indices, as well as for Greek indices.

Formula (3) written with the real part of the time component has the form

$$a_x = \frac{a'_x + \frac{V}{c}a'_t}{(1 - V^2/c^2)^{1/2}}, \quad a_y = a'_y, \quad a_z = a'_z, \quad a_t = \frac{a'_t + \frac{V}{c}a'_x}{(1 - V^2/c^2)^{1/2}}. \tag{13.5}$$

We have shown (see p. 43) that every transformation from one 4-dimensional frame to another represents a rotation in the 4-space. Hence the transformation formulas (3) can in general be written in a form analogous to (I,67.4)

$$a_\alpha = \sum_{\beta=1}^{4} \alpha_{\alpha\beta} a'_\beta, \qquad \beta = 1,2,3,4, \tag{13.6}$$

or, using the summation convention, in the form

$$a_\alpha = \alpha_{\alpha\beta} a'_\beta. \tag{13.7}$$

Comparing this with (3), we obtain the following values for the coefficients in the case of a special transformation (see (3.22))

$$\|\alpha_{\alpha\beta}\| = \begin{Vmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{Vmatrix} = \begin{Vmatrix} \gamma & 0 & 0 & -i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ i\beta\gamma & 0 & 0 & \gamma \end{Vmatrix}, \tag{13.8}$$

which represent the cosines of the angles between the corresponding coordinate axes of both frames (cf. table I,56.1).

xxxxx

Analogically to the vector products of 3-vectors, we can introduce vector products of 4-vectors (cf. I, §54A):

1. <u>The product of a scalar p and a 4-vector $\overleftrightarrow{a}$</u> is a 4-vector

$$\overleftrightarrow{b} = p\overleftrightarrow{a} \tag{13.9}$$

with components

$$b_\alpha = p\,a_\alpha. \tag{13.10}$$

The same laws which are valid for the product $p\vec{a}$ are also valid for the product $p\overleftrightarrow{a}$ (see (I,54.4), (I,54.5), (I,54.6), and (I,54.7)).

2. <u>The scalar product of two 4-vectors $\overleftrightarrow{a}$, $\overleftrightarrow{b}$</u> is a scalar

$$p = \overleftrightarrow{a}.\overleftrightarrow{b} \tag{13.11}$$

which can be expressed by the components of both 4-vectors as follows

$$p = a_\alpha b_\alpha. \tag{13.12}$$

The same laws which are valid for the product $\vec{a}.\vec{b}$ are also valid for the product $\overleftrightarrow{a}.\overleftrightarrow{b}$ (see (I,54.14), (I,54.15), and (I,54.16)).

The square of the 4-vector $\overleftrightarrow{a}$

$$\overleftrightarrow{a}^2 = a_\alpha^2 = a^2 + a_4^2 = a^2 - a_t^2 \tag{13.13}$$

can be positive, negative, or zero. Thus the magnitude of $\overleftrightarrow{a}$ will be real, imaginary, or zero, respectively. In these three cases we call the 4-vector SPACELIKE, TIMELIKE, or NULL, respectively (see §16).

A 4-dimensional analogue to the vector product of two 3-vectors cannot be built. As we have seen, the vector product of two 3-vectors represents, as a matter of fact, an antisymmetric tensor (see I, p. 240), and it is better to call such a product a "tensor product of two vectors". This 3-dimensional tensor product of two 3-vectors has its 4-dimensional analogue (see §14).

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With respect to the space rotations of the 4-dimensional frame (i.e., rotations in the planes (xy), (xz), and (yz)), the three space components of the 4-vector $\overleftrightarrow{a}$ represent a space vector. With respect to those rotations the fourth (time) component represents a scalar (we can say a "3-dimensional scalar"). Thus the 4-vector $\overleftrightarrow{a}$ can also be written in the form (cf. (10.2))

$$\overleftrightarrow{a} = (\vec{a}, a_\tau) = (\vec{a}, i a_t) = (\vec{a}, i\bar{a}). \tag{13.14}$$

xxxxx

Analogically to the scalar and 3-vector functions of scalar and vector arguments (see I, §57 and §59), we can introduce scalar and 4-vector functions of scalar and 4-vector arguments.

We can then consider differentiation of 4-vectors with respect to a scalar argument and of scalars and 4-vectors with respect to a 4-vector argument (cf. I, §58 and §60).

Of course, under a scalar we must now understand a "4-dimensional scalar", i.e., a quantity which preserves its value under any Lorentz transformation. Such a quantity, i.e. a 4-dimensional scalar, is called a LORENTZ INVARIANT QUANTITY, too.

xxxxx

The 4-dimensional analogues to the Hamilton operator ∇ and to the Laplace operator Δ (see I, §61) are:

1. The ERMA OPERATOR (the symbol was proposed by the Bulgarian physicist Erma Gerova)

$$\triangledown = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, -\frac{i}{c}\frac{\partial}{\partial t}\right), \tag{13.15}$$

which represents a symbolical 4-vector.

2. The d'ALEMBERT OPERATOR

$$\triangle = \triangledown \,.\, \triangledown = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}, \tag{13.16}$$

which represents a symbolical 4-dimensional scalar. We write the d'Alembert operator not as a square $\square$, as it is commonly done, but as a trapezium $\triangle$. So one comes to a perfect parallelism between the Hamilton and Laplace operators, on the one hand, and the Erma and d'Alembert operators, on the other hand.

xxxxx

Taking into account formulas (3.16), (3.19), and (10.2), we can write the transformation formulas for the space and time parts of any 4-vector $\overleftrightarrow{a}$ under a general Lorentz transformation in the form

$$\vec{a} = \vec{a}' + [\{\frac{1}{(1 - V^2/c^2)^{1/2}} - 1\}\frac{\vec{a}'.\vec{V}}{V^2} + \frac{\bar{a}'}{c(1 - V^2/c^2)^{1/2}}]\vec{V},$$

$$\bar{a} = \frac{1}{(1 - V^2/c^2)^{1/2}}(\bar{a}' - \frac{\vec{a}'.\vec{V}}{c}). \qquad (13.17)$$

§14. 4-TENSORS

In the 4-dimensional space we can consider 4-tensors by analogy with the 3-tensors in the 3-space. Their definition is analogical to that of the 3-tensors (see I, §67).

4-TENSOR $\overleftrightarrow{A}$ is any set of 16 quantities $A_{\alpha\beta}$ which must be transformed under a special Lorentz transformation according to the following formulas

$$A_{\alpha\beta} = \alpha_{\alpha\gamma}\alpha_{\beta\delta}A'_{\gamma\delta}, \qquad (14.1)$$

where the coefficients $\alpha_{\alpha\beta}$ are given by the matrix (13.8). We designate the 4-tensor by putting above its letter symbol the sign $\leftrightarrow$. The components of the 4-tensor $\overleftrightarrow{A}$ can be arranged in the following table

$$\overleftrightarrow{A} = \begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{vmatrix} = (A_{\alpha\beta}). \qquad (14.2)$$

All components A_{a4} and A_{4a} are imaginary quantities and the others are real.

With respect to the space rotations of the 4-dimensional frame, the nine space-space components A_{ab} of the 4-tensor represent a space tensor

$$\overleftrightarrow{A} = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix} = (A_{ab}). \qquad (14.3)$$

The time components $A_{\alpha4}$ and $A_{4\alpha}$ represent two 4-vectors

$$(A_{\alpha4}) = i(\vec{A}, i\bar{A}) \qquad (14.4)$$

and

$$(A_{4\alpha}) = i(\tilde{\vec{A}}, i\bar{A}). \qquad (14.5)$$

The 4-vector $(A_{\alpha 4})$ consists of a space-time 3-vector

$$i\vec{A} = (A_{14}, A_{24}, A_{34}) \qquad (14.6)$$

and of a time-time scalar

$$- A = A_{44}. \qquad (14.7)$$

The 4-vector $(A_{4\alpha})$ consists of a time-space 3-vector

$$i\tilde{\vec{A}} = (A_{41}, A_{42}, A_{43}) \qquad (14.8)$$

and of the same time-time scalar (7).

Therefore the 4-tensor (2) can be written in the following form

$$\overleftrightarrow{A} = \begin{vmatrix} \overleftrightarrow{A} & i\vec{A} \\ i\tilde{\vec{A}} & -\bar{A} \end{vmatrix}. \qquad (14.9)$$

xxxxx

By analogy with the 3-tensors of third rank (see I, p.236) we can introduce a 4-TENSOR OF THIRD RANK $(A_{\alpha\beta\gamma})$. The transformation formulas for its components under a special Lorentz transformation will be

$$A_{\alpha\varepsilon\gamma} = \alpha_{\alpha\delta}\alpha_{\beta\varepsilon}\alpha_{\gamma\zeta}A'_{\delta\varepsilon\zeta}. \qquad (14.10)$$

If we consider the 4-dimensional scalars, 4-vectors, and 4-tensors from a common point of view, we can call the 4-scalars 4-TENSORS OF ZERO RANK, the 4-vectors - 4-TENSORS OF FIRST RANK, and the 4-tensors - 4-TENSORS OF SECOND RANK.

UNIT 4-TENSOR $\overleftrightarrow{\delta}$ is called the 4-tensor with components

$$A_{\alpha\beta} = \delta_{\alpha\beta}, \qquad (14.11)$$

where $\delta_{\alpha\beta}$ is the 4-Kronecker symbol (see I, p. 77). In any 4-frame of reference the unit 4-tensor has the following components

$$\overleftrightarrow{\delta} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}. \qquad (14.12)$$

ZERO 4-TENSOR $\overleftrightarrow{0}$ is called the 4-tensor with conmonents

$$A_{\alpha\beta} = 0. \qquad (14.13)$$

In the same way as for 3-tensors (see I, §67) we can introduce the transposed, symmetric, and antisymmetric 4-tensors.

The symmetric 4-tensor has 10 different components and the antisymmetric one only 6.

The antisymmetric 4-tensor $\overleftrightarrow{A}$ can be written in the following form

$$\overleftrightarrow{A} = \begin{vmatrix} 0 & a_z & -a_y & -ip_x \\ -a_z & 0 & a_x & -ip_y \\ a_y & -a_x & 0 & -ip_z \\ ip_x & ip_y & ip_z & 0 \end{vmatrix}, \tag{14.14}$$

and with respect to the transformation in the 3-space it consists of an antisymmetric 3-tensor which represents an axial vector

$$\vec{a} = (a_x, a_y, a_z) \tag{14.15}$$

and of a polar vector

$$\vec{p} = (p_x, p_y, p_z). \tag{14.16}$$

Remark. We have to note that the antisymmetric 4-tensor $\overleftrightarrow{A}$ given with table (14) can be presented in a form where all its components are multiplied by -1 (cf. formulas (I,67.27) and (I,67.32))

$$\overleftrightarrow{A} = \begin{vmatrix} 0 & -a_z & a_y & ip_x \\ a_z & 0 & -a_x & ip_y \\ -a_y & a_x & 0 & ip_z \\ -ip_x & -ip_y & -ip_z & 0 \end{vmatrix}. \tag{14.16'}$$

Following our restriction (see I, p. 239), we shall always write the antisymmetric 4-tensor in the form (14).

If we use (1) and (13.8), we can easily find the transformation formulas for an antisymmetric 4-tensor, which, as we know (see I, p. 238), remains antisymmetric in any frame of reference,

$$A_{11} = A'_{11} = 0, \qquad A_{22} = A'_{22} = 0, \qquad A_{33} = A'_{33} = 0, \qquad A_{44} = A'_{44} = 0,$$

$$A_{12} = \frac{A'_{12} + i\frac{V}{c}A'_{24}}{(1 - V^2/c^2)^{1/2}}, \qquad A_{13} = \frac{A'_{13} + i\frac{V}{c}A'_{34}}{(1 - V^2/c^2)^{1/2}}, \qquad A_{14} = A'_{14},$$

$$A_{23} = A'_{23} \qquad A_{24} = \frac{A'_{24} + i\frac{V}{c}A'_{21}}{(1 - V^2/c^2)^{1/2}}, \qquad A_{34} = \frac{A'_{34} + i\frac{V}{c}A'_{31}}{(1 - V^2/c^2)^{1/2}}. \tag{14.17}$$

xxxx

By analogy with the different products in which 3-tensors take a part (see I, §68), we can introduce the corresponding products in which 4-tensors take a part:

1. The product of a scalar p and a 4-tensor $\overleftrightarrow{A}$ is a 4-tensor

$$\overleftrightarrow{B} = p\,\overleftrightarrow{A} \tag{14.18}$$

with components

$$B_{\alpha\beta} = pA_{\alpha\beta}. \tag{14.19}$$

2. The dyadic product of two 4-vectors $\vec{a}$, $\vec{b}$ is a 4-tensor

$$\overleftrightarrow{A} = \vec{a}\,\vec{b} \qquad (14.20)$$

with components

$$A_{\alpha\beta} = a_\alpha b_\beta. \qquad (14.21)$$

3. The tensor product of two 4-vectors $\vec{a}$, $\vec{b}$ is an antisymmetric 4-tensor

$$\overleftrightarrow{A} = \vec{a}\times\vec{b} \qquad (14.22)$$

with components

$$A_{\alpha\beta} = a_\alpha b_\beta - a_\beta b_\alpha. \qquad (14.23)$$

4. The vector product of a 4-tensor $\overleftrightarrow{A}$ and a 4-vector $\vec{a}$ from the left is a 4-vector

$$\vec{b} = \vec{a}.\overleftrightarrow{A} \qquad (14.24)$$

with components

$$b_\alpha = a_\beta A_{\alpha\beta}, \qquad (14.25)$$

and from the right is a 4-vector

$$\vec{c} = \overleftrightarrow{A}.\vec{a} \qquad (14.26)$$

with components

$$c_\alpha = a_\beta A_{\beta\alpha}. \qquad (14.27)$$

5. The scalar product of two 4-tensors $\overleftrightarrow{A}$, $\overleftrightarrow{B}$ is a scalar

$$p = \overleftrightarrow{A} : \overleftrightarrow{B} \qquad (14.28)$$

whose value is

$$p = A_{\alpha\beta} B_{\beta\alpha}. \qquad (14.29)$$

6. The tensor product of two 4-tensors $\overleftrightarrow{A}$, $\overleftrightarrow{B}$ is a 4-tensor

$$\overleftrightarrow{C} = \overleftrightarrow{A} . \overleftrightarrow{B} \qquad (14.30)$$

with components

$$C_{\alpha\beta} = A_{\alpha\gamma} B_{\gamma\beta}. \qquad (14.31)$$

For the products of 4-tensors the same rules are valid as for the products of 3-tensors

§15. FUNDAMENTAL 4-VECTOR EQUATIONS

Everey material system is built of material points (particles). We call (see IIA, p. 11) those space points material, the energy of which is different from zero, and non-material those ones, the energy of which is equal to zero. The material points can move with respect to the totality of non-material points which build "the space" of the used reference frame.

All physical quantities can be divided in two great classes: particles and fields. If the physical quantity is attached to a certain material point, we call it a PARTICLE PHY-

SICAL QUANTITY, and if the physical quantity is attached to a non-material point, we call it a FIELD PHYSICAL QUANTITY. As a rule, the particle quantities are designated by small letters and the field quantities by capital letters.

We can compose (see V, §9 and §10) physical quantities which have particle and field parts and we call such a coupled quantity FULL PHYSICAL QUANTITY, designating it by the same letter as the corresponding particle quantity but putting above it the sign "~".

The 4-vectors have two essentially different parts: a space part and a time part. The FUNDAMENTAL 4-VECTOR EQUATIONS give the connections which exist between the space and time derivatives of these two parts. We write these equations on axiomatical grounds, i.e., we do not deduce them logically from our axioms. But we shall always show that any specific equation which can be written when applying the fundamental 4-vector equations to different particle and field 4-vectors can be deduced from our axioms.

We suppose that the 4-tensors which have space-space, space-time, time-space, and time-time parts must satisfy some similar fundamental 4-tensor equations.

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Now we shall postulate the two fundamental 4-vector equations which every particle and field 4-vector must satisfy:

1. $\overset{\mapsto}{a}$ is a particle 4-vector

$$\overset{\mapsto}{a} = (\vec{a}, a_\tau) = (\vec{a}, i\bar{a}). \tag{15.1}$$

The fundamental equation for a particle 4-vector runs

$$\frac{da_\tau}{d\vec{r}} = -\frac{d\vec{a}}{d\tau}, \tag{15.2}$$

or

$$\frac{d\bar{a}}{d\vec{r}} = \frac{1}{c}\frac{d\vec{a}}{dt}, \tag{15.3}$$

where $d\tau = icdt$ is the independent time differential, $d\vec{r}$ is the dependent space differential and we have

$$d\vec{r} = \vec{v}dt, \tag{15.4}$$

where $\vec{v}$ is the velocity of the particle to which the 4-vector $\overset{\mapsto}{a}$ is attached. Thus we can also write equation (3) in the form

$$\frac{d\bar{a}}{dt} = \frac{\vec{v}}{c}.\frac{d\vec{a}}{dt}, \tag{15.5}$$

which is the mathematically correct form of equation (3). We give the fundamental equation for the particle 4-vectors in the form (2) because the equation in this form presents more clearly the connection between the space and time derivatives of the time and space parts of the particle 4-vectors.

2. $\overset{\mapsto}{A}$ is a field 4-vector

$$\overset{\mapsto}{A} = (\vec{A}, A_\tau) = (\vec{A}, i\bar{A}). \tag{15.6}$$

The fundamental equation for a field 4-vector runs

$$\frac{\partial \vec{A}}{\partial \vec{r}} = - \frac{\partial A_\tau}{\partial \tau}, \tag{15.7}$$

or

$$\text{div}\vec{A} = - \frac{1}{c}\frac{\partial \bar{A}}{\partial t}, \tag{15.8}$$

where $\partial\tau = ic\partial t$ is the independent time differential and $\partial\vec{r}$ is the independent space differential.

Those are the fundamental 4-vector equations, with the help of which we can obtain the fundamental physical laws.

In §22 we shall demonstrate the validity of equation (5), applying it to the particle 4-vectors introduced in ch. II.

Demonstrations of the validity of equation (8) can be found in part V where field 4-vectors will be introduced.

§16. 4-INTERVAL

In §10 we introduced the notion 4-interval which represents the 4-dimensional distance between two world points.

The 4-interval represents a conglomerate of space and time intervals, and only in special cases, as we shall see further on, it can be reduced either to a space interval or to a time interval.

In a given 4-dimensional frame of reference the 4-interval may have real, imaginary, or zero value. In these three cases we call the 4-interval SPACELIKE, TIMELIKE, and NULL, respectively (see p. 48).

According to (11.3), the 4-intervals between two world points registered in two different 4-frames have the same magnitude, i.e., they have the same "4-dimensional length". Thus, the 4-interval is a Lorentz invariant quantity (see p. 48), and we conclude that the property of the 4-interval to be spacelike, timeleike, or null is a Lorentz invariant property.

If two events occur with the same particle (moving with a constant velocity with respect to an inertial frame of reference), the 4-interval between the corresponding world points must be imaginary perforce. Indeed, it is imaginary in the 4-frame where the particle is at rest.

If two events occur with two different material points (moving with constant velocities with respect to an inertial frame of reference), the 4-interval between the corresponding world points may be real, imaginary, or zero.

The real (spacelike) 4-interval cannot be reduced to a time interval in whatever 4-frame; in a certain 4-frame it can be reduced to a space interval.

The imaginary (timelike) 4-interval cannot be reduced to a space interval in whatever 4-frame; in a certain 4-frame it can be reduced to a time interval, which will be the proper time interval. Thus the proper time interval is a Lorentz invariant quantity.

The zero (null) 4-interval can be reduced neither to a space interval nor to a time interval; in a certain frame it can be reduced to a 4-interval between two identical world points, i.e., to a zero space and to a zero time interval.

§17. THE LIGHT CONE

If the 4-interval between two world points is timelike, it is possible for one and the same particle to be a "witness" of the corresponding two events; if the 4-interval is real, this is not possible. Therefore two events, between which the distance is imaginary, can be arranged on the "age-scale" of a given particle in chronological order. This is not possible for two events between which the 4-interval is real. Let us consider these statements in detail:

For that purpose let us take an event 0 as origin of the 4-frame and let us consider, for better visualization, only one space dimension and time, plotting them on two axes (see fig. 17.1).

If a particle rests at the origin of the corresponding space-frame, its world points (i.e., the consequent events which occur with it, say, the ticks of a clock put at the origin) will be presented in the 4-frame by the points of the vertical straight line Oct. If the particle moves with a uniform velocity v and crosses the origin at t = 0, the events close to one another, of which this particle is a witness, will be presented by the points of the straight line m'm (called WORLD LINE). This line passes through 0 and is inclined to the ct-axis under an angle ϕ whose tangent is equal to v/c. Indeed,

$$OT = ct, \qquad OM' = TM = vt, \tag{17.1}$$

and we obtain (see also (11.8))

$$\tan\phi = \frac{TM}{OT} = \frac{v}{c}. \tag{17.2}$$

Since the maximum possible velocity is c, there is a maximum angle $\phi_{max} = \pi/4$ which the world line m'm can make with the ct-axis. In fig. 17.1 the lines a'a and b'b are shown which represent the propagation of two signals with the velocity of light in opposite directions and passing through the world point 0. The world lines, representing the motion of particles which cross the origin at t = 0, can only lie in the regions aOb and a'Ob'; if the velocities of these particles are not constant, the corresponding world lines will be curved. On the lines aa' and bb'

$$x = \pm\, ct. \tag{17.3}$$

First consider events whose world points lie within the region aOb. It is obvious that

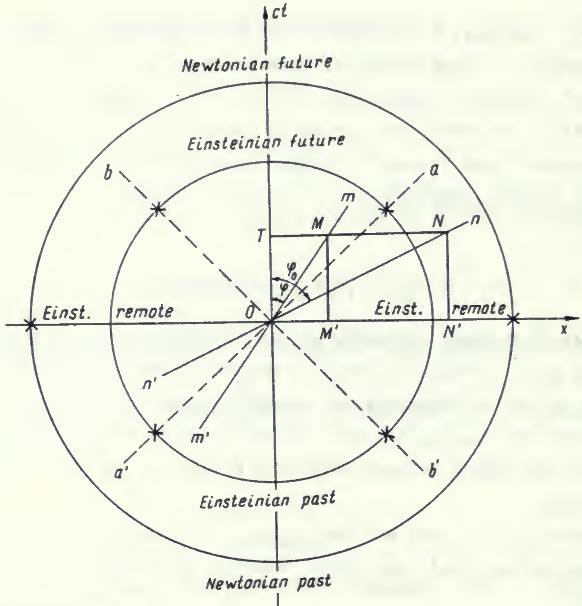

Fig. 17.1

for all the points in this region

$$x^2 - (ct)^2 < 0. \tag{17.4}$$

In other words, the 4-interval between any event in this region and the event O is timelike. In this region $t > 0$, i.e., all the events here occur after the event O. But if (4) is valid in one 4-frame, in any other frame K'

$$x'^2 - (ct')^2 < 0. \tag{17.5}$$

Thus all events in the region aOb are future events relative to O in all reference frames. We repeat that this must be understood in the sense that if some particle was the witness of the event O, it can be (in the future) a witness of any event in the region aOc. We call this region EINSTEINIAN FUTURE.

Exactly in the same way, all events in the region a'Ob' are in the EINSTEINIAN PAST in respect of the event O, i.e., events in this region occur before the event O in all frames of reference; we repeat, this is to be understood in the same sense as above.

Next consider regions a'Ob and aOb'. The 4-interval between any event in these regions and the event O is spacelike. Thus there is no particle which can be a witness of the event O and of whatever event in these regions. We call these two regions EINSTEINIAN REMOTE in respect of the event O.

Let us consider a particle which moves with a universal velocity (see (4.4))

$$\vec{v} = d\vec{r}/dt. \tag{17.6}$$

The space interval $d\vec{r}$ is covered in absolute space during the time dt, read on a clock which rests in absolute space. Dividing $d\vec{r}$ by the proper time interval (see (7.4))

$$dt_o = dt(1 - v^2/c^2)^{1/2}, \tag{17.7}$$

which is read on a clock attached to the particle in question, we obtain the proper velocity (see (4.22))

$$\vec{v}_o = \frac{d\vec{r}}{dt_o} = \frac{\vec{v}}{(1 - v^2/c^2)^{1/2}}. \tag{17.8}$$

If we assume that the said particle moves with the uniform proper velocity v_o along the x-axis and crosses the origin at t = 0, then the events close to one another occurring with this particle will be presented by the straight line n'n (called the PROPER WORLD LINE). This line passes through O and is inclined to the ct-axis under an angle ϕ_o whose tangent is equal to v_o/c. Indeed,

$$OT = ct, \qquad ON' = TN = v_o t, \tag{17.9}$$

and we obtain

$$\tan\phi_o = \frac{TN}{OT} = \frac{v_o}{c} = \frac{v}{c(1 - v^2/c^2)^{1/2}}. \tag{17.10}$$

Since for v = c, we have $v_o = \infty$, then $\phi_{omax} = \pi/2$, and the proper world lines of the light signals crossing the world point O coincide with the x-axis.

Obviously, if we operate with the proper quantities, then all events lying on the x-axis proceed at the same moment with the event O. This corresponds to the purely Newtonian space-time conceptions. Thus we call the region above the x-axis NEWTONIAN FUTURE, while the region beneath the x-axis is called NEWTONIAN PAST.

This analysis shows that if we work in high-velocity physics with the proper quantities, then we have to think in a Newtonian manner. We have to bear in mind this conclusion throughout the whole of high-velocity physics, and then many of its aspects which seemed strange and peculiar to the physicists during the dark "relativity age" will become clear and comprehensible.

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The diagram shown in fig. 17.1 is called a SPACE-TIME DIAGRAM.

If we consider all three space dimensions, then instead of the intersecting lines a'a and b'b in fig. 17.1, given by equation (3), we should have a "cone" (obviously the equation of a circular cone in the 3-space is $x^2 + y^2 - z^2 = 0$)

$$x^2 + y^2 + z^2 - c^2t^2 = 0 \tag{17.11}$$

in the 4-space. The axis of the 4-dimensional cone will coincide with the ct-axis and its apex will lie at the origin of the 4-frame. This cone is called the LIGHT CONE.

The light cone and the corresponding space-time diagrams are very useful when working in high-velocity physics. However, the light cone is treated by the theory of special relativity not only as a convenient mathematical tool but as a real "physical" result of the "space-time relativity", and many scholastical constructions and conclusions have been erected over the light cone. To save the reader's time, we will not criticize them.

§18. 4-RADIUS VECTOR

The most important 4-vector is the 4-radius vector $\overleftrightarrow{r}$ (see (10.2)). It consists of the radius vector $\vec{r}$ of a space point P and of the time coordinate t of an event which occurs at the point P, multiplied by ic.

The radius vector $\vec{r}$ (the space part of $\overleftrightarrow{r}$) is registered in a space frame K which is attached to absolute space. The moment t (the time part of $\overleftrightarrow{r}$ divided by ic) is registered on a clock which is at rest in absolute space. If we want to work in another inertial frame of reference K' which moves with velocity V along the positive direction of the x-axis of frame K, and at the initial zero moment both frames coincide, then the same event will be registered with a new radius vector $\overleftrightarrow{r}'$ with respect to the new space-frame K' and with a new moment t' registered on a clock which is at rest in K'. We must emphasize once more that if the transformation between the 4-radius vectors is Lorentzian (i.e., in the form (3.21)), then the synchronization between the K-and K'-clock is Einsteinian. The transformations in the 4-space are always Lorentzian.

§19. 4-VELOCITY

Let us consider the difference $d\overleftrightarrow{r}$ between two 4-radius vectors of two close events which occur with the same particle.

If we divide $d\overleftrightarrow{r}$ by the corresponding proper time interval dt_o (i.e., the time interval registered on a clock which is at rest in this frame of reference in which both events occur at the <u>same</u> space point), then, since dt_o is a Lorentz invariant quantity (see p. 55), we obtain a new 4-vector

$$\overleftrightarrow{v}_o = \frac{d\overleftrightarrow{r}}{dt_o} = \left(\frac{\vec{v}}{(1 - v^2/c^2)^{1/2}}, \frac{ic}{(1 - v^2/c^2)^{1/2}}\right), \tag{19.1}$$

which we call the 4-VELOCITY.

Obviously, the first three (space) components of the 4-velocity are the components of the proper velocity (see (17.8)) and the fourth (time) component is

$$v_{o4} = v_{o\tau} = i\bar{v}_o = ic_o = \frac{ic}{(1 - v^2/c^2)^{1/2}}, \tag{19.2}$$

the real part of which is the proper light velocity (see (4.29))

$$v_{ot} = \bar{v}_o = c_o = \frac{c}{(1 - v^2/c^2)^{1/2}}. \tag{19.3}$$

The square of the 4-velocity is a constant

$$\overleftrightarrow{v}_o^2 = - c^2. \tag{19.4}$$

The 4-velocity is a 4-vector since the nominator in (1) is a 4-vector and the denominator is a 4-dimensional scalar.

We can verify that the 4-velocity is a 4-vector if we show that its components must be transformed under a special Lorentz transformation according to formulas (13.5), namely,

$$v_{o1} = \frac{v'_{o1} - i\frac{V}{c}v'_{o4}}{(1 - V^2/c^2)^{1/2}}, \quad v_{o2} = v'_{o2}, \quad v_{o3} = v'_{o3}, \quad v_{o4} = \frac{v'_{o4} + i\frac{V}{c}v'_{o1}}{(1 - V^2/c^2)^{1/2}}. \tag{19.5}$$

Using here the identity (4.15) and having in mind that in the case of a special transformation $\vec{v}'.\vec{V} = v'_x V$, we obtain formulas (4.20) from the first three formulas (5) and the identity

$$c = c \tag{19.6}$$

from the last formula (5).

Remark. In the 4-space we work only with the Lorentz relative velocity $\vec{v}'_L$ (see (4.6)) and we do not use the relative velocity $\vec{v}'$ (see (4.5)). Thus when working in the 4-space we shall omit the index "L" of the Lorentz relative velocity, but we have to never forget this.

We can introduce a 4-vector

$$\overleftrightarrow{v} = (\vec{v}, i\bar{v}) = (\vec{v}, ic) \tag{19.7}$$

which has only the form of a 4-vector and which we shall call the UNIVERSAL 4-VELOCITY. The direct and inverse transformation formulas for its components are given by formulas (4.21), (4.22) and (6). Now the 4-velocity (1) can be called the PROPER 4-VELOCITY.

§20. 4-ACCELERATION

We introduce the following three different accelerations of a particle:

The UNIVERSAL ACCELERATION

$$\vec{u} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\frac{d\vec{r}}{dt}. \tag{20.1}$$

The FIRST PROPER ACCELERATION

$$\vec{u}_o = \frac{d\vec{v}_o}{dt_o} = \frac{d}{dt}\frac{d\vec{r}}{dt_o}. \tag{20.2}$$

The SECOND PROPER ACCELERATION

$$\vec{u}_{oo} = \frac{d\vec{v}_o}{dt_o} = \frac{d}{dt_o}\frac{d\vec{r}}{dt_o}. \tag{20.3}$$

Let us consider the difference $d\overleftrightarrow{v}_o$ between two 4-velocities of a particle with which two close events occur.

If we divide $d\overleftrightarrow{v}_o$ by the corresponding proper time interval dt_o, we obtain a new 4-vector

$$\overleftrightarrow{u}_{oo} = d\overleftrightarrow{v}/dt_o \tag{20.4}$$

which we call the 4-ACCELERATION.

The space and time parts of the 4-acceleration can be written in the form

$$\vec{u}_{oo} = \frac{\vec{u}}{1 - v^2/c^2} + \frac{\vec{v}}{c^2}\frac{\vec{v}.\vec{u}}{(1 - v^2/c^2)^2}, \tag{20.5}$$

$$\bar{u}_{oo} = \frac{\vec{v}.\vec{u}}{c(1 - v^2/c^2)^2} \tag{20.6}$$

The 4-acceleration can also be written in the following 4-dimensional form

$$\overleftrightarrow{u}_{oo} = \frac{d^2\overleftrightarrow{r}}{dt_o^2} + \frac{\overleftrightarrow{v}_o}{c^2}(\overleftrightarrow{v}_o.\frac{d^2\overleftrightarrow{r}}{dt_o^2}). \tag{20.7}$$

We can introduce the 4-vector

$$\overleftrightarrow{u}_o = d\overleftrightarrow{v}_o/dt \tag{20.8}$$

with space and time parts

$$\vec{u}_o = \frac{\vec{u}}{(1 - v^2/c^2)^{1/2}} + \frac{\vec{v}}{c^2}\frac{\vec{v}.\vec{u}}{(1 - v^2/c^2)^{3/2}}, \tag{20.9}$$

$$\bar{u}_o = \frac{\vec{v}.\vec{u}}{c(1 - v^2/c^2)^{3/2}}, \tag{20.10}$$

which has only the form of a 4-vector and which we shall call the FIRST PROPER 4-ACCELERATION. The quantity $\overleftrightarrow{u}_{oo}$ can be called the SECOND PROPER 4-ACCELERATION and only this quantity is a legitimate 4-vector.

We introduce also the UNIVERSAL 4-ACCELERATION $\overleftrightarrow{u}$; its fourth (time) components is identically equal to zero, i.e.,

$$\bar{u} = d\bar{v}/dt = dc/dt = 0. \tag{20.11}$$

Taking into account (11), we can write formulas (5) and (6) in the following form

$$\vec{u}_{oo} = \frac{\vec{u}}{1 - v^2/c^2} + \frac{\vec{v}}{c^2}\frac{\overleftrightarrow{v}.\overleftrightarrow{u}}{(1 - v^2/c^2)^2}, \tag{20.12}$$

$$\bar{u}_{oo} = \frac{\bar{u}}{1 - v^2/c^2} + \frac{\bar{v}}{c^2}\frac{\overleftrightarrow{v}.\overleftrightarrow{u}}{(1 - v^2/c^2)^2}. \tag{20.13}$$

After the differentiation of (19.4) with respect to time, we obtain

$$\overleftrightarrow{v}_o.\overleftrightarrow{u}_o = 0. \tag{20.14}$$

This equality can be proved as correct by substituting here (17.8), (19.3), (9), and (10).

§21. 4-SUPER-ACCELERATION

We introduce the following four different super-accelerations of a particle:

The UNIVERSAL SUPER-ACCELERATION

$$\vec{w} = \frac{d\vec{u}}{dt} = \frac{d^2\vec{v}}{dt^2} = \frac{d^3\vec{r}}{dt^3}. \tag{21.1}$$

The FIRST PROPER SUPER-ACCELERATION

$$\vec{w}_o = \frac{d\vec{u}_o}{dt} = \frac{d^2\vec{v}_o}{dt^2}. \tag{21.2}$$

The SECOND PROPER SUPER-ACCELERATION

$$\vec{w}_{oo} = \frac{d\vec{u}_{oo}}{dt} = \frac{d^2\vec{v}_o}{dtdt_o}. \tag{21.3}$$

The THIRD PROPER SUPER-ACCELERATION

$$\vec{w}_{ooo} = \frac{d\vec{u}_{oo}}{dt_o} = \frac{d}{dt_o}\frac{d\vec{v}_o}{dt_o}. \tag{21.4}$$

Let us consider the difference $d\overleftrightarrow{u}_{oo}$ between two 4-accelerations of a particle with which two close events occur.

If we divide $d\overleftrightarrow{u}_{oo}$ by the corresponding proper time interval dt_o, we obtain a new 4-vector

$$\overleftrightarrow{w}_{ooo} = \frac{d\overleftrightarrow{u}_{oo}}{dt_o} \tag{21.5}$$

which we call 4-SUPER-ACCELERATION; obviously $\overleftrightarrow{w}_{ooo}$ must be called the THIRD PROPER 4-SUPER-ACCELERATION.

Substituting (20.7) into (5) and performing the differentiation, we obtain an expression that can be written in the form

$$\overleftrightarrow{w}_{ooo} = \frac{d^2\overleftrightarrow{v}_o}{dt_o^2} + \frac{1}{c^2}\left(\overleftrightarrow{v}_o\cdot\frac{d^2\overleftrightarrow{v}_o}{dt_o^2}\right)\overleftrightarrow{v}_o + \frac{1}{c^2}\left(\overleftrightarrow{v}_o\cdot\frac{d^2\overleftrightarrow{r}}{dt_o^2}\right)\frac{d^2\overleftrightarrow{r}}{dt_o^2} + \frac{2}{c^4}\left(\overleftrightarrow{v}_o\cdot\frac{d^2\overleftrightarrow{r}}{dt_o^2}\right)^2\overleftrightarrow{v}_o + \frac{1}{c^2}\left(\frac{d^2\overleftrightarrow{r}}{dt_o^2}\right)^2\overleftrightarrow{v}_o. \tag{21.6}$$

For $v \ll c$ the first term on the right side of (6) is predominant and for $v \to c$ the second term is the predominant one. Indeed, after carrying out the differentiation, we establish that the second term has the factor $(1 - v^2/c^2)^{9/2}$ in its denominator, while in the denominators of all other terms the factor $(1 - v^2/c^2)$ has a smaller exponent. Therefore we can write the 4-super-acceleration preserving only the first two terms in (6) and neglecting the last three, namely, in the form

$$\overleftrightarrow{w}_{ooo} = \frac{d^2\overleftrightarrow{v}_o}{dt_o^2} + \frac{\overleftrightarrow{v}_o}{c^2}\left(\overleftrightarrow{v}_o\cdot\frac{d^2\overleftrightarrow{v}_o}{dt_o^2}\right). \tag{21.7}$$

Taking into account (2), we can write this in the form

$$\overleftrightarrow{w}_{ooo} = \frac{1}{1 - v^2/c^2}\{\overleftrightarrow{w}_o + \frac{\overleftrightarrow{v}_o}{c}(\frac{\overleftrightarrow{v}_o}{c}.\overleftrightarrow{w}_o)\}, \qquad (21.8)$$

where $\overleftrightarrow{w}_o$ is the FIRST PROPER 4-SUPER-ACCELERATION. Thus we can write the space and time parts of the 4-super-acceleration in the form

$$\vec{w}_{ooo} = \frac{\vec{w}_o}{1 - v^2/c^2} + \frac{\vec{v}}{c^2}\frac{\overleftrightarrow{v}.\overleftrightarrow{w}_o}{(1 - v^2/c^2)^2}, \qquad (21.9)$$

$$\bar{w}_{ooo} = \frac{\bar{w}_o}{1 - v^2/c^2} + \frac{\bar{v}}{c^2}\frac{\overleftrightarrow{v}.\overleftrightarrow{w}_o}{(1 - v^2/c^2)^2}. \qquad (21.10)$$

These formulas give the expressions for the components of the third proper 4-super-acceleration by the components of the first proper 4-super-acceleration. They have the same character as formulas (20.12) and (20.13).

We can consider the SECOND PROPER 4-SUPER-ACCELERATION whose connection with $\overleftrightarrow{w}_{ooo}$ is

$$\overleftrightarrow{w}_{oo} = \overleftrightarrow{w}_{ooo}(1 - v^2/c^2)^{1/2}. \qquad (21.11)$$

Finally, we can introduce the UNIVERSAL 4-SUPER-ACCELERATION $\overleftrightarrow{w}$; its fourth (time) component is identically equal to zero (see (1) and (20.11))

$$\bar{w} = d\bar{u}/dt = d^2c/dt^2 = 0. \qquad (21.12)$$

§22. APPLICATION OF THE FUNDAMENTAL 4-VECTOR EQUATION FOR PARTICLE 4-VECTORS

Now we shall apply the fundamental 4-vector equation (15.5) to the particle 4-vectors introduced in this chapter.

1. 4-radius vector. The 4-radius vector can be either a particle or a field 4-vector. Thus none of the fundamental 4-vector equations can be applied to it.

2. 4-velocity. The space and time parts of the 4-velocity are given by formulas (17.8) and (19.3).

Equation (15.5) applied to the 4-velocity runs as follows

$$\frac{d\bar{v}_o}{dt} = \frac{\vec{v}}{c}.\frac{d\vec{v}_o}{dt}. \qquad (22.1)$$

The validity of this equation can immediately be proved right if one substituted here (17.8) and (19.3).

3. 4-acceleration. The space and time parts of the 4-acceleration are given by formulas (20.5) and (20.6).

Equation (15.5) applied to the 4-acceleration runs as follows

$$\frac{d\bar{u}_{oo}}{dt} = \frac{\vec{v}}{c}.\frac{d\vec{u}_{oo}}{dt}. \qquad (22.2)$$

The validity of this equation can be proved right if we substitute here (20.5) and (20.6), but at this differentiation we have to consider velocity $\vec{v}$ as a constant. We must consider here $\vec{v}$ as a constant because (2) gives the connection between the space and time derivatives of the 4-acceleration, and the velocity figures in the 4-acceleration only as a result of the specific frame in which we consider the 4-acceleration. At the differentiation in equation (1) we did not consider the velocity as a constant because this equation gives the connection between the space and time derivatives of the velocity itself.

4. 4-super-acceleration. The space and time parts of the 4-super-acceleration are given by formulas (21.9) and (21.10).

Equation (15.5) applied to the 4-super-acceleration runs as follows

$$\frac{d\bar{w}_{ooo}}{dt} = \frac{\vec{v}}{c} \cdot \frac{d\vec{w}_{ooo}}{dt}. \qquad (22.3)$$

The validity of this equation can be proved right if we substitute here (21.9) and (21.10), considering at the differentiation $\vec{v}$ as a constant (see above).

Chapter III

EQUATIONS OF MOTION

§23. ESTABLISHMENT OF THE FORM OF TIME ENERGY

The establishment of the form of time energy at low velocities of the particles is given in IIB, §6. According to the assertion of our sixth axiom, the differential of the time energy of a particle is proportional to the scalar product of its velocity and the differential of the velocity, the mass of the particle being the coupling constant. In low-velocity mechanics this assertion was written in the form (see (IIA.3.8))

$$de_0 = m\vec{v}.d\vec{v}. \tag{23.1}$$

Now we shall establish the form of time energy in high-velocity mechanics and we shall proceed from the axiomatical assertion (1), where we have to replace the universal velocity $\vec{v}$ by the proper velocity $\vec{v}_0$, because such is the postulate of our third axiom.

There are three possibilities, and we shall respectively obtain three forms of time energy in high-velocity mechanics

$$de^0 = m\vec{v}_0.d\vec{v}, \tag{23.2}$$

$$de_0 = m\vec{v}.d\vec{v}_0, \tag{23.3}$$

$$de_{00} = m\vec{v}_0.d\vec{v}_0. \tag{23.4}$$

After the integration of these three equations, we obtain

$$e^0 = - mc^2(1 - v^2/c^2)^{1/2} + C^0, \tag{23.5}$$

$$e_0 = \frac{mc^2}{(1 - v^2/c^2)^{1/2}} + C_0, \tag{23.6}$$

$$e_{00} = \frac{1}{2}\frac{mc^2}{1 - v^2/c^2} + C_{00}, \tag{23.7}$$

where C^0, C_0, and C_{00} are the corresponding constants of integration.

If we assume $C^0 = C_0 = C_{00} = 0$, we can write these formulas in the form

$$e^0 = - mc^2(1 - v^2/c^2)^{1/2}, \tag{23.8}$$

$$e_0 = \frac{mc^2}{(1 - v^2/c^2)^{1/2}}, \tag{23.9}$$

$$e_{00} = \frac{1}{2}\frac{mc^2}{1 - v^2/c^2}. \tag{23.10}$$

These are the forms of time energy in high-velocity mechanics. We call them as follows

e^0 - LAGRANGE TIME ENERGY,

e_0 - HAMILTON TIME ENERGY,

e_{00} - MARINOV TIME ENERGY.

All three forms of time energy are used in our theory; however, the Hamilton energy is the most convenient, as the proper time momentum (see (IIA,2.5) and (19.3)) is proportional to it

$$\bar{p}_o = \frac{e_o}{c} = m_o c = mc_o = \frac{mc}{(1 - v^2/c^2)^{1/2}}. \qquad (23.11)$$

Further on, when talking about particle's energy, we shall always mean the Hamilton energy e_o. But, as the reader will see, very often the use of the Lagrange energy will permit to give a more compact form to many formulas, and the Marinov energy can help to understand certain subtleties of the theory.

When $v \ll c$, we obtain for these energies, within an accuracy of second order in v/c, the following expressions

$$e_f^o = - mc^2 + mv^2/2, \qquad (23.12)$$

$$e_{of} = e_f = mc^2 + mv^2/2, \qquad (23.13)$$

$$e_{oof} = mc^2/2 + mv^2/2, \qquad (23.14)$$

which we call, respectively, LAGRANGE, HAMILTON, and MARINOV FACTUAL ENERGIES. As the formulas show, the REST LAGRANGE, HAMILTON, and MARINOV ENERGIES (those ones for $v = 0$) are different quantities. When talking about rest energy we shall always mean the Hamilton rest energy, i.e., the universal energy, $e = mc^2$.

The kinetic energy of a particle is the difference between its time and rest energies (cf. IIB, p. 54). In high-velocity mechanics there are three different kinetic energies, which we call, respectively, LAGRANGE, HAMILTON, and MARINOV KINETIC ENERGIES. Their expressions as infinite series are (see (I,44.2), (I,44.3), and (I,44.1))

$$e_k^o = - mc^2\{(1 - v^2/c^2)^{1/2} - 1\} = mc^2 \sum_{n=1}^{\infty} \frac{(2n-3)!!}{(2n)!!} \frac{v^{2n}}{c^{2n}}, \qquad (23.15)$$

$$e_{ok} = mc^2\{\frac{1}{(1 - v^2/c^2)^{1/2}} - 1\} = mc^2 \sum_{n=1}^{\infty} \frac{(2n-1)!!}{(2n)!!} \frac{v^{2n}}{c^{2n}}, \qquad (23.16)$$

$$e_{ook} = \frac{mc^2}{2}(\frac{1}{1 - v^2/c^2} - 1) = \frac{mc^2}{2} \sum_{n=1}^{\infty} \frac{v^{2n}}{c^{2n}}. \qquad (23.17)$$

The first term in all three different kinetic energies is the same

$$e_k = mv^2/2, \qquad (23.18)$$

and we call this LOW-VELOCITY KINETIC ENERGY. The Hamilton kinetic energy e_{ok} given by formula (16) can be called HIGH-VELOCITY KINETIC ENERGY.

The energy of a system of n particles, according to our sixth axiom, is the sum of the energies of every single particle, i.e.,

$$E_o = \sum_{i=1}^{n} e_{oi} = \sum_{i=1}^{n} \frac{m_i c^2}{(1 - v_i^2/c^2)^{1/2}}. \qquad (23.19)$$

xxxxx

From equality (11) we obtain the relation between proper and universal masses

$$m_o = \frac{m}{(1 - v^2/c^2)^{1/2}}. \qquad (23.20)$$

Remark. We must emphasize that the notion "proper mass" is introduced for mathematical convenience only, as is the case with the notion "mass" itself. In IIA, p. 12 we noted that it is better to construct physics without introducing the notion "mass" at all, and that we introduce this notion in our theory only for historical reasons. On the other hand, the factor $(1 - v^2/c^2)^{-1/2}$ in formula (11) is attached to the velocity of light c and not to the mass m. Thus the notion "proper mass" is an artificial one. We must never forget this when operating with proper mass, and keep in mind that the mass of the particle m is a quantity independent of velocity which can always be substituted by the quantity e/c^2.

§24. THE FUNDAMENTAL EQUATIONS OF MOTION

As the high-velocity forms of space and space-time energies in gravimagretism and electromagnetism are different, the Lagrange equations in those two physical domains will be slightly different. We shall deduce the more complicated equations in gravimagretism, from which the equations in electromagnetism can be obtained immediately.

A. The fundamental equations in gravimagretism

In high-velocity gravimagretism the space energy U depends also on the velocities of the particles and equation (IIB,7.1) is to be replaced by the following one (see formulas (IIA,3.2) and (23.20))

$$dU = \sum_{i=1}^{n} \left(\frac{\partial U}{\partial \vec{r}_i}.d\vec{r}_i + \frac{\partial U}{\partial \vec{v}_i}.d\vec{v}_i\right) = \sum_{i=1}^{n} \left\{\frac{\partial U}{\partial \vec{r}_i} + \frac{U_i \vec{v}_i.d\vec{v}_i}{c^2(1 - v^2/c^2)^{3/2}}\right\} = \sum_{i=1}^{n} \left(\frac{\partial U}{\partial \vec{r}_i}.d\vec{r}_i + \frac{U_i}{c^2}\vec{v}_i.d\vec{v}_{oi}\right), \qquad (24.1)$$

where U_i is the part of the space energy in which the i-th particle takes part. Note that in (1) the energy U_i must be written without the factor $(1 - v_i^2/c^2)^{-1/2}$; the reason for this becomes clear in §24C.

In high-velocity physics equation (IIB,7.2) is to be replaced by the following one, as now the time energy is given by formula (23.19),

$$dE_o = \sum_{i=1}^{n} \frac{\partial E_o}{\partial \vec{v}_i}.d\vec{v}_i = \sum_{i=1}^{n} \frac{\partial e_{oi}}{\partial \vec{v}_i}.d\vec{v}_i = \sum_{i=1}^{n} \frac{d}{dt}\left(\frac{\partial e_i^o}{\partial \vec{v}_i}\right).d\vec{r}_i = \sum_{i=1}^{n} m\vec{u}_{oi}.d\vec{r}_i, \qquad (24.2)$$

where e_{0i} and e_i^0 are the Hamilton and Lagrange time energies of the i-th particle. Indeed, we have, on the one hand,

$$\frac{\partial e_0}{\partial \vec{v}}.d\vec{v} = \frac{m\vec{v}.\vec{u}\,dt}{(1 - v^2/c^2)^{3/2}}, \qquad (24.3)$$

and, on the other hand,

$$\frac{d}{dt}\left(\frac{\partial e^0}{\partial \vec{v}}\right).d\vec{r} = m\left\{\frac{\vec{u}}{(1 - v^2/c^2)^{1/2}} + \frac{\vec{v}(\vec{v}.\vec{u})}{c^2(1 - v^2/c^2)^{3/2}}\right\}.\vec{v}\,dt = \frac{m\vec{v}.\vec{u}\,dt}{(1 - v^2/c^2)^{3/2}}. \qquad (24.4)$$

In high-velocity gravimagretism we have to take into account also the space-time energy W. As the space-time energy depends on the distances between the material points and on their velocities, we shall have (see formulas (IIA,3.9) and (23.20))

$$dW = \sum_{i=1}^{n}\left(\frac{\partial W}{\partial \vec{r}_i}.d\vec{r}_i + \frac{\partial W}{\partial \vec{v}_i}.d\vec{v}_i\right) = \sum_{i=1}^{n}\left(\frac{\partial W}{\partial \vec{r}_i}.d\vec{r}_i + \frac{\partial_1 W}{\partial_1 \vec{v}_i}.d\vec{v}_i + \frac{\partial_2 W}{\partial_2 \vec{v}_i}.d\vec{v}_i\right) =$$

$$\sum_{i=1}^{n}\left\{\frac{\partial W}{\partial \vec{r}_i}.d\vec{r}_i + d\left(\frac{\partial_1 W}{\partial_1 \vec{v}_i}.\vec{v}_i\right) - d\left(\frac{\partial_1 W}{\partial_1 \vec{v}_i}\right).\vec{v}_i + \frac{W_i}{c^2}\vec{v}_i.d\vec{v}_{0i}\right\}, \qquad (24.5)$$

where $\partial_1 W/\partial_1 \vec{v}_i$ signifies that the differentiation is to be carried out on the velocities in the nominators of W, while $\partial_2 W/\partial_2 \vec{v}_i$ signifies that the differentiation is to be carried out on the velocities in the denominators of W. W_i is the part of the space-time energy in which the i-th particle takes part. In (5) the energy W_i must be written without the factor $(1 - v_i^2/c^2)^{-1/2}$; the reason for this becomes clear in §24C.

We have

$$\sum_{i=1}^{n} d\left(\frac{\partial_1 W}{\partial_1 \vec{v}_i}.\vec{v}_i\right) = \sum_{i=1}^{n} dW_i = d\sum_{i=1}^{n} W_i = 2\,dW. \qquad (24.6)$$

This transition is <u>extremely important</u>. We beg the reader to become <u>persuaded</u> in the correctness of this formula by taking into account formula (IV,2.4).

Using (6) in (5), we obtain

$$dW = \sum_{i=1}^{n}\left\{- \frac{\partial W}{\partial \vec{r}_i}.d\vec{r}_i + d\left(\frac{\partial_1 W}{\partial_1 \vec{v}_i}\right).\vec{v}_i - \frac{W_i}{c^2}\vec{v}_i.d\vec{v}_{0i}\right\}. \qquad (24.7)$$

Substituting equations (1), (2), and (7) into the energy conservation law (IIA,4.1) and dividing by dt, we obtain, by the same reasoning as in IIB, p. 55 and p. 56, the FUNDAMENTAL EQUATIONS OF MOTION IN GRAVIMAGRETISM (cf. (IIB,7.7))

$$\frac{d}{dt}\left\{\frac{\partial(E^0 + W)}{\partial \vec{v}_i}\right\} + \frac{U_i - W_i}{c^2}\vec{u}_{0i} = - \frac{\partial(U - W)}{\partial \vec{r}_i}, \qquad i = 1,2,\ldots..n, \qquad (24.8)$$

which we also call the FULL LAGRANGE EQUATIONS IN GRAVIMAGRETISM. Here we have written the normal partial derivative $\partial W/\partial \vec{v}_i$ instead of the derivative $\partial_1 W/\partial_1 \vec{v}_i$ acting only on the velocities in the nominators of W, but the reader must <u>never</u> forget this.

The quantity (cf. (IIB,7.14))

$$\tilde{\vec{F}}_i = - \frac{\partial(U - W)}{\partial \vec{r}_i} \tag{24.9}$$

is called the FULL POTENTIAL FORCE.

The quantity (cf. (IIB,8.3))

$$\tilde{\vec{f}}_{oi} = (m + \frac{U_i - W_i}{c^2})\vec{u}_{oi} + \frac{d}{dt}\frac{\partial W}{\partial \vec{v}_i} = \tilde{m}\vec{u}_{oi} + \frac{d}{dt}\frac{\partial W}{\partial \vec{v}_i} = \vec{f}_{oi} + \frac{d}{dt}\frac{\partial W}{\partial \vec{v}_i} \tag{24.10}$$

is called the PROPER FULL KINETIC FORCE.

The quantity

$$\tilde{m} = m + (U_i - W_i)/c^2 \tag{24.11}$$

is called the FULL MASS, and the ordinary mass m can be called the ISOLATED MASS.

The FULL NEWTON EQUATIONS (i.e., the FULL NEWTON'S SECOND LAW) are (cf. (IIB,7.15))

$$\tilde{\vec{f}}_{oi} = \tilde{\vec{F}}_i, \qquad i = 1,2,.....n. \tag{24.12}$$

The FULL NEWTON'S THIRD LAW is (cf. (IIB,8.1))

$$\frac{\partial(U_{ij} - W_{ij})}{\partial \vec{r}_i} = - \frac{\partial(U_{ij} - W_{ij})}{\partial \vec{r}_j}, \qquad \text{i.e.,} \qquad \tilde{\vec{F}}_i = - \tilde{\vec{F}}_j. \tag{24.13}$$

B. The fundamental equations in electromagnetism

In electromagnetism space energy is not velocity dependent and space-time energy has no "velocity dependent denominators". Thus, it is easy to see that the FULL LAGRANGE EQUATIONS IN ELECTROMAGNETISM will have the form

$$\frac{d}{dt}\left\{\frac{\partial(E^o + W)}{\partial \vec{v}_i}\right\} = - \frac{\partial(U - W)}{\partial \vec{r}_i}, \qquad i = 1,2,.....n. \tag{24.14}$$

Correspondingly the PROPER FULL KINETIC FORCE will have the form

$$\tilde{\vec{f}}_{oi} = m\vec{u}_{oi} + \frac{d}{dt}\frac{\partial W}{\partial \vec{v}_i} = \vec{f}_{oi} + \frac{d}{dt}\frac{\partial W}{\partial \vec{v}_i}, \tag{24.15}$$

and the notion full mass cannot be introduced, as the electric and magnetic energies have <u>no</u> influence on the mass of the particle. Thus in gravimagretism the kinetic force $\vec{f}_o$ is to be written with the <u>full mass</u> $\tilde{m}$ (see formula (10)), while in electromagnetism the kinetic force $\vec{f}_o$ is to be written with the <u>isolated mass</u> m.

C. The Newton-Marinov equation

Now we shall give another form of the full Lagrange (Newton) equations in gravimagretism.

Let us have a system of n masses m_i moving with velocities $\vec{v}_i$, whose distances from a given reference point are r_i. The quantities

$$\Phi = - \gamma \sum_{i=1}^{n} \frac{m_{oi}}{r_i}, \qquad \vec{A} = - \gamma \sum_{i=1}^{n} \frac{m_{oi}\vec{v}_i}{cr_i} \tag{24.16}$$

are called GRAVITATIONAL POTENTIAL and MAGRETIC POTENTIAL at the reference point (see also IV, §2).

If a particle with mass m, called a TEST MASS, crosses the reference point with a velocity $\vec{v}$, then the gravitational and magretic energies of the whole system of n + 1 masses in which mass m takes part will be

$$U = m\Phi \quad (U = m_o\Phi), \qquad W = m\frac{\vec{v}}{c}.\vec{A} \quad (W = m_o \frac{\vec{v}}{c}.\vec{A}). \tag{24.17}$$

In equation (8) we can write U_i, W_i instead of U, W, and e_i^o instead of E^o, and then suppress the index "i". Substituting now into (8) the gravitational and magretic energies from (17) taken in their first forms, we obtain

$$\frac{m}{c^2}(c^2 + \Phi - \frac{\vec{v}}{c}.\vec{A})\dot{\vec{u}}_o + \frac{m}{c}\frac{d\vec{A}}{dt} = - m\,\mathrm{grad}(\Phi - \frac{\vec{v}}{c}.\vec{A}). \tag{24.18}$$

Whether we have to take U and W in their first or second forms (17) depends on the form of the time energy term in (8). U and W are to be taken in their second forms (17) if the time energy term in (8) is taken in its Marinov form (see (23.10))

$$dE_{oo} = \sum_{i=1}^{n} \frac{\partial e_{ooi}}{\partial \vec{v}_i}.d\vec{v}_i = \sum_{i=1}^{n} \frac{1}{(1 - v^2/c^2)^{1/2}} \frac{d}{dt}(\frac{\partial e_i^o}{\partial \vec{v}_i}).d\vec{r}_i = \sum_{i=1}^{n} m\dot{\vec{u}}_{ooi}.d\vec{r}_i. \tag{24.19}$$

As in (2) we have taken the time energy in its Hamilton form, we had to take U_i in (1) and W_i in (5) without the factor $(1 - v^2/c^2)^{-1/2}$.

Equation (18) represents the full Newton (Lagrange) equation in gravimagretism written with the help of the potentials, and we call it the NEWTON-MARINOV EQUATION.

When deducing the Newton-Marinov equation we suppose that our material system is isolated. But it is impossible to construct a gravitationally isolated system, as one cannot suppress the gravitational action of all celestial bodies. Looking at formula (18), logic demands to suppose that the term c^2 in the nominator on its left side represents the gravitational potential generated by all celestial bodies at the reference point taken with a negative sign (let us assume $\vec{v}$ small, so that we can neglect the magretic potential of the celestial bodies), i.e.,

$$c^2 = - \Phi_w = \gamma \sum_{i=1}^{n} \frac{m_i}{r_i}, \tag{24.20}$$

where n is the number of the particles in the world, or the number of the celestial bodies (in this case m_i is the mass of the i-th celestial body). From this point of view the mystery of time energy disappears, as time energy represents nothing else but the negative gravitational energy of the particle with the mass of the whole world, i.e.,

$$m_o c^2 = - m_o\Phi_w. \tag{24.21}$$

So we reduce the energy forms to two kinds - space energy and space-time energy, and it becomes clear that the "volume" and the "materiality" of the particles can never be established, as such "particles", i.e., "drops of energy", do not exist. The time energy of any particle is its gravitational energy dispersed in the whole world. Thus, accepting the undefinable notions "space" and "time" es self-evident, the unique enigmatic notion in physics remains the notion "space energy". (N.B. May be in this link of any particle with the whole universe one has to look for the explanation of the paraphysical phenomena.)

If we should look at physics from this point of view, we can cancel the notion "time energy" in our axiomatics and operate only with the notions "space energy" and "space-time energy" (let us remind - see p. 66 - that in a similar manner we can cancel the notion "mass" and operate only with the notion "energy").

The notion "time energy" can be excluded from our axiomatics if we replace the sixth and the ninth axiom by the following two:

Axiom VI. The energy e_o of any particle is its gravitational energy with the mass of the whole world, taking it with a negative sign. We call this energy WORLD ENERGY and denote it by U_w. The world energy of a unit mass which rests in absolute space is equal to $-c^2$ energy units. Thus the world energy of a mass m moving in absolute space is

$$U_w = - m_o c^2. \tag{24.22}$$

Axiom IX. The change in time of the space and space-time energies of an isolated material system is equal to the change in time of its world energy, that is

$$dU + dW = dU_w. \tag{24.23}$$

So we see that the discussion of the problem about the equality of "inertial" and "gravitating" masses is senseless, as "inertal mass" does not exist. The mass is only gravitational. Thus all costly experiments with which one tries to establish whether there is a difference between the "inertial" and "gravitating" masses have been and continue to be a waste of time, efforts, and money.

D. The Newton-Lorentz equation

The full Newton equation in electromagnetism has the same form as the Newton-Marinov equation, however the mass in the proper space and time momenta is not the full mass of the particle but its isolated mass. We call it the NEWTON-LORENTZ EQUATION and we shall write it in a form analogical to (18)

$$m\vec{u}_o + \frac{q}{c}\frac{d\vec{A}}{dt} = - q\,\mathrm{grad}(\Phi - \frac{\vec{v}}{c}.\vec{A}), \tag{24.24}$$

where q is the electric charge of a test mass m moving with a velocity $\vec{v}$, and

$$\Phi = \sum_{i=1}^{n} \frac{q_i}{r_i}, \qquad \vec{A} = \sum_{i=1}^{n} \frac{q_i \vec{v}_i}{c r_i} \tag{24.25}$$

are the ELECTRIC POTENTIAL and MAGNETIC POTENTIAL at the reference point (see also V, §2).

E. The Lagrange equations

Now, assuming that the full masses of the particles are approximately equal to their isolated masses (as it is always the case in reality), we shall write the fundamental equations of motion in a general form valid for both, gravimagretism and electromagnetism, making use of the Lagrange function.

The LAGRANGE ENERGY of the system is E^0. The FULL LAGRANGE ENERGY is (cf. (IIB,7.10))

$$L = E^0 - U. \tag{24.26}$$

The TOTAL LAGRANGE ENERGY is

$$\tilde{L} = E^0 - U + W = L + W. \tag{24.27}$$

Thus the fundamental equations of motion (8) and (14) can be written in the form of LAGRANGE EQUATIONS similar to that in low-velocity physics (see (IIB,7.12))

$$\frac{d}{dt} \frac{\partial \tilde{L}}{\partial \vec{v}_i} = \frac{\partial \tilde{L}}{\partial \vec{r}_i}, \qquad i = 1,2,\ldots..n. \tag{24.28}$$

The HAMILTON ENERGY of the system is E_0. The FULL HAMILTON ENERGY is

$$H = E_0 + U. \tag{24.29}$$

The TOTAL HAMILTON ENERGY is

$$\tilde{H} = E_0 + U + W = H + W. \tag{24.30}$$

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If we use the generalized coordinates (see IIB, p. 58), we can write the Lagrange equations (28) in a form analogical to (IIB,7.24)

$$\frac{d}{dt} \frac{\tilde{L}}{\partial \dot{q}_k} = \frac{\tilde{L}}{\partial q_k}, \qquad k = 1,2,\ldots..s, \tag{24.31}$$

where q_k is the k-th generalized coordinate, $\dot{q}_k$ is the k-th generalized velocity, s is the numbers of degrees of freedom of the system, and $\tilde{L}$ is the Lagrangian (27) of the considered material system, where the Cartesian coordinates and velocities are expressed through the generalized ones (see (IIB,7.17)).

Remark. Further on in part III, except for §25, we shall not define the specific form of the space energy and we shall ignore the space-time energy, as we do in mechanics (see IIA, p. 6).

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The PROPER MOMENTUM of one particle of the system is

$$\vec{p}_0 = \partial L/\partial\vec{v} = \partial e^0/\partial\vec{v} = m\vec{v}_0. \quad (24.32)$$

The 4-MOMENTUM of the particle is

$$\overleftrightarrow{p}_0 = \partial L/\partial\overleftrightarrow{v} = \partial e^0/\partial\overleftrightarrow{v} = m\overleftrightarrow{v}_0 = (\vec{p}_0, i\bar{p}_0). \quad (24.33)$$

The space part and the time part of the 4-momentum represent the space momentum and time momentum introduced in our third axiom (see (IIA,2.5)).

The 4-dimensional generalization of the Lagrange equations (28) is

$$\frac{d\overleftrightarrow{p}_{0i}}{dt} = \frac{\partial L}{\partial\overleftrightarrow{r}_i}, \qquad i = 1,2,\ldots.n, \quad (24.34)$$

where in the fourth component of this equation the radius vectors in L are to be considered as explicit functions of time, while the velocities are not to be considered as explicit functions of time. Thus the fourth component of this equation is to be written in the form

$$-\frac{d\bar{p}_{0i}}{dt} = \frac{1}{c}\frac{\partial L(t)}{\partial t_i}, \quad (24.35)$$

where the index "i" in the time differentiation denotes that we have to differentiate only the radius vector of the i-th particle in L.

F. Kinetic and potential forces

As in high-velocity mechanics there are three different types of acceleration, there are also three different types of kinetic forces

$$\vec{f} = m\vec{u}, \qquad \vec{f}_0 = m\vec{u}_0, \qquad \vec{f}_{00} = m\vec{u}_{00}, \quad (24.36)$$

which are called, respectively, UNIVERSAL KINETIC FORCE, FIRST PROPER KINETIC FORCE, and SECOND PROPER KINETIC FORCE.

The UNIVERSAL, FIRST PROPER, and SECOND PROPER 4-KINETIC FORCES are

$$\overleftrightarrow{f} = d\overleftrightarrow{p}/dt = m\overleftrightarrow{u}, \qquad \overleftrightarrow{f}_0 = d\overleftrightarrow{p}_0/dt = m\overleftrightarrow{u}_0, \qquad \overleftrightarrow{f}_{00} = d\overleftrightarrow{p}_0/dt_0 = m\overleftrightarrow{u}_{00}. \quad (24.37)$$

Obviously, only $\overleftrightarrow{f}_{00}$ is a legitimate 4-vector, which we shall briefly call 4-POTENTIAL FORCE.

The POTENTIAL FORCE

$$\vec{F} = -\partial L/\partial\vec{r} \quad (24.38)$$

is equal to the first proper kinetic force, as it follows from equations (12) and (28).

When presenting equation (12) in the form

$$\vec{f}_{00} = \vec{F}_0, \quad (24.39)$$

we have to call $\vec{F}_0$ the PROPER POTENTIAL FORCE, and when calculating it according to formula (9), we have to take U (and, of course, W, if space-time energy will be also considered) in its second form (17), independent of whether we work in gravimagretism or electromagnetism. In the latter case we have to write in (17) q_o instead of m_o, considering the "proper charge" q_o only formally, as the factor $(1 - v_o^2/c^2)^{-1/2}$ must be attached to the space and time velocities.

The 4-POTENTIAL FORCE is

$$\overleftrightarrow{F} = \partial L/\partial \overleftrightarrow{r}, \tag{24.40}$$

and the 4-dimensional generalization of Newton's equation (39) is

$$\overleftrightarrow{f}_{oo} = \overleftrightarrow{F}_o. \tag{24.41}$$

§25. THE LABORATORY FUNDAMENTAL EQUATIONS OF MOTION

In §24A and §24B we gave the fundamental equations of motion in gravimagretism and electromagnetism considered in absolute space. In the following section we shall write these equations in the laboratory which moves with a certain velocity $\vec{V}$ in absolute space.

A. Gravimagretism

The Newton-Marinov equation (24.18) for the motion of a mass m in a gravimagretic field in absolute space can be written also in the form

$$\frac{m}{c^3}(c_o c^2 + c_o\Phi - \vec{v}_o.\vec{A})\vec{u}_o + \frac{m}{c}\frac{d\vec{A}}{dt_o} = -\frac{m}{c}\mathrm{grad}(c_o\Phi - \vec{v}_o.\vec{A}), \tag{25.1}$$

where Φ, $\vec{A}$ are the gravitational and magretic potentials produced by a certain material system at a reference point where a test mass m moving with a proper space velocity $\vec{v}_o$ and a proper time velocity c_o acquires during a proper time interval dt_o a first proper acceleration $\vec{u}_o$.

Let us suppose now that the same material system generating the gravimagretic field and the reference point with the test mass move together with a velocity $\vec{V}$ in absolute space.

First we shall show that the difference between the gravitational and magretic energies of a material system, within an accuracy of second order in "velocity/light velocity", is independent of the velocity of the system as a whole in absolute space.

Indeed, let us consider two particles of the system with masses m_i and m_k moving with velocities $\vec{v}_i$ and $\vec{v}_k$ in absolute space. Calculating within an accuracy of second order in v_i/c and v_k/c, we obtain for the difference of their gravitational and magretic energies (see (IIA,3.2) and (IIA,3.9))

$$U - W = -\gamma\frac{m_i m_k(1 - \vec{v}_i.\vec{v}_k/c^2)}{\{(1 - v_i^2/c^2)(1 - v_k^2/c^2)\}^{1/2}\, r} = -\gamma\frac{m_i m_k}{r}\{1 + \frac{(\vec{v}_i - \vec{v}_k)^2}{2c^2}\}. \tag{25.2}$$

If we observe this system from a laboratory which moves with a velocity $\vec{V}$ in absolute space, we obtain for the difference of the gravitational and magretic energies, calculating again within an accuracy of second order in v_i'/c, v_k'/c, and V/c,

$$U - W = -\gamma \frac{m_i m_k \{1 - (\vec{v}_i' + \vec{V}).(\vec{v}_k' + \vec{V})/c^2\}}{\{1 - (\vec{v}_i' + \vec{V})^2/c^2\}^{1/2}\{1 - (\vec{v}_k' + \vec{V})^2/c^2\}^{1/2}\, r} = -\gamma \frac{m_i m_k}{r}\{1 + \frac{(\vec{v}_i' - \vec{v}_k')^2}{2c^2}\}. \qquad (25.3)$$

where $\vec{v}_i'$, $\vec{v}_k'$ are the velocities in the laboratory. We have used the Galilean formula for velocity addition (4.1), but the same result will be obtained, within the necessary accuracy, if the Marinov formula for velocity addition (4.25) should be used.

Thus, within the necessary accuracy, we can write the expression $c_o\Phi - \vec{v}_o.\vec{A}$ in the Newton-Marinov equation, taking the quantities c_o, $\vec{v}_o$, Φ, $\vec{A}$, not with respect to absolute space but with respect to the laboratory, i.e., measuring the proper elements of motion of the particles of the system not in universal but in laboratory time. Consequently the conclusion can be drawn that the gravimagretic interaction of a system of particles does not depend on the velocity of the system as a whole.

Hence the Newton-Marinov equation (1), after cancelling the common factor $(1 - v'^2/c^2)^{-1/2}$, can be written in the form

$$\frac{m}{c^2}(c^2 + \Phi' - \frac{\vec{v}'}{c}.\vec{A}')\frac{d}{dt}\frac{\vec{v}' + \vec{V}}{\{1 - (\vec{v}' + \vec{V})^2/c^2\}^{1/2}} + \frac{m}{c}\frac{d\vec{A}'}{dt} = -m\,\mathrm{grad}(\Phi' - \frac{\vec{v}'}{c}.\vec{A}'), \qquad (25.4)$$

where $\vec{v}'$, Φ', and $\vec{A}'$ are measured in the laboratory.

We call (24.18) the ABSOLUTE NEWTON-MARINOV EQUATIONS and (4) the RELATIVE (or LABORATORY) NEWTON-MARINOV EQUATION. The unique difference between equation (24.18) where all quantities are taken with respect to absolute space and equation (4) where all quantities are taken with respect to the laboratory is that the laboratory's absolute velocity $\vec{V}$ appears in the proper laboratory acceleration of the test mass.

B. Electromagnetism

Since the electromagnetic potentials are not "velocity dependent" as the gravimagretic potentials (24.16), the difference between the electric and magnetic energies of a material system will depend on the velocity of the system as a whole in absolute space.

Indeed, let us consider two particles of the system with electric charges q_i and q_k moving with velocities $\vec{v}_i$ and $\vec{v}_k$ in absolute space. We have for the difference of their electric and magnetic energies (see (IIA,3.5) and (IIA,3.10) for $1/\varepsilon_o = \mu_o = 1$)

$$U - W = q_i q_k (1 - \vec{v}_i.\vec{v}_k/c^2). \qquad (25.5)$$

If we observe this system from a laboratory which moves with a velocity $\vec{V}$ in absolute space, we obtain for the difference of the electric and magnetic energies, making use of the exact Marinov formula for velocity addition (4.25)

$$U - W = q_i q_k \{1 - \frac{\vec{v}_i' . \vec{v}_k'}{c^2} - \frac{(\vec{v}_i' + \vec{v}_k').\vec{V}}{c^2}(1 - \frac{V^2}{c^2})^{1/2} - \frac{V^2}{c^2} + \frac{(\vec{v}_i'.\vec{V})(\vec{v}_k'.\vec{V})}{c^4}\} \cong$$

$$q_i q_k \{1 - \frac{\vec{v}_i' . \vec{v}_k'}{c^2} - \frac{(\vec{v}_i' + \vec{v}_k').\vec{V}}{c^2} - \frac{V^2}{c^2}\}, \qquad (25.6)$$

where $\vec{v}_i'$ and $\vec{v}_k'$ are the velocities in the laboratory.The last approximate result, accurate within an accuracy of second order in v_i'/c, v_k'/c, and V/C can be obtained by a direct application of the Galilean formula for velocity addition.

If we denote by U' and W' the space energy and space-time energy calculated in the laboratory, then comparing (2) and (3), we conclude that in gravimagretism

$$U - W = U' - W', \qquad (25.7)$$

while comparing (5) and (6), we conclude that in electromagnetism

$$U - W \neq U' - W'. \qquad (25.8)$$

Consequently the conclusion can be drawn that the electromagnetic interaction of a system of particles depends on the velocity of the system as a whole. Moreover, equation (6) shows that this dependence is of first order in V/c.

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To find the relative Newton-Lorentz equation, let us consider the motion of a mass m with electric charge q and laboratory velocity $\vec{v}'$ in the electromagnetic field of a system of charges q_i having laboratory velocities $\vec{v}_i'$. Let the laboratory move with velocity $\vec{V}$ in absolute space, and assume $V \ll c$, so that we shall pay attantion only to terms of first order in V/c; the velocities v' and v_i' will be assumed arbitrarily large.

The difference of the electric and magnetic energies in which charge q takes part can be written as follows

$$U - W = q\sum\frac{q_i}{r_i} - q(\vec{v}' + \vec{V}).\sum\frac{q_i(\vec{v}_i' + \vec{V})}{c^2 r_i} = q\sum\frac{q_i}{r_i}(1 - \frac{\vec{v}_i'.\vec{V}}{c^2} - \frac{\vec{v}'.\vec{V}}{c^2} - \frac{V^2}{c^2}) - q\vec{v}'.\sum\frac{q_i\vec{v}_i'}{c^2 r_i} =$$

$$q\Phi'(1 - \frac{\vec{v}'.\vec{V}}{c^2}) - \frac{q\vec{v}'}{c}.\vec{A}'. \qquad (25.9)$$

where

$$\Phi' = \sum\frac{q_i}{r_i}(1 - \frac{V^2}{c^2}) - \frac{\vec{V}}{c}.\vec{A}', \qquad \vec{A}' = \sum\frac{q_i\vec{v}_i'}{cr_i} \qquad (25.10)$$

will be called the RELATIVE (or LABORATORY) ELECTRIC AND MAGNETIC POTENTIALS, while Φ and $\vec{A}$ (see formulas (24.25)) will be called ABSOLUTE ELECTRIC AND MAGNETIC POTENTIALS. As formulas (9) show, the laboratory magnetic potential is the one which would be measured if the laboratory were at rest in absolute space; the laboratory electric potential, however, is not the one which would be measured if the laboratory were at rest in absolute space, but depends on the absolute velocity of the laboratory.

Substituting (9) into (24.14) and taking into account that

$$d\vec{A}/dt = d\vec{A}'/dt, \tag{25.11}$$

we can write the RELATIVE (or LABORATORY) NEWTON-LORENTZ EQUATION in the form

$$\frac{d}{dt}\frac{m(\vec{v}' + \vec{V})}{\{1 - (\vec{v}' + \vec{V})^2/c^2\}^{1/2}} + \frac{q}{c}\frac{d\vec{A}'}{dt} = - q\,\mathrm{grad}\{\Phi'(1 - \frac{\vec{v}'.\vec{V}}{c^2}) - \frac{\vec{v}'}{c}.\vec{A}'\}, \tag{25.12}$$

meanwhile (24.24) will be called the ABSOLUTE NEWTON-LORENTZ EQUATION. It is obvious from this equation that the potential force acting on the test charge does not only depend on the relative electric and magnetic potentials but also on the product $\vec{v}'.\vec{V}$. Let us also emphasize that while the relative magnetic potential has the form of the absolute magnetic potential, the relative electric potential has not the form of the absolute electric potential.

We consider the introduction of the relative Newton-Lorentz equation in the theory as very important. We are surprised that this equation was not proposed by Lorentz or by other scientists before the beginning of the dark "relativity age".

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The theory of relativity denies the existence of absolute space and consequently it does not even discuss the problem which must be the fundamental equations of motion in the laboratory. The theory of relativity is able to analyse and solve only the following problem: "Let the electromagnetic field in a certain inertial frame be given: which will be then the electromagnetic field for an observer who moves with a velocity $\vec{V}$ in this frame?" - Applying the Lorentz transformation formulas, relativity obtains definite results which are obtained also in our absolute space-time theory (see part V). Whether one considers the case of the observer in motion and the system at rest or vice versa is the same for relativity, as only the relative motion is of importance.

According to our absolute space-time theory, the cases "system at rest in absolute space, observer in motion" and "observer at rest in absolute space, system in motion" are not identical and we discuss the problem at the following pages (see §§38, 40, 59, 63, 64, 68). Furthermore our theory considers the problem "system and observer moving in absolute space", a problem which relativity cannot even pose, as for relativity absolute space does not exist.

The Lorentz transformation is a convenient mathematical tool for the theory of relativity, as by application of a Lorentz transformation one can always exclude the constant velocity of the center-of-mass of the observed material system and, forgetting that the new time coordinates contain not only old time but also old space coordinates, one can consider the moving material system as being at rest as a whole. Thus a relativist will write instead of equation (12) the equation (24.24) independent of the motion of the laboratory, i.e., he will write a wrong equation. This shows that the Lorentz transformation is to be used with caution because, if treated "relativistically", it leads to errors. We point out an obvious error of the Lorentz transformation in §42. However, the most impressive "error" of the Lorentz transformation when treated relativistically is the result of the "coupled mirrors" experiment (see §52).

Nevertheless the errors to which lead the Lorentz transformation formulas when treated relativistically are scarce, as in the predominent number of cases the absolute effects annihilate each other and the effects which can be observed are null. The fact that anisotropy effects in inertially moving frames have been observed for the first time in the fourth-quarter of the 20th century shows clearly that for almost all phenomena observed by the physicists the inertially moving laboratory can practically be considered as being at rest in absolute space.

We shall frequently use the Lorentz transformation in our investigation, as the mathematical conviniences which it offers are enormous. Especially this concerns the transition from a "low-velocity observer" to a "high-velocity observer", as the reader will see in part V.

C. Mechanics

In §44 we reveal the differences between the absolute and relativistic approaches in the most simple and clear way. Many subsections of §44 dedicated to mechanics are to be presented now but we prefer to give the whole matter together when the reader will become familiar with the kinematics of light exposed in ch. VII.

Here we shall only emphasize that, according to our absolute conceptions, the time energy e_0 of a particle (see (23.9)) is the same in any inertial frame in which one observes its motion, as e_0 depends not on the particle's laboratory velocity (as assumed by the theory of relativity), but on the particle's absolute velocity. Thus the time energy which is a 3-dimensional scalar is a frame independent quantity. Any 3-dimensional scalar which preserves its value in any inertial frame will be called a MARINOV INVARIANT QUANTITY. The space energy which depends on the distances between the particles is also a Marinov invariant quantity. Thus the full Hamilton and full Lagrange energies are also Marinov (frame) independent quantities.

One must make a substantial difference between a "Lorentz invariant quantity" (see p. 48) and a "Marinov invariant quantity". The Lorentz invariance concerns only 4-scalars (as 4-interval, 4-momentum, the scalar product of two 4-vectors) and is a result of the 4-dimensional formalism. The Marinov invariance concerns 3-scalars (as 3-interval, time energy); beside the distances which are registered at a single moment and thus are equal in any frame, the Marinov invariant quantities are always connected with the "gravitational charges" of the particles which are their proper masses written with the particles' absolute velocities. Both the Lorentz and the Marinov invariance are extremely important for the investigation of the high-velocity phenomena, as the reader will see when reading this book. The Lorentz invariance is used when there is an observer at rest in absolute space and a particle first considered at rest and then in motion, while the Marinov invariance is used when there is a particle moving in absolute space and two different observers.

Chapter IV

LAWS OF CONSERVATION

§26. ENERGY AND MOMENTUM (4-MOMENTUM)

The ninth axiom asserts that the full energy of an isolated material system is constant in time. In IIB, §10 we showed that this is true also for the low-velocity momentum.

Now, proceeding from the homogeneity of the 4-space, we shall show that the momentum and the full energy (i.e., the 4-momentum) of an isolated material system are constant in time also at high velocities of the particles.

Indeed, let us consider our isolated material system in two different 4-frames and let us suppose that the second is shifted parallel to the first one. The space parts of both 4 frames are immobile to one another, and thus the full Lagrange energy of the system will have the same value in both frames. Hence, if we designate the difference between the full Lagrange energies registered in both frames by dL, we shall have

$$dL = 0. \tag{26.1}$$

The velocities of the particles in both frames, whose space parts are at rest with respect to one another, are equal, and we can write (cf. IIB, p. 68)

$$dL = \sum_{i=1}^{n} \frac{\partial L}{\partial \overleftrightarrow{r}_i} . d\overleftrightarrow{r} = 0, \tag{26.2}$$

where $d\overleftrightarrow{r}_i$ is the difference between the 4-radius vectors of some event occured with the particle m_i and registered in both 4-frames.

At a parallel shift all changes of the 4-radius vectors of the particles will be equal to the displacement of the origin of the second 4-frame with respect to the origin of the first one, i.e.,

$$d\overleftrightarrow{r}_i = d\overleftrightarrow{R}, \qquad i = 1,2,\ldots..n. \tag{26.3}$$

Thus we can write (2) in the form

$$d\vec{R} . \sum_{i=1}^{n} \frac{\partial L}{\partial \overleftrightarrow{r}_i} = 0. \tag{26.4}$$

As the displacement $d\overleftrightarrow{R}$ is arbitrary, we conclude

$$\sum_{i=1}^{n} \partial L/\partial \overleftrightarrow{r}_i = 0. \tag{26.5}$$

Using the Lagrange equations (24.34), we can write the space part of this equation in the form

$$\frac{d}{dt} \sum_{i=1}^{n} \vec{p}_{oi} = 0, \tag{26.6}$$

or

$$\vec{P}_o \equiv \sum_{i=1}^{n} \vec{p}_{oi} = \vec{Const}, \qquad (26.7)$$

i.e., the proper momentum of an isolated material system, which is the sum of the proper momenta of all its particles is constant in time.

Using the time part of the Lagrange equations (24.34), we can write the time part of equation (5), after its multiplication by -2ic, in the form

$$-2\sum\frac{\partial L}{\partial t_i} = \sum\frac{\partial U}{\partial t_i} + \sum\frac{\partial U}{\partial t_i} = \frac{d}{dt}\sum e_{oi} + \sum\frac{\partial U}{\partial t_i} = \frac{d}{dt}(E_o + U) = 0, \qquad (26.8)$$

or

$$H = E_o + U = \sum_{i=1}^{n} e_{oi} + U = Const, \qquad (26.9)$$

i.e., the full energy of an isolated material system is constant in time. N.B. We can, of course, give for the energy conservation law a similar deduction to that presented in IIB, p. 66.

Remark. When obtaining the time component of the 4-momentum according to formula (24.33), we have to perform the calculation as follows

$$i\frac{\bar{p}_o}{c} \equiv \frac{im^* c}{(1 - v^2/c^2)^{1/2}} = \frac{\partial(L/c)}{i\,\partial c} \equiv im\{(1 - v^2/c^2)^{1/2} + \frac{v^2/c^2}{(1 - v^2/c^2)^{1/2}}\}, \qquad (26.10)$$

where $m^* = m/c$ is the Marinov mass (see IIA, p. 18).

If the space energy of the considered material system is equal to zero, or if the system consists of parts isolated one from another, the laws of momentum and energy conservation (7) and (9) can be written in a 4-dimensional form as a LAW OF 4-MOMENTUM CONSERVATION

$$\overleftrightarrow{P}_o = \sum_{i=1}^{n} \overleftrightarrow{p}_{oi} = \overleftrightarrow{Const}. \qquad (26.11)$$

If we wish to return to low-velocity mechanics using formula (23.13), then equation (9) must be written in the form

$$c^2\frac{d}{dt}\sum_{i=1}^{n} m_i + \frac{d}{dt}\{\sum_{i=1}^{n} m_i v_i^2/2 + U\} = 0. \qquad (26.12)$$

The first term on the left side of this equation gives the law of mass conservation in low-velocity mechanics (see (IIB,11.2)) and the second term the law of energy conservation in low-velocity mechanics (see (IIB,9.4)). By using the notations introduced in p. 65, this law can be written

$$E_k + U = Const. \qquad (26.13)$$

xxxxx

Let us have a system of n particles with time energy E_o and space energy U. We can consider this system as a single particle placed at the center of inertia (see IIB, p. 71) of the system. The full energy of this hypothetical particle, considered only as time energy, will be

$$H = \frac{Mc^2}{(1 - V^2/c^2)^{1/2}}, \tag{26.14}$$

where M is the mass of the whole system considered as a particle and V is the velocity of its center of inertia.

Taking into account (23.19), we conclude that

$$M \neq \sum_{i=1}^{n} m_i. \tag{26.15}$$

The difference

$$\Delta M = M - \sum_{i=1}^{n} m_i \tag{26.16}$$

between the mass of the whole system considered as a single particle and the sum of the masses of all elementary particles of the system is called the MASS DEFECT.

If the velocities v_i do not differ too much from the velocity of the center of inertia V, we obtain from (16), (14), (23.19), and (24.29)

$$\frac{\Delta M\, c^2}{(1 - V^2/c^2)^{1/2}} = U. \tag{26.17}$$

If we consider our material system in absolute space, we shall have

$$\Delta M\, c^2 = U. \tag{26.18}$$

This quantity is called BINDING ENERGY of the system.

From this equation we can make the following conclusions:

1. If $\Delta M > 0$, the system can disintegrate spontaneously into parts isolated from one abother. Indeed, let us assume that the system has disintegrated into two parts. Then

$$M c^2 = \frac{m_1 c^2}{(1 - v_1^2/c^2)^{1/2}} + \frac{m_2 c^2}{(1 - v_2^2/c^2)^{1/2}}, \tag{26.19}$$

where m_1 and m_2 are the masses of the two isolated systems obtained after the desintegration and considered as particles. Since $v_1 \neq 0$, $v_2 \neq 0$, equality (19) is possible only when

$$M > m_1 + m_2. \tag{26.20}$$

2. If $\Delta M < 0$, with the help of similar considerations, we conclude that the system cannot disintegrate spontaneously into parts isolated from one another. This disintegration is only possible if the system is supplied from outside with an energy quantity at least equal to its binding energy $|\Delta M| c^2$.

At the spontaneous disintegration the binding energy, which in this case is positive, transforms into kinetic energy of the material systems obtained after the desintegration.

xxxxx

The space and time parts of the 4-momentum, i.e., the proper space momentum and the proper time momentum, can be written in the form

$$\vec{p}_o = \frac{m\vec{v}}{(1 - v^2/c^2)^{1/2}} = \frac{e_o}{c^2}\vec{v}, \tag{26.21}$$

$$\bar{p}_o = \frac{mc}{(1 - v^2/c^2)^{1/2}} = \frac{e_o}{c}. \tag{26.22}$$

If the particle moves with a velocity near to c, the magnitude of its proper momentum will be

$$p_o \cong e_o/c, \tag{26.23}$$

and we conclude that in this case the magnitude of the proper space momentum is equal to the proper time momentum.

Using formulas (13.5), we can write the transformation formulas for the components of the 4-momentum as follows (for the fourth component we take the proper energy)

$$p_{ox} = \frac{p'_{ox} + Ve'_o/c^2}{(1 - V^2/c^2)^{1/2}}, \quad p_{oy} = p'_{oy}, \quad p_{oz} = p'_{oz}, \quad e_o = \frac{e'_o + Vp'_{ox}}{(1 - V^2/c^2)^{1/2}}. \tag{26.24}$$

Taking into account formula (19.4), we can write

$$\overset{\leftrightarrow}{p}{}_o^2 \equiv p_o^2 - \bar{p}_o^2 = - m^2c^2. \tag{26.25}$$

Substituting here (22), we obtain the Hamiltonian of a free particle

$$H = e_o = (p_o^2c^2 + m^2c^4)^{1/2}, \tag{26.26}$$

as we call Hamiltonian the full energy of a system expressed through the momenta of its particles (see II, p. 80).

If $v \ll c$,

$$p_o \cong p \ll mc, \tag{26.27}$$

and we obtain from (26)

$$H = e_f = mc^2 + p^2/2m, \tag{26.28}$$

which is equation (23.13).

Taking into account (23.8), (23.9), and (21), we can arrive at the relation

$$e_o = \vec{v}.\vec{p}_o - e^o, \tag{26.29}$$

so that putting here $e_o = H$, $e^o = L$, and taking into account (24.32), we obtain for a free particle

$$H = \vec{v}.\frac{\partial L}{\partial \vec{v}} - L. \tag{26.30}$$

In the case of a system (with $U \neq 0$) we shall arrive at the same relation, as the Lagrange energies of the free particles e_i^o figure as additive quantities in the Lagrangian L of the whole system, i.e.,

$$H = \sum_{i=1}^{n} \vec{v}_i . \frac{\partial L}{\partial \vec{v}_i} - L. \tag{26.31}$$

§27. ACTION

We introduce the notion action analogically as in low-velocity mechanics (see IIB, §17). The ACTION of a material system is the quantity

$$S = \int_{t_0}^{t} L\, dt, \tag{27.1}$$

where t_0 and t are two successive moments.

From (1) we obtain

$$dS/dt = L. \tag{27.2}$$

Let us take partial derivatives with respect to $\vec{r}_i$ from both sides of equation (2)

$$\frac{\partial}{\partial \vec{r}_i} \frac{dS}{dt} = \frac{\partial L}{\partial \vec{r}_i}, \qquad i = 1,2,\ldots..n. \tag{27.3}$$

On the grounds of the Lagrange equations (24.28), we shall have

$$\frac{\partial}{\partial \vec{r}_i} \frac{dS}{dt} = \frac{d}{dt} \frac{\partial L}{\partial \vec{v}_i}. \tag{27.4}$$

Taking into account (24.32), we obtain

$$\frac{d}{dt} \frac{\partial S}{\partial \vec{r}_i} = \frac{d}{dt} \vec{p}_{oi}, \tag{27.5}$$

or

$$\frac{\partial S}{\partial \vec{r}_i} = \vec{p}_{oi}, \qquad i = 1,2,\ldots..n, \tag{27.6}$$

i.e., the partial derivative of the action S with respect to the radius vector $\vec{r}_i$ is equal to the proper momentum $\vec{p}_{oi}$, and, possibly, can differ with a quantity constant in time.

The action S is a function of $\vec{r}_i$ and t only, so that we can write

$$\frac{dS}{dt} = \frac{\partial S}{\partial t} + \sum_{i=1}^{n} \frac{\partial S}{\partial \vec{r}_i} . v_i . \tag{27.7}$$

Taking into consideration (2), (6), and (26.31), we obtain

$$\frac{\partial S}{\partial t} = L - \sum_{i=1}^{n} \vec{v}_i . \vec{p}_{oi} = - H. \tag{27.8}$$

For a free material point we get from (8) and (26.22)

$$\frac{\partial S}{\partial \tau} = i \bar{p}_o . \tag{27.9}$$

Unifying formulas (6) and (9), we can write the relation between the 4-momentum and the 4-space derivative of the action

$$\overleftrightarrow{p}_o = \frac{\partial S}{\partial \overleftrightarrow{r}}. \qquad (27.10)$$

xxxxx

If H does not depend on time (i.e., H = Const), then, as S depends only on the coordinates and time, formula (8) yields

$$S = - Ht + S_c, \qquad (27.11)$$

where the part of the action S_c depends only on the coordinates and is called CURTAILED ACTION (see IIB, p. 87).

xxxxx

In high-velocity mechanics we can also consider the principle (or the theorem) of least action, as we did in low-velocity mechanics (see IIB, §18). The analogy is almost complete.

§28. 4-ANGULAR MOMENTUM

Let us consider a given material system in two space frames of reference which are arbitrarily rotated upon a small angle $d\vec{\phi}$ one with respect to the other. Let the difference between the radius vectors $\vec{r}$ and $\vec{r}'$ of an arbitrary particle of the system registered in frames K and K', respectively, be

$$d\vec{r} = \vec{r} - \vec{r}', \qquad (28.1)$$

where, for brevity, we do not write the index "i" showing the number of the particle.

Using formula (IIB,2.12), we can write

$$d\vec{r} = d\vec{\phi}\times\vec{r}. \qquad (28.2)$$

From (1) and (2) we obtain

$$\vec{r}' = \vec{r} - d\vec{\phi}\times\vec{r}. \qquad (28.3)$$

If we resolve rotation $d\vec{\phi}$ into three rotations: a) $d\phi_{12}$ in the plane $(r_1 r_2)$, i.e, (xy), b) $d\phi_{23}$ in the plane $(r_2 r_3)$, i.e., (yz), c) $d\phi_{31}$ in the plane $(r_3 r_1)$, i.e., (zx), we can write the vector formula (3) in the form of the following three scalar equations

$$r_1' = r_1 - (d\phi_{31} r_3 - d\phi_{12} r_2),$$
$$r_2' = r_2 - (d\phi_{12} r_1 - d\phi_{23} r_3),$$
$$r_3' = r_3 - (d\phi_{23} r_2 - d\phi_{31} r_1). \qquad (28.4)$$

If we take into consideration that

$$d\phi_{ab} = - d\phi_{ba}, \qquad (28.5)$$

i.e., that the rotation in the plane $(r_a r_b)$ from axis (r_a) to axis (r_b) is equal with opposite sign to the rotation in the same plane from axis (r_b) to axis (r_a), we can write formulas (4) as follows

$$\begin{aligned} r_1' &= r_1 + \qquad\quad + d\phi_{12}x_2 + d\phi_{13}x_3, \\ r_2' &= r_2 + d\phi_{21}x_1 + \qquad\quad + d\phi_{23}x_3, \\ r_3' &= r_3 + d\phi_{31}x_1 + d\phi_{32}x_2 + \qquad\quad , \end{aligned} \tag{28.6}$$

or (we use the summation convention - see the remark on p. 46)

$$r_a' = r_a + d\phi_{ab}r_b. \tag{28.7}$$

The angles $d\phi_{ab}$ build a 3-dimensional tensor of second rank

$$(d\phi_{ab}) = \begin{vmatrix} d\phi_{11} & d\phi_{12} & d\phi_{13} \\ d\phi_{21} & d\phi_{22} & d\phi_{23} \\ d\phi_{31} & d\phi_{32} & d\phi_{33} \end{vmatrix}, \tag{28.8}$$

which we call the TENSOR OF THE ELEMENTARY ROTATION. This tensor is antisymmetric (see (5)), i.e., it has the form

$$(d\phi_{ab}) = \begin{vmatrix} 0 & d\phi_{12} & d\phi_{13} \\ -d\phi_{12} & 0 & d\phi_{23} \\ -d\phi_{13} & -d\phi_{23} & 0 \end{vmatrix}. \tag{28.9}$$

xxxxx

If we work in the 4-dimensional space and we consider two 4-frames rotated with respect to each other, we can introduce, by an analogy with the 3-dimensional case, the 4-TENSOR OF THE ELEMENTARY ROTATION

$$(d\phi_{\alpha\beta}) = \begin{vmatrix} d\phi_{11} & d\phi_{12} & d\phi_{13} & id\phi_{14} \\ d\phi_{21} & d\phi_{22} & d\phi_{23} & id\phi_{24} \\ d\phi_{31} & d\phi_{32} & d\phi_{33} & id\phi_{34} \\ id\phi_{41} & id\phi_{42} & id\phi_{43} & -d\phi_{44} \end{vmatrix}. \tag{28.10}$$

This 4-tensor is antisymmetric, too, i.e.,

$$d\phi_{\alpha\beta} = - d\phi_{\beta\alpha}, \tag{28.11}$$

and the relation between the coordinates registered in the primed and unprimed frames will be

$$r_\alpha' = r_\alpha + d\phi_{\alpha\beta}r_\beta. \tag{28.12}$$

Under a special Lorentz transformation, taking into account (11.8), we shall have

$$d\phi_{14} = d\phi = \tan(d\phi) = V/c, \tag{28.13}$$

and the 4-tensor (10) will obtain the form

$$(d\phi_{\alpha\beta}) = \begin{vmatrix} 0 & 0 & 0 & i\frac{V}{c} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -i\frac{V}{c} & 0 & 0 & 0 \end{vmatrix}. \qquad (28.14)$$

Now formulas (12) yield

$$x' = x - Vt, \qquad y' = y, \qquad z' = z, \qquad t' = t - \frac{V}{c^2}x. \qquad (28.15)$$

We see that these formulas coincide with the Lorentz transformation formulas (3.20) at the assumption $V \ll c$, which must be done having in mind relation (13). Note that in the last formula (15) the term Vx/c^2 is not neglected as small, since the quantity x may be large.

xxxxx

Now we shall deduce the law of conservation of the angular momentum in high-velocity mechanics, making use of the formulas introduced.

From formulas (11) and (12) we obtain

$$dr_\alpha = r_\alpha - r'_\alpha = - d\phi_{\alpha\beta} r_\beta = d\phi_{\beta\alpha} r_\beta. \qquad (28.16)$$

Let us take the time derivatives of these formulas; since the elementary rotations $d\phi_{\beta\alpha}$ must be considered as constant quantities, we shall have

$$dv_\alpha = d\phi_{\beta\alpha} v_\beta. \qquad (28.17)$$

As we said (see p. 77), the full Lagrange energy L of the material system must have the same value in the new 4-frame of reference as in the old one; thus the increase of the Lagrangian must be equal to zero

$$dL = 0. \qquad (28.18)$$

Remark. The theory of relativity asserts (see, for example, Landau and Lifshitz, "The Classical Theory of Fields", Pergamon Press, 1961, the bottom of p. 38) that "because of the isotropy of space and time, the Lagrangian does not change under a rotation in four-space". This assertion is not true, as the Lagrange energy (consider for simplicity's sake the time energy of a free particle) is not a 4-dimensional scalar and is not a Lorentz invariant quantity. Thus, under the relativistic approach, the Lagrangian changes under a rotation in 4-space.

Since under a 4-rotation there change the components both of the radius vectors and the velocities, we shall have

$$dL = \sum_{i=1}^{n} \left(\frac{\partial L}{\partial \vec{r}_i} . d\vec{r}_i + \frac{\partial L}{\partial \vec{v}_i} . d\vec{v}_i\right) = \sum_{i=1}^{n} \left(\frac{\partial L}{\partial r^i_\beta} dr^i_\beta + \frac{\partial L}{\partial v^i_\beta} dv^i_\beta\right) = 0. \qquad (28.19)$$

The index "i" shows the number of the material point of the system (i = 1,2,.....n), while the index "β" shows the number of the component (β = 1,2,3,4) and is submitted to the summation convention.

Using formulas (16) and (17), we can write (19) in the form

$$d\phi_{\alpha\beta}\sum_{i=1}^{n}\left(\frac{\partial L}{\partial r_{\beta}^{i}} r_{\alpha}^{i} + \frac{\partial L}{\partial v_{\beta}^{i}} v_{\alpha}^{i}\right) = 0. \tag{28.20}$$

Substituting here (24.34) and (24.33), we get

$$d\phi_{\alpha\beta}\sum_{i=1}^{n}(r_{\alpha}^{i}\dot{p}_{o\beta}^{i} + \dot{r}_{\alpha}^{i}p_{o\beta}^{i}) = 0, \tag{28.21}$$

or

$$\frac{d}{dt}\{d\phi_{\alpha\beta}\sum_{i=1}^{n} r_{\alpha}^{i}p_{o\beta}^{i}\} = 0. \tag{28.22}$$

Recalling that every tensor $\overleftrightarrow{A}$ can be presented as a sum of both a symmetric and an antisymmetric tensor according to the relation (see I, p. 239)

$$A_{\alpha\beta} = \frac{1}{2}(A_{\alpha\beta} + A_{\beta\alpha}) + \frac{1}{2}(A_{\alpha\beta} - A_{\beta\alpha}), \tag{28.23}$$

we can write formula (22) in the form

$$\frac{1}{2}\,\frac{d}{dt}\{d\phi_{\alpha\beta}\sum_{i=1}^{n}(r_{\alpha}^{i}p_{o\beta}^{i} + r_{\beta}^{i}p_{o\alpha}^{i})\} + \frac{1}{2}\,\frac{d}{dt}\{d\phi_{\alpha\beta}\sum_{i=1}^{n}(r_{\alpha}^{i}p_{o\beta}^{i} - r_{\beta}^{i}p_{o\alpha}^{i}) = 0. \tag{28.24}$$

The first term on the left side is a scalar product of antisymmetric tensor (see (11)) and a symmetric tensor, and such a product is equal to zero (see I, p. 241). Therefore we can write (24) in the form

$$\frac{1}{2}\,d\phi_{\alpha\beta}\frac{d}{dt}\sum_{i=1}^{n}(r_{\alpha}^{i}p_{o\beta}^{i} - r_{\beta}^{i}p_{o\alpha}^{i}) = 0. \tag{28.25}$$

From here, having in mind that the elementary tensor of rotation $d\overleftrightarrow{\phi}$ is an arbitrary quantity, we obtain

$$\frac{d}{dt}\sum_{i=1}^{n}(r_{\alpha}^{i}p_{o\beta}^{i} - r_{\beta}^{i}p_{o\alpha}^{i}) = 0. \tag{28.26}$$

Thus at the motion of an isolated material system the tensor quantity

$$\overleftrightarrow{L}_{o} = \sum_{i=1}^{n}(\overleftrightarrow{r}_{i}\overleftrightarrow{p}_{oi} - \overleftrightarrow{p}_{oi}\overleftrightarrow{r}_{i}), \tag{28.27}$$

representing the difference of the dyadic products of the 4-vectors $\overleftrightarrow{r}_{i}$ and $\overleftrightarrow{p}_{oi}$ summed over all particles of the system, remains constant, i.e.,

$$\overleftrightarrow{L}_{o} = \overleftrightarrow{Const}. \tag{28.28}$$

xxxxx

The antisymmetric 4-tensor $\overleftrightarrow{L}_o$ is called the 4-ANGULAR MOMENTUM. The 4-angular momentum $\overleftrightarrow{L}_o$ of a system is the sum of the 4-angular momenta

$$\overleftrightarrow{T}_{oi} = \overleftrightarrow{r}_i \overleftrightarrow{p}_{oi} - \overleftrightarrow{p}_{oi} \overleftrightarrow{r}_i, \qquad i = 1,2,\ldots..n, \tag{28.29}$$

of all its particles.

We can easily establish that the space components of the 4-angular momentum represent the components of the PROPER ANGULAR MOMENTUM (see (IIB,12.8))

$$\overleftrightarrow{L}_o \equiv \sum_{i=1}^{n} \overleftrightarrow{T}_{oi} = \sum_{i=1}^{n} \vec{r}_i \times \vec{p}_{oi}, \tag{28.30}$$

where $\overleftrightarrow{T}_{oi}$ are the proper angular momenta of the single particles. $\overleftrightarrow{L}_o$ represents an antisymmetric 3-tensor of second rank (see (14.14) and (14.15)) and thus can be considered as an axial vector, as we did in IIB, §12.

The relations between the components of the space part of the 4-angular momentum and the components of the 3-vector of the proper angular momentum are as follows (cf. (I,67.31))

$$L_{ox} = L_{oyz} = - L_{ozy}, \qquad L_{oy} = L_{ozx} = - L_{oxz}, \qquad L_{oz} = L_{oxy} = - L_{oyx}. \tag{28.31}$$

The time-space components of the 4-tensor $\overleftrightarrow{L}_o$ are

$$L_{o4a} = i \sum_{i=1}^{n} (ctp_{oa}^{i} - r_a^{i} \frac{e_o^{i}}{c}). \tag{28.32}$$

Therefore, they are components of the 3-vector (see (14.8))

$$\tilde{\vec{L}}_o = \sum_{i=1}^{n} (ct\vec{p}_{oi} - \vec{r}_i \frac{e_{oi}}{c}). \tag{28.33}$$

On the grounds of (28), this vector is constant in time. Since the energy H of the whole system is also constant, we can write

$$\frac{c}{H} \tilde{\vec{L}}_o = \frac{c^2}{H} \sum_{i=1}^{n} \vec{p}_{oi} t - \frac{1}{H} \sum_{i=1}^{n} e_{oi} \vec{r}_i = \vec{Const}. \tag{28.34}$$

If we denote

$$\vec{R} = \frac{1}{H} \sum_{i=1}^{n} e_{oi} \vec{r}_i \tag{28.35}$$

and (see (26.11)

$$\vec{V} = \frac{c^2}{H} \sum_{i=1}^{n} \vec{p}_{oi} = \frac{c^2}{H} \vec{P}_o = \vec{Const}, \tag{28.36}$$

we obtain

$$\vec{R} = \vec{V}t - \vec{Const}, \tag{28.37}$$

i.e., the fictitious point, called CENTER OF INERTIA, whose radius vector is $\vec{R}$, moves with a constant velocity $\vec{V}$; thus $\vec{V}$ can be considered as the velocity of the whole system (cf. IIB, §11).

If the velocities of all particles of the system are low, we can write

$$\sum_{i=1}^{n} e_{oi}\vec{r}_i \cong c^2 \sum_{i=1}^{n} m_i\vec{r}_i, \tag{28.38}$$

and

$$H \cong Mc^2, \tag{28.39}$$

where M is the mass of the whole system considered as a particle.

From (35), (38), and (39) we obtain the low-velocity value of the radius vector of the center of inertia (see (IIB,11.8))

$$\vec{R} = \sum_{i=1}^{n} m_i\vec{r}_i/M, \tag{28.40}$$

but we must not forget that if $U \neq 0$, inequality (26.15) will be found.

xxxxx

Using formulas (14.17), we can write the transformation formulas for the space components of the 4-angular momentum under special Lorentz transformation as follows

$$L_{oxy} = \frac{L'_{oxy} + \frac{V}{c}L'_{oty}}{(1 - V^2/c^2)^{1/2}}, \qquad L_{oxz} = \frac{L'_{oxz} + \frac{V}{c}L'_{otz}}{(1 - V^2/c^2)^{1/2}}, \qquad L_{yz} = L'_{yz}. \tag{28.41}$$

If we assume that the system as a whole is at rest with respect to frame K', we shall have

$$\vec{P}'_o = M\vec{V}' = 0, \tag{28.42}$$

and if we assume moreover

$$\vec{R}' = 0, \tag{28.43}$$

i.e., if we put the center of inertia at the origin of frame K', on the grounds of (33), we obtain

$$\tilde{\vec{L}}'_o = (L'_{otx}, L'_{oty}, L'_{otz}) = 0. \tag{28.44}$$

In this case the transformation formulas (41) will be reduced to the following

$$L'_{oxy} = L_{oxy}(1 - V^2/c^2)^{1/2}, \qquad L'_{oxz} = L_{oxz}(1 - V^2/c^2)^{1/2}, \qquad L'_{oyz} = L_{oyz}. \tag{28.45}$$

Chapter V

CANONICAL EQUATIONS

§29. HAMILTON'S EQUATIONS

Let us work with s generalized coordinates q connected with the Cartesian coordinates according to formulas (IIB,7.17).

When in formulas (IIB,10.14) we write instead of the low-velocity Lagrangian (II,7.10) the high-velocity Lagrangian (24.26), we obtain the PROPER GENERALIZED MOMENTA

$$p_{ok} = \partial L/\partial \dot{q}_k, \qquad k = 1,2,.....s. \tag{29.1}$$

The PROPER GENERALIZED KINETIC FORCES are

$$f_{ok} = \dot{p}_{ok}, \qquad k = 1,2,.....s. \tag{29.2}$$

The GENERALIZED POTENTIAL FORCES are the same as in low-velocity mechanics

$$F_k = \partial L/\partial q_k, \qquad k = 1,2,.....s. \tag{29.3}$$

With the generalized forces Newton's second law (the Lagrange equations) can be written in the form

$$\dot{p}_{ok} = \partial L/\partial q_k, \qquad k = 1,2,.....s. \tag{29.4}$$

xxxxx

The Hamilton equations can be introduced in high-velocity mechanics exactly in the same manner as in low-velocity mechanics. As an exercise we shall repeat the whole deduction.

Since the Lagrangian depends on the generalized coordinates and velocities, its total differential will be

$$dL = \sum_{k=1}^{s} \frac{\partial L}{\partial q_k} dq_k + \sum_{k=1}^{s} \frac{\partial L}{\partial \dot{q}_k} d\dot{q}_k. \tag{29.5}$$

Making use of Newton's equations (4) and of formulas (1), we can write (5) in the form

$$dL = \sum \dot{p}_{ok} dq_k + \sum p_{ok} d\dot{q}_k, \tag{29.6}$$

or

$$d(\sum p_{ok}\dot{q}_k - L) = -\sum \dot{p}_{ok} dq_k + \sum \dot{q}_k dp_{ok}. \tag{29.7}$$

Considering H as a function of q and p_o, we obtain from here the HAMILTON (or CANONICAL) EQUATIONS, as according to (26.31) the left side of (7) is the differential of H,

$$\dot{q}_k = \frac{\partial H}{\partial p_{ok}}, \qquad \dot{p}_{ok} = -\frac{\partial H}{\partial q_k}, \qquad k = 1,2,.....s. \tag{29.8}$$

The canonical transformations can be introduced in high-velocity mechanics in the same manner as in low-velocity mechanics (see IIB, §19).

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The Poisson brackets can be defined in high-velocity mechanics in the same manner as in low-velocity mechanics (see IIB, §20).

In high-velocity mechanics we can work in a phase space as in low-velocity mechanics, and the deduction of the Liouville theorem is almost the same (see IIB, §22).

§30. HAMILTON-JACOBI'S EQUATION

Let us write equation (27.8) in the form

$$H(q_1,.....q_s,p_{o1},.....p_{os},t) + \frac{\partial S}{\partial t} = 0, \tag{30.1}$$

supposing that the considered system is not isolated (i.e., its energy may depend on time), and operating with generalized coordinates.

The HAMILTON-JACOBI EQUATION is obtained if in (1) we substitute all proper generalized momenta by the partial derivatives of action S with respect to the corresponding generalized coordinates according to formulas (cf. (27.6))

$$p_{ok} = \partial S/\partial q_k, \qquad k = 1,2,.....s. \tag{30.2}$$

So we obtain

$$H(q_1,.....q_s,\frac{\partial S}{\partial q_1},.....\frac{\partial S}{\partial q_s},t) + \frac{\partial S}{\partial t} = 0. \tag{30.3}$$

If the Hamiltonian H does not depend on time, the action is given by formula (27.11), and the Hamilton-Jacobi equation is to be written in the form

$$H(q_1,.....q_s,\frac{\partial S_c}{\partial q_1},.....\frac{\partial S_c}{\partial q_s}) - H = 0, \tag{30.4}$$

where S_c is the curtailed action.

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The Hamilton-Jacobi equation for one non-isolated particle can be obtained by substituting (27.10) into (26.25)

$$(\frac{\partial S}{\partial x})^2 + (\frac{\partial S}{\partial y})^2 + (\frac{\partial S}{\partial z})^2 - \frac{1}{c^2}(\frac{\partial S}{\partial t})^2 + m^2c^2 = 0. \tag{30.5}$$

Let us, for low velocity of the particle, make a transition from equation (5) to the corresponding Hamilton-Jacobi equation in low-velocity mechanics (cf. (IIB,23.2))

$$\frac{1}{2m}\{(\frac{\partial S'}{\partial x})^2 + (\frac{\partial S'}{\partial y})^2 + (\frac{\partial S'}{\partial z})^2\} + \frac{\partial S'}{\partial t} = 0. \tag{30.6}$$

For low velocities, the high-velocity Lagrangian L is related with the low-velocity Lagrangian L' by the formula

$$L = - mc^2 + L'. \tag{30.7}$$

Hence, according to formula (27.1), the high-velocity action S is related with the low-velocity action S' by the formula

$$S = - mc^2 + S'. \tag{30.8}$$

Substituting (8) into (5), we get

$$\left(\frac{\partial S'}{\partial x}\right)^2 + \left(\frac{\partial S'}{\partial y}\right)^2 + \left(\frac{\partial S'}{\partial z}\right)^2 + 2m\frac{\partial S}{\partial t} - \frac{1}{c^2}\left(\frac{\partial S'}{\partial t}\right)^2 = 0, \tag{30.9}$$

and by putting $c \to \infty$, we come to equation (6).

§31. PARTICLES AND WAVES

A. The de Broglie relations

In our third axiom (see IIA, p. 12) we introduced the quantities wave-vector, wave-scalar, wavelength, and wave-period of a particle, which we call its WAVE CHARACTERISTICS. The space and time momentum are called its PARTICLE CHARACTERISTICS. In this section we shall consider the wave characteristics of the particles.

The relations between the proper and universal wave-vectors and wave-scalars of a particle are (see (IIA,2.6), (17.8), and (19.3))

$$\vec{k}_o = \frac{m\vec{v}_o}{h} = \frac{m\vec{v}}{h(1 - v^2/c^2)^{1/2}} = \frac{\vec{k}}{(1 - v^2/c^2)^{1/2}},$$

$$\bar{k}_o = \frac{mc_o}{h} = \frac{mc}{h(1 - v^2/c^2)^{1/2}} = \frac{\bar{k}}{(1 - v^2/c^2)^{1/2}}. \tag{31.1}$$

The relations between the proper and universal wavelengths and wave-periods are

$$\lambda_o = \frac{1}{k_o} = \frac{h}{mv_o} = \frac{h}{mv}(1 - v^2/c^2)^{1/2} = \lambda(1 - v^2/c^2)^{1/2},$$

$$\tau_o = \frac{1}{c\bar{k}_o} = \frac{h}{mcc_o} = \frac{h}{mc^2}(1 - v^2/c^2)^{1/2} = \tau(1 - v^2/c^2)^{1/2}. \tag{31.2}$$

The magnitude k of the wave-vector is called the WAVE-NUMBER. If we denote by $\vec{n}$ the unit vector along the direction of propagation of the particle, its wave-vector can be expressed by its wave-number and wavelength as follows

$$\vec{k}_o = k_o\vec{n} = \frac{1}{\lambda_o}\vec{n} \qquad \left(\vec{k} = k\vec{n} = \frac{1}{\lambda}\vec{n}\right). \tag{31.3}$$

The reciprocal value of the wave-period is called the WAVE-FREQUENCY (in short, FREQUENCY) of the particle

$$\nu_o = 1/\tau_o = c\bar{k}_o \qquad (\nu = 1/\tau = c\bar{k}). \tag{31.4}$$

The relations between the space momentum and the wave-vector (wave-length) and between the time momentum and the wave-scalar (wave-frequency) are the following

$$\vec{p}_0 = h\vec{k}_0 = \frac{h\vec{n}}{\lambda_0} = \frac{h\bar{k}_0}{c}\,\vec{v} = \frac{h\nu_0}{c^2}\,\vec{v},$$

$$\bar{p}_0 = h\bar{k}_0 = \frac{h\nu_0}{c} = \frac{hc}{v}\,k_0 = \frac{hc^2}{\lambda_0 v}. \qquad (31.5)$$

These are the famous DE BROGLIE RELATIONS.

The relation between the energy of the particle and its frequency is the following

$$e_0 \equiv \frac{mc^2}{(1 - v^2/c^2)^{1/2}} = h\nu_0. \qquad (31.6)$$

The relation between the frequency and the wavelength is the following

$$\nu_0\lambda_0 = c^2/v \qquad (\nu\lambda = c^2/v). \qquad (31.7)$$

The name PHASE VELOCITY is often used for the quantity

$$v_{ph} = c^2/v, \qquad (31.8)$$

and when considering the wave character of the particles the name GROUP VELOCITY is often attached to the ordinary velocity v. The phase velocity is always larger than c and for a particle at rest becomes equal to infinity. Only for photons (i.e., for particles moving with the velocity of light) the phase velocity becomes equal to the group velocity, and thus equal to the velocity of propagation of the photons. In our theory the phase velocity has no physical meaning.

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Taking into account that the space and time momentum form a 4-vector, we see that the quantity

$$\overleftrightarrow{k}_0 = (\vec{k}_0, i\bar{k}_0) = \left(\frac{\nu_0}{c^2}\vec{v},\ i\frac{\nu_0}{c}\right) = \left(\frac{\vec{n}}{\lambda_0},\ i\frac{c}{v\lambda_0}\right) \qquad (31.9)$$

represents a 4-vector which we call the 4-WAVE-VECTOR.

The 4-wave-vector is a material 4-vector and thus it must satisfy the fundamental 4-vector equation (15.5)

$$\frac{d\nu_0}{dt} = \frac{\vec{v}}{c^2}.\frac{d}{dt}(\nu_0\vec{v}). \qquad (31.10)$$

Indeed, performing the differentiation, we get

$$(1 - v^2/c^2)\,\frac{d\nu_0}{dt} = \frac{\nu_0}{c^2}\,\vec{v}.\vec{u}, \qquad (31.11)$$

and using here (6), we obtain the identity

$$\frac{de_0}{dt} = \frac{m\vec{v}.\vec{u}}{(1 - v^2/c^2)^{3/2}}. \qquad (31.12)$$

xxxxx

According to our conceptions, the wave character of a particle cannot be explained referring to waves in material media (say, water waves). First of all the particles propagate in vacuum. Thus there is no material substance in which the "wave process" can take place. The artificial ad hoc introduction of such a hypothetical substance (to which, when considering light phenomena, the name AETHER was given) has caused many confusions in physics.

The mathematical parallels which Fresnel established between the light interference and difraction phenomena and the same phenomena for water and acoustic waves, have forced the scientific community to accept the aether as a physical reality. For this reason the interference and diffraction phenomena of massive particles discovered in the twenties of this century have been accepted as a great puzzle, as the massive particles, obviously, do not represent vibrations of the "aether".

Both, the photoeffect (i.e., the extrication of electrons when illuminating metals) and the Compton effect (see §35) heavily attacked the aether conceptions. Another blow against the aether was struck by Einstein's theory of relativity which asserts that there is no possibility of establishing experimentally the existence of the aether. However, our high-velocity experiments (see ch. VIII) showed that in a laboratory moving in absolute space the velocity of light is direction dependent, and certain scientists consider our experiments as a physical confirmation of the aether. We disagree with such opinions and object firmly to the aether conceptions.

As we show in this book, the light velocity direction dependence in a moving laboratory is due to the attachment of the photons to absolute space through their proper masses. All massive particles are attached to absolute space in the same manner, and their motion in moving laboratories is also direction dependent (see §44). The attachment of the particles to absolute space is due to the fact that the time energies of the particles represent their gravitational energy with the mass of the whole world (see p. 69). Thus our experiments must in no way be considered as an experimental evidence leading to a resurrection of the aether conceptions.

How then can one explain the wave character of the particles? According to us, the wave character of the massive and massless particles is to be explained by the phenomenon INTERFERENCE to which any two particles are subjected when they are close enough to one another in space and time. The phenomenon "interference" is to be considered in quantum physics. On this topic our third axiom, which is classical, says only the following: Any particle has a certain extention is space (for a given moment) determined by its proper wavelength λ_0 and a certain duration in time (for a given space point) determined by its proper wave-period τ_0. The particles are independent of one another if they are separated in space and time over "distances" larger than λ_0 and τ_0. If they are within those space and time intervals, the phenomenon "interference" appears which is postulated and investigated in quantum physics.

The waves attached to massive particles are called DE BROGLIE WAVES. (N.B. The expression "wave attached to a particle" is not good, as the particles simply display a certain periodicity, but following the historical tradition we also use this expression.) The term "de Broglie wave" was introduced because the waves attached to photons and electromagnetic quanta (i.e., to massless particles) were supposed to be substantially different from the waves attached to electrons and neutrons (i.e., to massive particles). We do not make a qualitative difference between massive and massless particles (see §31C) and thus we do not make a qualitative difference between light waves and de Broglie waves.

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From equation (2) we obtain the following relation

$$\lambda_o = v(\tau_o c^2/v^2). \tag{31.13}$$

As v is the velocity of the particle, the time in which the particle covers a distance equal to its wavelength λ_o is (see (2))

$$\tau^* = \frac{c^2}{v^2}\tau_o = \frac{h}{mv^2}(1 - v^2/c^2)^{1/2}. \tag{31.14}$$

For low velocities ($v \to 0$) this formula results in $\tau^* \to \infty$. For high velocities ($v \to c$) it gives $\tau^* \to \tau_o$.

We see that the wavelength sets the limits of the "space extention" of the particle along its direction of motion, while the wave-period sets the limits for its "time duration"; thus the 4-wave-vector sets the limits for the "space-time extention" of the particle. Obviously, the particles with short wavelengths and small wave-periods are more concentrated in space-time, i.e., they have a more obvious "particle character", while those ones with long wavelengths and large wave-periods are more extended in space-time, i.e., their "wave character" is more apparent.

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Now we shall show the validity of the following relation, known under the name of REYLEIGH'S FORMULA

$$v = v_{ph} - \lambda_o \frac{dv_{ph}}{d\lambda_o}, \tag{31.15}$$

where the phase velocity v_{ph} is given by (8).

Indeed, substituting the first formula (5) and formula (6) into (26.25), we obtain

$$\nu_o = c\left(\frac{1}{\lambda_o^2} + \frac{m^2c^2}{h^2}\right)^{1/2}. \tag{31.16}$$

Taking into account (7) and (8), we find

$$v_{ph} = c(1 + m^2c^2\lambda_o^2/h^2)^{1/2}, \qquad v = \frac{c}{(1 + m^2c^2\lambda_o^2/h^2)^{1/2}}, \tag{31.16}$$

and substituting these expressions in (15), we arrive at an identity.

B. The Heisenberg uncertainty relations

Now we shall search for the limits in which the dynamic characteristis of a particle (momentum, energy) can be measured.

First we shall show with which precision the particle's energy can be established. The energy of a particle can be found by measuring the wave-frequency ν_0. Consequently let us see with which precision the frequency ν_0 can be measured. To do this, we present a "time snap-shot" of identical particles which cross a certain space-point in fig. 37.1, where the time axis is taken as the abscissa. The black circles formally represent the particles and the diameters are equal to their proper wave-periods τ_0. The particles must follow one after another with time τ_0, i.e., the black circles must follow tightly one after another (in the figure, for clarity's sake, wide spaces are left between the circles). Counting the number of the particles which have crossed the space point in a unit of time, one finds their frequency ν_0. Let us suppose that N particles have crossed the space point for a time Δt. As it can be seen from the figure, during time Δt one will count N ± 1 par-

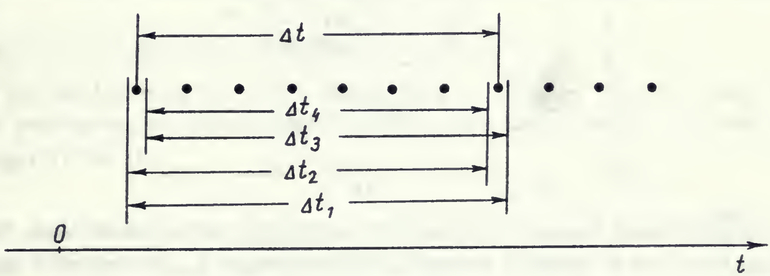

Fig. 31.1

ticles. Indeed, as the time in which a particle crosses a space point is equal to its wave-period, and one can always count a whole number of particles, we shall have (see the figure)

$$\Delta t \cong \Delta t_1 \cong \Delta t_2 \cong \Delta t_3 \cong \Delta t_4, \tag{31.17}$$

but in the time Δt_1 one counts N + 1 particles, in times Δt_2 and Δt_3 one counts N particles, and in time Δt_4 one counts N - 1 particles. Thus if the number of particles which one counts in the time Δt is N, the actual number, which we shall denote by $\bar{N}$, will be such a one that

$$\bar{N} = N \pm 1. \tag{31.18}$$

Taking into account that

$$\bar{N} = \Delta t/\bar{\tau}_0 = \Delta t\, \bar{\nu}_0 , \qquad N = \Delta t/\tau_0 = \Delta t\, \nu_0 , \tag{31.19}$$

where $\bar{\tau}_0$ and $\bar{\nu}_0$ are the actual wave-period and frequency of the particles, while τ_0 and

ν_0 are the measured ones, we obtain

$$(\nu_0 - \bar{\nu}_0)\Delta t = \pm 1. \tag{31.20}$$

Multiplying this equation by the Planck constant, we obtain, having in mind formula (6

$$(e_0 - \bar{e}_0)\Delta t = \pm h. \tag{31.21}$$

Denoting $\Delta e_0 = e_0 - \bar{e}_0$, we find eventually

$$\Delta e_0 \Delta t = \pm h. \tag{31.22}$$

This relation shows that when we want to measure the energy of a particle with a highe precision, we have to do this for a longer interval of time. The limits set by formula (22) cannot be surpassed: they are set by nature herself, and when these limits are achieved, any further refinement of the experimental techniques is senseless.

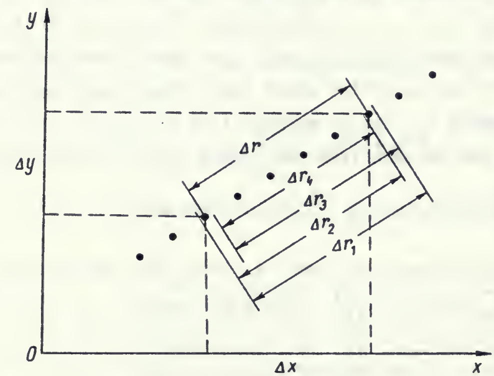

Fig. 31.2

Now we shall show with which precision the particle's momentum can be established. The momentum of a particle can be found by measuring the wave-vector $\vec{k}_0$. Consequently let us see with which precision the wave-vector $\vec{k}_0$ can be measured. To do this, in fig. 37.2, where for the sake of simplicity a two-dimensional case is shown, we present a "space snap-shot" of identical particles which proceed along a certain direction. The black circles represent the particles, and the diameters are equal to their proper wavelengths λ_0. The particles must follow one after another in a distance λ_0, i.e., the black circles must be placed tightly one after another (in the figure, for clarity's sake, wides spaces are left between the circles). Counting the number of particles over a unit of length, one finds their wave number k_0.

In the same way as above we can show that if one counts N particles along the distance Δr, the actual number $\bar{N}$ will be such a one that relation (18) is valid. Taking into account that

$$\bar{N} = \Delta r/\bar{\lambda}_0 = \Delta r\, \bar{k}_0, \qquad N = \Delta r/\lambda = \Delta r\, k_0, \tag{31.23}$$

where $\bar{\lambda}_0$ and $\bar{k}_0$ are the actual wavelength and wave-number of the particle, while λ_0 and k are the measured ones, we obtain

$$(\hbar_o - \bar{\hbar}_o)\Delta r = \pm 1. \qquad (31.24)$$

Multiplying this equation by the Planck constant, we obtain, having in mind the first formula (5)

$$(p_o - \bar{p}_o)\Delta r = \pm h. \qquad (31.25)$$

Denoting $\Delta p_o = p_o - \bar{p}_o$, we find eventually

$$\Delta p_o \Delta r = \pm h. \qquad (31.26)$$

This formula shows that when we want to measure the momentum of a particle with a higher precision, we have to do this over a longer space interval. The limits set by formula (26) cannot be surpassed: they are set by nature herself, and when these limits are achieved, any further refinement of the experimental techniques is senseless.

It is clear from fig. 31.2 that when we measure the components of the wave-vector (and respectively the components of the momentum), then formula (26) is to be replaced by the following three formulas

$$\Delta p_{ox}\Delta x = \pm h, \qquad \Delta p_{oy}\Delta y = \pm h, \qquad \Delta p_{oz}\Delta z = \pm h. \qquad (31.27)$$

Formulas (22) and (27) represent the famous HEISENBERG'S UNCERTAINTY RELATIONS.

C. Massive and massless particles

If the considered particle is a photon ($v = c$), then, as the proper wavelength and the proper wave-period are finite quantities, we obtain from formulas (2) and from the second formulas (3) and (4)

$$\lambda = \infty, \qquad \tau = \infty, \qquad \hbar = 0, \qquad \nu = 0. \qquad (31.28)$$

Thus some of the photon's universal characteristics become equal to zero (i.e., they are equal for all photons) and some become equal to infinity (i.e., they have no mathematical sense). On the other hand, relations (2) show that the universal masses of the photons are equal to zero. So we can give exact definition of the notions massless and massive particles, already used on p. 93: Any particle (as the photon) with universal mass equal to zero is called MASSLESS PARTICLE, while any particle (as the electron) with universal mass different from zero is called MASSIVE PARTICLE.

Remark. We shall only work with those photon characteristics which are different from zero and have mathematical sense. Thus we shall not make use of the photon's universal quantities λ, τ, $\hbar$, ν, and we shall write the proper quantities λ_o, τ_o, $\hbar_o$, ν_o for photons without the index "o".

Hence for a photon formulas (5) and (6) will be written in the form

$$\vec{p}_o = h\vec{\hbar}, \qquad e_o = h\nu, \qquad (31.29)$$

and the first formula (7) obtains the form

$$\nu\lambda = c. \qquad (31.30)$$

The 4-wave-vector for a photon will be written (see formula (9))

$$\overset{\leftrightarrow}{k} = (\vec{k}, i\bar{k}) = (\frac{\nu}{c}\vec{n}, i\frac{\nu}{c}) = (\frac{1}{\lambda}\vec{n}, i\frac{1}{\lambda}). \qquad (31.31)$$

The fundamental 4-vector equation (15.5), applied to the photon 4-wave-vector, runs

$$\frac{1}{c}\frac{d\nu}{dt} = \vec{n}.\frac{d\vec{k}}{dt}, \qquad (31.32)$$

which represents an identity. On the other hand, from (31) we obtain immediately

$$\overset{\leftrightarrow}{k}^2 = 0, \qquad (31.33)$$

while for massive particles we have from (9)

$$\overset{\leftrightarrow}{k}_o^2 = - \frac{\nu_o^2}{c^2}(1 - v^2/c^2). \qquad (31.34)$$

Let us write the Hamilton equations of motion for a photon. For this purpose we write equations (29.8) in Cartesian coordinates

$$\dot{\vec{r}} = \partial H/\partial \vec{p}_o, \qquad \dot{\vec{p}}_o = - \partial H/\partial \vec{r}. \qquad (31.35)$$

Putting here $H = e_o$ and substituting the expressions for $\vec{p}_o$ and e_o from (29), we get

$$\dot{\vec{r}} = \partial\nu/\partial\vec{k}, \qquad \dot{\vec{k}} = - \partial\nu/\partial\vec{r}. \qquad (31.36)$$

xxxxx

Parallel to the frequency ν, which is called LINEAR FREQUENCY, we consider also the so-called ANGULAR FREQUENCY

$$\omega = 2\pi\nu, \qquad (31.37)$$

which is introduced for mathematical convenience.

The wave-vector $\vec{k}$ will be called the LINEAR WAVE-VECTOR, while

$$\vec{k} = 2\pi\vec{k} \qquad (31.38)$$

will be called the ANGULAR WAVE-VECTOR.

The Planck constant h will be called the LINEAR PLANCK CONSTANT, while

$$\hbar = h/2\pi \qquad (31.39)$$

will be called the ANGULAR PLANCK CONSTANT.

Chapter VI

C O L L I S I O N S

§32. DISINTEGRATION OF PARTICLES

Let us consider the SPONTANEOUS DISINTEGRATION (see IIB, p. 120) of a particle with mass M into two particles m_1 and m_2 from a high-velocity point of view.

The law of energy conservation used in the c-frame (center-of-mass reference frame) gives

$$Mc^2 = (e_{o1})_c + (e_{o2})_c, \tag{32.1}$$

where $(e_{o1})_c$, $(e_{o2})_c$ are the energies of the first and second daughter particles. Further on in this chapter (as in IIB, ch. V) we shall denote all quantities considered in the c-frame by the index "c".

One must have

$$(e_{o1})_c > m_1c^2, \qquad (e_{o2})_c > m_2c^2, \tag{32.2}$$

and therefore inequality (26.20) must be valid.

If

$$M < m_1 + m_2, \tag{32.3}$$

the spontaneous disintegration (1) is not possible (see p. 80).

Beside the law (1), also the law of momentum conservation is valid

$$0 = (\vec{p}_{o1})_c + (\vec{p}_{o2})_c, \tag{32.4}$$

from where

$$(p_{o1})_c^2 = (p_{o2})_c^2. \tag{32.5}$$

Taking into consideration (26.26), we can write this equation in the form

$$(e_{o1})_c^2/c^2 - m_1c^2 = (e_{o2})_c^2/c^2 - m_2c^2. \tag{32.6}$$

From (1) and (6) the energies of the daughter particles can be found

$$(e_{o1})_c = \frac{M^2 + m_1^2 - m_2^2}{2M}c^2, \qquad (e_{o2})_c = \frac{M^2 - m_1^2 + m_2^2}{2M}c^2. \tag{32.7}$$

xxxxx

Now we shall analyse the process of spontaneous disintegration in an l-frame (laboratory reference frame) in which the initial particle M moves with a velocity $\vec{V}$.

Let us consider one of the particles obtained after the disintegration, denoting its mass by m and its energy in the c-frame by e_{oc}. The energy of this daughter particle in the l-frame will be denoted by e_o; as we have settled in IIB, p. 122, we denote the quantities considered in the l-frame without any index.

Let us denote by θ the angle between the velocity $\vec{v}$ of particle m in the l-frame and the velocity $-\vec{V}$ of the 1-frame with respect to the c-frame (since the velocoty of particle M in the 1-frame is $\vec{V}$, the velocity of the 1-frame with respect to the c-frame, i.e to the particle M, is $-\vec{V}$). Assuming that the transformation between the 1- and c-frames is a special one, we can write, using the last formula (26.24)

$$e_{oc} = \frac{e_o - Vp_o\cos\theta}{(1 - V^2/c^2)^{1/2}}. \qquad (32.8)$$

From here we find the dependence of angle θ on the energy e_o

$$\cos\theta = \frac{e_o - e_{oc}(1 - V^2/c^2)^{1/2}}{V(e_o^2/c^2 - m^2c^2)^{1/2}}. \qquad (32.9)$$

If we want to establish the dependence of e_o on angle θ, we have to solve the followir quadratic equation obtained from (9)

$$(1 - \frac{V^2}{c^2}\cos^2\theta)e_o^2 - 2e_{oc}(1 - \frac{V^2}{c^2})^{1/2}e_o + e_{oc}^2(1 - \frac{V^2}{c^2}) + m^2c^2V^2\cos^2\theta = 0. \qquad (32.10)$$

This equation has one positive root if $v_c > V$, or two positive roots if $v_c < V$, where v_c is the velocity of particle m in the c-frame.

These conclusions can be made on the grounds of the following considerations:

According to the first two of formulas (26.24), we have the following relations betwee the components of the momentum of particle m considered in the 1-frame and its momentum and energy considered in the c-frame (the velocity of the c-frame with respect to the 1-frame is $\vec{V}$, and we assume that the disintegration proceeds in the xy-plane)

$$p_{ox} = \frac{p_{oc}\cos\theta_c + \frac{V}{c^2}e_{oc}}{(1 - V^2/c^2)^{1/2}}, \qquad p_{oy} = p_{oc}\sin\theta_c, \qquad (32.11)$$

where θ_c is the angle between the velocity $\vec{v}_c$ of particle m in the c-frame and the velocity $\vec{V}$ of the c-frame with respect to the 1-frame.

Excluding frome these two equations θ_c, we obtain

$$\{p_{ox}(1 - V^2/c^2)^{1/2} - \frac{V}{c^2}e_{oc}\}^2 + p_{oy}^2 = p_{oc}^2. \qquad (32.12)$$

This equation can also be written in the form

$$\{v_{ox} - \frac{e_{oc}}{mc^2}\frac{V}{(1 - V^2/c^2)^{1/2}}\}^2 \frac{1}{\frac{v_{oc}^2}{1 - V^2/c^2}} + \frac{v_{oy}^2}{v_{oc}^2} = 1, \qquad (32.13)$$

which represents the equation of an ellipse (see (I,15.7)) with respect to the variables v_{ox} and v_{oy}. The semi-axes a and b of this ellipse are

$$a = \frac{v_{oc}}{(1 - V^2/c^2)^{1/2}}, \qquad b = v_{oc}, \qquad (32.14)$$

and its center O is shifted from point A, for which

$$v_{ox} = v_{oy} = 0, \tag{32.15}$$

over a distance

$$AO = \frac{e_{oc}V}{mc^2(1 - V^2/c^2)^{1/2}}. \tag{32.16}$$

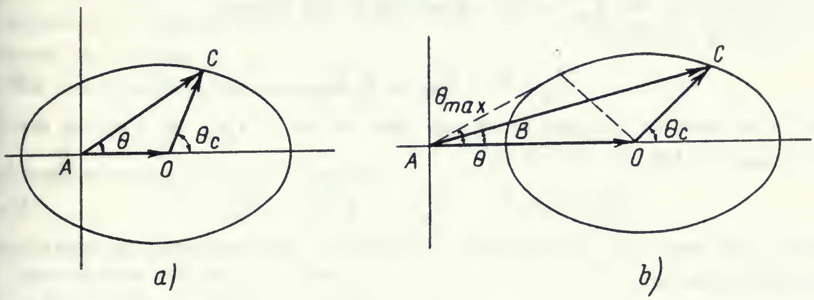

Fig. 32.1

We have (see fig. 32.1)

$$\vec{AC} = \vec{v}_o \equiv \frac{\vec{v}}{(1 - v^2/c^2)^{1/2}}, \qquad \vec{AO} = \frac{\vec{V}}{(1 - V^2/c^2)^{1/2}} \frac{1}{(1 - v_c^2/c^2)^{1/2}},$$

$$\vec{OC} = \frac{\vec{v}_c}{(1 - v_c^2/c^2)^{1/2}} \frac{1}{(1 - V^2\cos^2\theta_c/c^2)^{1/2}}, \tag{32.17}$$

where the last equality is written on the grounds of (14).

If the velocities are low with respect to light velocity, the ellipse in fig. 32.1 becomes a circle (see fig. IIB,28.1).

If $v_c > V$, point A lies outside the ellipse, and, for a given angle θ, velocity v_o (and energy e_o) can have two different values, because the radius of the ellipse $v_{oc}/(1 - V^2\cos^2\theta_c/c^2)^{1/2}$ can be drawn from the center of the ellipse to point C as well as to point B (see fig. 32.1b). It is clear that in this case angle θ cannot surpass a certain angle θ_{max}, when velocity v_o is tangent to the ellipse. The value of θ_{max} can be found if we equate to zero the discriminant of the quadratic equation (10), thus obtaining

$$\sin\theta_{max} = \frac{v_c}{V}\left(\frac{1 - V^2/c^2}{1 - v_c^2/c^2}\right)^{1/2}. \tag{32.18}$$

xxxxx

Now we shall consider the spontaneous disintegration of a particle when one of the daughter particles, say m_2, is a photon (i.e., $m_2 = 0$). This phenomenon can be called EMISSION OF A PHOTON FROM A PARTICLE. If we assume that the particle is an atom, this will be emission of a photon from an atom.

The law of 4-momentum conservation (26.11) used in the c-frame gives (further on in this section we do not write the index "c" denoting that the quantities are considered in the c-frame)

$$\overleftrightarrow{p}_o = \overleftrightarrow{p}_{o1} + \overleftrightarrow{p}_{o2}, \tag{32.19}$$

where $\overleftrightarrow{P}_o$ is the 4-momentum of the atom before the radiation, $\overleftrightarrow{p}_{o1}$ is the 4-momentum of the atom after the radiation, and $\overleftrightarrow{p}_{o2}$ is the 4-momentum of the photon.

From this equation we obtain

$$\overleftrightarrow{p}_{o2}^2 = \overleftrightarrow{P}_o^2 + \overleftrightarrow{p}_{o1}^2 - 2\overleftrightarrow{P}_o \cdot \overleftrightarrow{p}_{o1}. \tag{32.20}$$

If we denote the mass of the atom after the radiation by m (i.e., $m_1 = m$), we shall have on the grounds of (26.25) and (31.33)

$$\overleftrightarrow{P}_o^2 = - M^2c^2, \qquad \overleftrightarrow{p}_{o1}^2 = - m^2c^2, \qquad \overleftrightarrow{p}_{o2}^2 = 0. \tag{32.21}$$

Substituting this into (20) and denoting the energy of the atom after the radiation by e_o (i.e., $e_{o1} = e_o$), we get

$$2Me_o = M^2c^2 + m^2c^2, \tag{32.22}$$

from where

$$e_o = \frac{M^2 + m^2}{2M} c^2. \tag{32.23}$$

On the other hand the energy conservation law gives

$$Mc^2 = e_o + h\nu, \tag{32.24}$$

where ν is the frequency of the photon (see (31.29)).

From (23) and (24) we obtain

$$h\nu = \frac{M^2 - m^2}{2M} c^2. \tag{32.25}$$

If we write equation (23) in the form

$$\frac{m}{(1 - v^2/c^2)^{1/2}} = \frac{M^2 - m^2}{2M}, \tag{32.26}$$

and if we solve it with respect to m (keeping in mind that it must be $m < M$), we obtain

$$m = M\left(\frac{1 - v/c}{1 + v/c}\right)^{1/2}. \tag{32.27}$$

Substituting that into (25), we find eventually

$$h\nu = M\frac{vc^2}{v + c}. \tag{32.28}$$

From this relation we can determine the velocity v of the atom after the radiation, if the frequency of the photon is known, or, conversely, the frequency of the photon, if the velocity of the atom is known.

§33. INELASTIC COLLISION OF PARTICLES

The process when two particles collide, and after this collision only one particle emerges, is called INELASTIC COLLISION (or COALESCENCE) of particles.

Let m_1 and m_2 be the masses of two colliding particles before the collision and M the mass of the joint particle after the collision. Correspondingly let $\overleftrightarrow{v}_{o1}$ and $\overleftrightarrow{v}_{o2}$ be the 4-velocities of the colliding particles and $\overleftrightarrow{V}_o$ the 4-velocity of the joint particle considered in a 1-frame.

The law of 4-momentum conservation gives

$$M\overleftrightarrow{V}_o = m_1\overleftrightarrow{v}_{o1} + m_2\overleftrightarrow{v}_{o2}, \qquad (33.1)$$

and having in mind (19.4), we obtain

$$M^2 = m_1^2 + m_2^2 - 2m_1m_2 \frac{\overleftrightarrow{v}_{o1}\cdot\overleftrightarrow{v}_{o2}}{c^2}. \qquad (33.2)$$

Here we have the mass M of the joint particle in terms of known quantities. Then (1) gives $\overleftrightarrow{V}_o$, and the problem is solved.

xxxxx

Let us express M in (2) in terms of the universal 3-velocities $\vec{v}_1$ and $\vec{v}_2$

$$M^2 = m_1^2 + m_2^2 + \frac{2m_1m_2}{(1 - v_1^2/c^2)^{1/2}(1 - v_2^2/c^2)^{1/2}}(1 - \vec{v}_1.\vec{v}_2/c^2). \qquad (33.3)$$

If $v_1 \ll c$, $v_2 \ll c$, this gives approximately

$$M^2 = m_1^2 + m_2^2 + 2m_1m_2 + m_1m_2 \frac{v_1^2 + v_2^2 - 2\vec{v}_1.\vec{v}_2}{c^2} = (m_1 + m_2)^2 + m_1m_2 \frac{(\vec{v}_1 - \vec{v}_2)^2}{c^2}, \qquad (33.4)$$

or

$$M = m_1 + m_2 + \frac{1}{2} \frac{m_1m_2}{m_1 + m_2} \frac{v^2}{c^2}, \qquad (33.5)$$

where

$$\vec{v} = \vec{v}_1 - \vec{v}_2 \qquad (33.6)$$

is the vector of the mutual velocity of both particles (see IIB, p. 124).

The mass defect of the joint particle is (see (26.16))

$$\Delta M = M - (m_1 + m_2) = \frac{1}{2} \frac{m_1m_2}{m_1 + m_2} \frac{v^2}{c^2}. \qquad (33.7)$$

Let us compare the binding energy ΔMc^2 corresponding to this mass defect with the energy dissipating into "heat" at the inelastic collision in low-velocity mechanics.

The law of momentum conservation (IIB,10.4) gives

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{V}, \qquad (33.8)$$

since in low-velocity mechanics

$$m_1 + m_2 = M. \tag{33.9}$$

Now from (8) we obtain

$$(m_1 + m_2)^2 V^2 = m_1^2 v_1^2 + m_2^2 v_2^2 + 2m_1 m_2 \vec{v}_1 . \vec{v}_2. \tag{33.10}$$

Thus the loss of kinetic energy (dissipating into "heat") is

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2}(m_1 + m_2)V^2 = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} v^2, \tag{33.11}$$

where $\vec{v}$ is the vector of the mutual velocity given by (6), and we have used (10) in (11)

Comparing (11) with (7), we see that the binding energy of the joint particle obtained after the inelastic collision under high-velocity consideration is equal to the loss of kinetic energy at the inelastic collision under low-velocity consideration. In low-velocity mechanics one <u>assumes</u> (but until now <u>nobody</u> has confirmed this experimentally) that this loss of kinetic energy is transformed into "heat", which is a notion from the domain of statistical physics. According to us this treatment must be revised. The energy ballance at elastic and inelastic collision of particles must be performed remaining in the domain of classical physics and operating with the changes in the masses of the particles.

xxxxx

Now we shall consider the inelastic collision when one of the particles, say, m_2, is a photon (i.e., $m_2 = 0$). This phenomenon can be called ABSORPTION OF A PHOTON BY A PARTICLE (by an atom).

This process can be treated in a similar way as the process of emission of a photon from a particle (see p. 101).

The law of 4-momentum conservation used in the 1-frame, in which the atom is at rest before the collision, gives

$$\overset{\mapsto}{p}_{o1} + \overset{\mapsto}{p}_{o2} = \overset{\mapsto}{P}_o, \tag{33.12}$$

where $\overset{\mapsto}{p}_{o1}$ is the 4-momentum of the atom before the collision, $\overset{\mapsto}{p}_{o2}$ is the 4-momentum of the photon, and $\overset{\mapsto}{P}_o$ is the 4-momentum of the atom after the collision.

From here we obtain equation (32.20), and denoting the mass and the energy of the atom before the absorption correspondingly by m and e_o (i.e., $m_1 = m$, $e_{o1} = e_o$), we can write (32.20) in the form

$$2\, m\, E_o = M^2 c^2 + m^2 c^2, \tag{33.13}$$

from where

$$E_o = \frac{m^2 + m^2}{2m} c^2. \tag{33.14}$$

On the other hand, the energy conservation law gives

$$E_o = mc^2 + h\nu, \tag{33.15}$$

where ν is the frequency of the photon.

From (14) and (15) we obtain

$$h\nu = \frac{M^2 - m^2}{2\,m} c^2. \tag{33.16}$$

If we write equation (14) in the form

$$\frac{M}{(1 - V^2/c^2)^{1/2}} = \frac{M^2 + m^2}{2\,m}, \tag{33.17}$$

and if we solve it with respect to M (keeping in mind that it must be $M > m$), we obtain

$$M = m\left(\frac{1 + V/c}{1 - V/c}\right)^{1/2}. \tag{33.18}$$

Substituting this into (16), we find eventually

$$h\nu = m\,\frac{Vc^2}{c - V}. \tag{33.19}$$

From this equation we can determine the velocity V of the atom after the absorption, if the frequency of the photon is known, or, conversely, the frequency of the photon if the velocity of the atom is known.

§34. ELASTIC COLLISION OF PARTICLES

A. Description in the 1-frame

The process when two particles collide, and after this collision their masses remain the same, is called ELASTIC COLLISION of particles (cf. IIB, p. 123).

Let us consider the elastic collision of two particles with masses m_1, m_2, momenta $\vec{p}_{o1}$, $\vec{p}_{o2}$ and energies e_{o1}, e_{o2} from a high-velocity point of view. As in IIB, §29, we shall denote the values of the momenta and energies after the collision by primes.

The law of 4-momentum conservation gives

$$\overleftrightarrow{p}_{o1} + \overleftrightarrow{p}_{o2} = \overleftrightarrow{p}'_{o1} + \overleftrightarrow{p}'_{o2}. \tag{34.1}$$

Let us write this equation in the form

$$\overleftrightarrow{p}_{o1} + \overleftrightarrow{p}_{o2} - \overleftrightarrow{p}'_{o1} = \overleftrightarrow{p}'_{o2}, \tag{34.2}$$

and let us square it; after taking into account (26.25), we obtain

$$-m_1^2c^2 + \overleftrightarrow{p}_{o1}\cdot\overleftrightarrow{p}_{o2} - \overleftrightarrow{p}_{o1}\cdot\overleftrightarrow{p}'_{o2} - \overleftrightarrow{p}_{o2}\cdot\overleftrightarrow{p}'_{o1} = 0. \tag{34.3}$$

Analogically squaring the equation

$$\overleftrightarrow{p}_{o1} + \overleftrightarrow{p}_{o2} - \overleftrightarrow{p}'_{o2} = \overleftrightarrow{p}'_{o1}, \tag{34.4}$$

we obtain

$$-m^2c^2 + \overleftrightarrow{p}_{o1}\cdot\overleftrightarrow{p}_{o2} - \overleftrightarrow{p}_{o1}\cdot\overleftrightarrow{p}'_{o2} - \overleftrightarrow{p}_{o2}\cdot\overleftrightarrow{p}'_{o2} = 0. \tag{34.5}$$

Let us work in the 1-frame in which before the collision one of the particles (say, particle m_2) is at rest. In this case we have

$$\vec{p}_{o2} = 0, \qquad e_{o2} = m_2c^2, \tag{34.6}$$

and we can write the three scalar products in (3) in the form

$$\overleftrightarrow{p}_{o1}\cdot\overleftrightarrow{p}_{o2} = -m_2e_{o1}, \qquad \overleftrightarrow{p}_{o2}\cdot\overleftrightarrow{p}'_{o1} = -m_2e'_{o1}, \qquad \overleftrightarrow{p}_{o1}\cdot\overleftrightarrow{p}'_{o1} = p_{o1}p'_{o1}\cos\theta_1 - \frac{e_{o1}e'_{o1}}{c^2}, \tag{34.7}$$

where θ_1 is the angle between the velocities of the first particle before and after the collision.

Substituting (7) into (3), we obtain

$$\cos\theta_1 = \frac{e'_{o1}(e_{o1}+m_2c^2) - m_2c^2e_{o1} - m_1^2c^4}{c^2p_{o1}p'_{o1}}. \tag{34.8}$$

The three scalar products in (5) can be written in the same 1-frame (i.e., at the conditions (6)) in the form

$$\overleftrightarrow{p}_{o1}\cdot\overleftrightarrow{p}_{o2} = -m_2e_{o1}, \qquad \overleftrightarrow{p}_{o2}\cdot\overleftrightarrow{p}'_{o2} = -m_2e'_{o2}, \qquad \overleftrightarrow{p}_{o1}\cdot\overleftrightarrow{p}'_{o2} = p_{o1}p'_{o2}\cos\theta_2 - \frac{e_{o1}e'_{o2}}{c^2}, \tag{34.9}$$

where θ_2 is the angle between the velocity of the first particle before the collision and the velocity of the second particle after the collision.

Substituting (9) into (5), we obtain

$$\cos\theta_2 = \frac{(e_{o1}+m_2c^2)(e'_{o2} - m_2c^2)}{c^2p_{o1}p'_{o2}}. \tag{34.10}$$

Formulas (8) and (10) give the connection between the scattering angles of both particles in the used 1-frame and the values of their energies and momenta before and after the collision. From these formulas the energies of the particles after the collision can be expressed through the angles θ_1 and θ_2.

Substituting

$$p_{o1} = (e_{o1}^2/c^2 - m_1^2c^2)^{1/2}, \qquad p'_{o2} = (e'^2_{o2}/c^2 - m_2^2c^2)^{1/2} \tag{34.11}$$

into (10) and squaring the obtained equation, we find after some manipulations

$$e'_{o2} = m_2c^2\,\frac{(e_{o1} + m_2c^2)^2 + (e_{o1}^2 - m_1^2c^4)\cos^2\theta_2}{(e_{o1} + m_2c^2)^2 - (e_{o1}^2 - m_1^2c^4)\cos^2\theta_2}. \tag{34.12}$$

If we try to express e'_{o1} through θ_1 from (8), we shall obtain a more cumbersome expression.

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If $m_1 > m_2$, that is, if the impringing particle is heavier than the one at rest, the scattering angle θ_1 cannot exeed a certain maximum value. To find this angle θ_{1max}, we write formula (8) in the form

$$\frac{m_1^2 v_1 v_2' \cos\theta_1}{c^2(1 - v_1^2/c^2)^{1/2}(1 - v_1'^2/c^2)^{1/2}} = \frac{m_1^2}{(1 - v_1^2/c^2)^{1/2}(1 - v_1'^2/c^2)^{1/2}} + \frac{m_1 m_2}{(1 - v_1'^2/c^2)^{1/2}} - \frac{m_1 m_2}{(1 - v_1^2/c^2)^{1/2}} - m_1^2, \quad (34.13)$$

and preserving only the terms of second order in v_1/c and v_1'/c, we can rewrite this formula in the form

$$(m_2/m_1 - 1)v_1'^2 + 2v_1 v_1' \cos\theta_1 - v_1^2 = 0. \quad (34.14)$$

This is a quadratic equation with respect to v_1' and its solution is

$$v_1' = v_1\{- \cos\theta_1 \pm(\cos^2\theta_1 + m_2/m_1 - 1)^{1/2}\}\frac{m_1}{m_2 - m_1} . \quad (34.15)$$

The maximum value θ_{1max} of the angle θ_1 is determined from the condition that the root of equation (14) must be double, i.e., the square root on the right side of (15) must be equal to zero, from where we obtain (see also fig. 34.1b)

$$\sin\theta_{1max} = m_2/m_1. \quad (34.16)$$

This angle coincides with the one obtained under low-velocity consideration (cf. (IIB,29.24)).

3. Description in the c-frame

The elastic collision looks most simply in the c-frame in which one has

$$(\vec{p}_{o1})_c = - (\vec{p}_{o2})_c = \vec{p}_{oc}, \qquad (\vec{p}'_{o1})_c = - (\vec{p}'_{o2})_c = \vec{p}'_{oc}, \quad (34.17)$$

and on the grounds of the energy conservation law the absolute values of the momenta must remain the same after the collision as before the collision, i.e.,

$$p_{oc} = p'_{oc}. \quad (34.18)$$

Let us designate the angle between the lines along which the particles approach one another and fly off (i.e., the angle of scattering in the c-frame) by χ. The collision is entirely defined after giving this angle, which cannot be determined if we consider the particles as points (see IIB, p. 125).

It is convenient to choose angle χ as a unique parameter which cannot be determined by using the laws of momentum and energy conservation.

Let us express the energies of the particles after the collision by this parameter in the l-frame in which mass m_2 is at rest. For this purpose we must return to equation (3) but we shall write there the scalar product $\overleftrightarrow{p}_{o1}\cdot\overleftrightarrow{p}_{o1}$ with the quantities in the c-frame, i.e.,

$$\overleftrightarrow{p}_{o1}\cdot\overleftrightarrow{p}'_{o1} = (\overleftrightarrow{p}_{o1})_c\cdot(\overleftrightarrow{p}'_{o1})_c = (\vec{p}_{o1})_c\cdot(\vec{p}'_{o1})_c - (e_{o1})_c(e'_{o1})_c/c^2 =$$

$$p_{oc}^2\cos\chi - (e_{o1})_c^2/c^2 = p_{oc}^2(\cos\chi - 1) - m_1^2c^2. \qquad (34.19)$$

We write both other scalar products in (3) in the l-frame, i.e., we take them from the first two of formulas (7). Now substituting (19) and both first formulas (7) into (3), we obtain

$$e'_{o1} - e_{o1} = \frac{p_{oc}^2}{m_2}(\cos\chi - 1). \qquad (34.20)$$

We have now to express p_{oc} by quantities registered in the l-frame. This can easily be done if we equate the invariant $\overleftrightarrow{p}_{o1}\cdot\overleftrightarrow{p}_{o2}$ written in the c- and l-frames, i.e.,

$$(\vec{p}_{o1})_c\cdot(\vec{p}_{o2})_c - (e_{o1})_c(e_{o2})_c/c^2 = - m_2e_{o1}, \qquad (34.21)$$

or (see (17) and (26.26))

$$\{(p_{oc}^2 + m_1^2c^2)(p_{oc}^2 + m_2^2c^2)\}^{1/2} = m_2e_{o1} - p_{oc}^2. \qquad (34.22)$$

Solving this equation with respect to p_{oc}^2, we get

$$p_{oc}^2 = \frac{m_2(e_{o1}^2 - m_1^2c^4)}{m_1^2c^2 + m_2^2c^2 + 2m_2e_{o1}}, \qquad (34.23)$$

and substituting this into (20), we obtain the energy of the first particle after the collision

$$e'_{o1} = e_{o1} - \frac{m_2(e_{o1}^2 - m_1^2c^4)}{m_1^2c^2 + m_2^2c^2 + 2m_2e_{o1}}(1 - \cos\chi). \qquad (34.24)$$

The energy of the second particle after the collision can be obtained from the energy conservation law

$$e_{o1} + m_2c^2 = e'_{o1} + e'_{o2}. \qquad (34.25)$$

Substituting here (24), we find

$$e'_{o2} = m_2c^2 + \frac{m_2(e_{o1}^2 - m_1^2c^4)}{m_1^2c^2 + m_2^2c^2 + 2m_2e_{o1}}(1 - \cos\chi). \qquad (34.26)$$

The second terms on the right side of equations (24) and (26) represent the energy which the first particle loses and the second particle acquires.

The exchanged energy is greatest when

$$\chi = \pi, \qquad (34.27)$$

and then we have

$$e'_{o2max} - m_2c^2 = e_{o1} - e'_{o1min} = \frac{2m_2(e_{o1}^2 - m_1^2c^4)}{m_1^2c^2 + m_2^2c^2+2m_2e_{o1}}. \qquad (34.28)$$

Let us assume that $m_1 \ll m_2$, that is, the mass of the impringing particle is small compared to the mass of the particle at rest. In this case, according to low-velocity mechanics, the light particle can transfer only a negligible part of its energy to the heavy one (see formulas (IIB,29.18)). This is not the case in high-velocity mechanics. From formula (28) we conclude that in this case the transferred energy is

$$e_{o1} - e'_{o1min} = \frac{2e_{o1}}{m_2c^2/e_{o1} + 2} = \begin{cases} e_{o1} & \text{for} \quad e_{o1} \gg m_2c^2, \\ \frac{2}{3}e_{o1} & \text{for} \quad e_{o1} = m_2c^2, \end{cases} \tag{34.29}$$

i.e., for sufficiently large energies e_{o1} the impringing particle transfers almost all its kinetic energy to the particle at rest.

An analogical situation exists for $m_1 \gg m_2$, that is, when a heavy particle impringes on a light one. Here, too, according to low velocity mechanics, only a negligible energy transfer would occur. From a high-velocity point of view, i.e., from formula (28), we deduce that in this case the transferred energy is

$$e_{o1} - e'_{o1min} = \frac{2e_{o1}}{2 + m_1^2c^2/m_2e_{o1}} = \begin{cases} e_{o1} & \text{for} \quad e_{o1} \gg m_1^2c^2/m_2 \\ \frac{2}{3}e_{o1} & \text{for} \quad e_{o1} = m_1^2c^2/m_2, \end{cases} \tag{34.30}$$

i.e., for sufficiently large energies e_{o1} the impringing particle transfers almost all its kinetic energy to the particle at rest.

C. Graphical presentation

We can interpret the obtained results graphically, as we did in low-velocity mechanics (see IIB, §29). The way which we shall follow is almost the same as in §32.

Assuming that the collision proceeds in the xy-plane, we shall have, according to the first two of formulas (26.24), the following relations between the components of the momentum of particle m_1 in the l-frame and its momentum and energy in the c-frame, all considered after the collision (with primes we denote here the quantities after the collision)

$$(p'_{o1})_x = \frac{(p'_{o1})_c\cos\theta_c + V(e'_{o1})_c/c^2}{(1 - V^2/c^2)^{1/2}}, \qquad (p'_{o1})_y = (p'_{o1})_c\sin\theta_c, \tag{34.31}$$

where $\vec{V}$ is the velocity of the c-frame with respect to the l-frame ($\vec{V}$ is equal with opposite sign to the velocity of particle m_2 with respect to the c-frame before the collision), and θ_c is the angle between the velocity $(\vec{v}'_1)_c$ of particle m_1 after the collision and the velocity $\vec{V}$ of the c-frame.

Taking into account (17) and (18), we conclude that V will be the absolute value of the velocity of particle m_2 after the collision registered in the c-frame. Therefore we can write

$$p'_{oc} = (p'_{o2})_c = \frac{m_2V}{(1 - V^2/c^2)^{1/2}}. \tag{34.32}$$

If we consider (in the l-frame) particles m_1 and m_2 as a system with momentum

$\vec{p}_{o1} + \vec{p}_{o2} = \vec{p}_{o1}$ and energy $e_{o1} + e_{o2} = e_{o1} + m_2c^2$ (since $\vec{p}_{o2} = 0$), we can write that the velocity of the center of inertia of this system will be (see (28.36))

$$V = \frac{p_{o1}c^2}{e_{o1} + m_2c^2}, \tag{34.33}$$

and this velocity will be equal to the velocity of particle m_2 after the collision, registered in the c-frame.

Excluding from both formulas (31) angle θ_c and writing there $(p'_{o1})_c = p'_{oc}$ (see the second formula (17)), we obtain the following equation

$$\{(p'_{o1})_x(1 - V^2/c^2)^{1/2} - \frac{V}{c^2}(e'_{o1})_c\}^2 + (p'_{o1})_y^2 = p'^2_{oc}, \tag{34.34}$$

or

$$\{(p'_{o1})_x - \frac{V}{c^2}\frac{(e'_{o1})_c}{(1 - V^2/c^2)^{1/2}}\}^2 \frac{1 - V^2/c^2}{p'^2_{oc}} + \frac{(p'_{o1})_y^2}{p'^2_{oc}} = 1. \tag{34.35}$$

This is the equation of an ellipse with respect to the variables $(p'_{o1})_x$ and $(p'_{o1})_y$. The semi-axes of this ellipse are (see fig. 34.1)

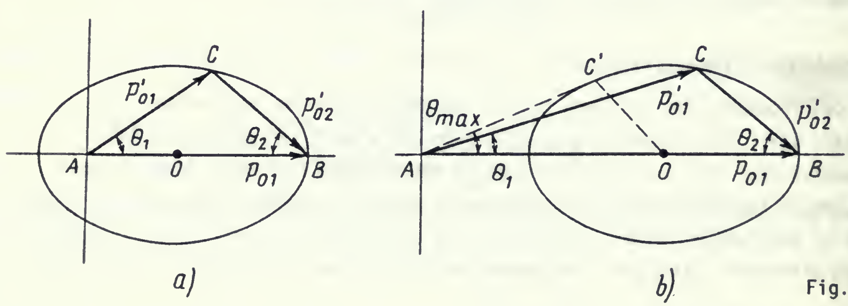

Fig. 34.1

$$a = \frac{p'_{oc}}{(1 - V^2/c^2)^{1/2}}, \qquad b = p'_{oc}, \tag{34.36}$$

and its center O is shifted from the point A (for which $(p'_{o1})_x = (p'_{o1})_y = 0$) over a distance

$$AO = \frac{V}{c^2}\frac{(e'_{o1})_c}{(1 - V^2/c^2)^{1/2}}. \tag{34.37}$$

We have from equation (35)

$$\vec{AC} = \vec{p}'_{o1} = \frac{m_1\vec{v}'_1}{(1 - v'^2_1/c^2)^{1/2}}. \tag{34.38}$$

Then equation (35) gives the following value for the distance AB (see formulas (17) and (18))

$$AB = OB + AO = \frac{p'_{oc}}{(1 - V^2/c^2)^{1/2}} + \frac{V}{c^2}\frac{(e'_{o1})_c}{(1 - V^2/c^2)^{1/2}} = \frac{(p_{o1})_c + (e_{o1})_c V/c^2}{(1 - V^2/c^2)^{1/2}}, \quad (34.39)$$

and since the velocity $(\vec{v}_1)_c$ of particle m_1 before the collision has the same direction as the velocity $\vec{V}$ of the c-frame, we conclude on the grounds of the first formula (26.24) that

$$\vec{AB} = \vec{p}_{o1} = \frac{m_1\vec{v}_1}{(1 - v_1^2/c^2)^{1/2}}. \quad (34.40)$$

Finally from the momentum conservation law

$$\vec{p}_{o1} = \vec{p}'_{o1} + \vec{p}'_{o2}, \quad (34.41)$$

we conclude

$$\vec{CB} = \vec{p}'_{o2} = \frac{m_2\vec{v}'_2}{(1 - v_2'^2/c^2)^{1/2}}. \quad (34.42)$$

Substituting (32) and (33) into (36), we can write the semi-axes of the ellipse in the form

$$b = \frac{m_2 p_{o1}}{(m_1^2c^2 + m_2^2c^2 + 2m_2e_{o1})^{1/2}},$$

$$a = \frac{m_2 p_{o1}(e_{o1} + m_2c^2)}{m_1^2c^2 + m_2^2c^2 + 2m_2e_{o1}}. \quad (34.43)$$

If $m_1 = m_2 = m$, we see that

$$a = p_{o1}/2. \quad (34.44)$$

Thus if $m_1 < m_2$, point A lies in the ellipse (see fig. 34.1a), and if $m_1 > m_2$, point A lies outside the ellipse (see fig. 34.1b).

§35. THE COMPTON EFFECT

If one of the particles performing elastic collision is a photon, we have what is called the COMPTON EFFECT.

Let us consider the elastic collision of a photon and a particle in a frame where the particle is at rest before the collision. Denoting the mass of the particle by m, the frequency of the photon by ν, and its direction of propagation by the unit vector $\vec{n}$, we we can write the 4-vector equation (34.1), dividing it into a space and a time part, i.e., in the form of momentum and energy conservation laws, as follows (see (31.29) and (31.31))

$$\frac{h\nu}{c}\vec{n} = \frac{m\vec{v}'}{(1 - v'^2/c^2)^{1/2}} + \frac{h\nu'}{c}\vec{n}', \quad (35.1)$$

$$mc^2 + h\nu = \frac{mc^2}{(1 - v'^2/c^2)^{1/2}} + h\nu'. \qquad (35.2)$$

Here the unknown quantities are the velocity $\vec{v}'$ of the particle, the frequency ν' of the photon, and the direction of its propagation $\vec{n}'$, all of them after the collision. Consequently there are six unknowns (a unit vector is determined by two parameters) and only four equations. Threfore two of the unknown quantities cannot be determined in the domain of mechanics (see IIB, p. 125). Usually, as an additional condition which we have to impose, we choose the direction of the photon's propagation after the collision $\vec{n}'$.

We take (see fig. 35.1) $\vec{n}$ to be directed along the x-axis of the used frame and we choose $\vec{n}'$ to lie in the xy-plane (first condition), and the angle between $\vec{n}$ and $\vec{n}'$ to be equal to θ (second condition), i.e.,

$$\vec{n}.\vec{n}' = \cos\theta. \qquad (35.3)$$

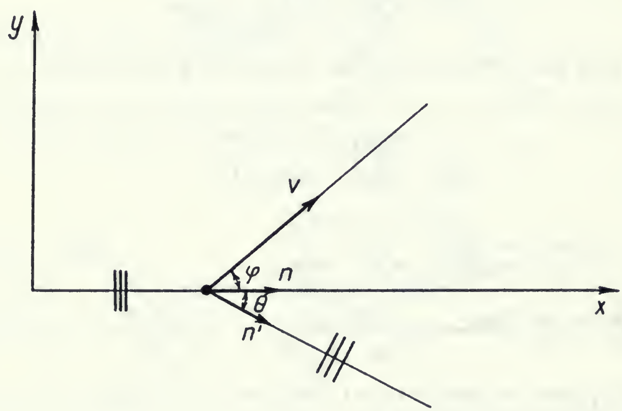

Fig. 35.1

If we transfer the second term in (1) from the right side to the left one, and then square the equation, the following scalar equation will be obtained

$$h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\theta = \frac{m^2v'^2c^2}{1 - v'^2/c^2}. \qquad (35.4)$$

Squaring now equation (2), after transferring the second term from the right side to the left one, and subtracting from it the equation (4), we can eliminate the velocity v' and obtain the following relation between ν and ν'

$$\nu - \nu' = \frac{2h\nu\nu'}{mc^2}\sin^2(\theta/2) \qquad (35.5)$$

or

$$\nu' = \frac{\nu}{1 + \frac{2h\nu}{mc^2}\sin^2(\theta/2)}. \qquad (35.6)$$

Using here (31.30), we get

$$\lambda' - \lambda = \frac{2h}{mc}\sin^2(\theta/2), \tag{35.7}$$

where λ and λ' are the photon's wavelengths respectively before and after the collision.

To find the speed of the recoiling particle, we substitute (6) into (2) and after some manipulations we get

$$v' = \frac{\frac{2h\nu}{mc}\sin^2(\frac{\theta}{2})\{1 + \frac{h\nu}{mc^2}(2 + \frac{h\nu}{mc^2})\sin^2(\frac{\theta}{2})\}^{1/2}}{1 + 2\frac{h\nu}{mc^2}(1 + \frac{h\nu}{mc^2})\sin^2(\frac{\theta}{2})}. \tag{35.8}$$

From formula (1) and fig. 35.1 we have

$$\frac{h\nu}{c} = \frac{mv'}{(1 - v'^2/c^2)^{1/2}}\cos\phi + \frac{h\nu'}{c}\cos\theta,$$

$$0 = \frac{mv'}{(1 - v'^2/c^2)^{1/2}}\sin\phi - \frac{h\nu'}{c}\sin\theta. \tag{35.9}$$

If we transfer here the second terms from the right sides to the left sides and divide the second of the equations obtained by the first one, after using (6), we arrive to the following relation

$$\tan\phi = \frac{\cot(\theta/2)}{1 + h\nu/mc^2}. \tag{35.10}$$

xxxxx

In view of the indeterminateness of the problem, it is interesting to examine the limits of the quantities involved when the direction of the photon after the collision is regarded as arbitrary.

The phton's frequency after the collision passes (see (6)) from a maximum

$$\nu'_{max} = \nu, \tag{35.11}$$

for $\theta = 0$, to a minimum

$$\nu'_{min} = \frac{\nu}{1 + 2h\nu/mc^2}, \tag{35.12}$$

for $\theta = \pi$. In the last case the photon is thrown straight back, and equation (7) gives

$$\lambda' - \lambda = \frac{2h}{mc}. \tag{35.13}$$

The quantity

$$\lambda_m = \frac{h}{mc} \tag{33.14}$$

is called the COMPTON WAVELENGTH of the particle m (cf. (IIA,6.4)).

By (10) $\tan\phi$ is positive; thus the particle is always thrown forward. The speed of recoil v' (see (8)) ranges from zero for $\theta = 0$ (in which case there is no collision) to a maximum for $\theta = \pi$, given by

$$v'_{max} = \frac{\frac{2h\nu}{mc}(1 + \frac{h\nu}{mc^2})}{(1 + \frac{h\nu}{mc^2})^2 + (\frac{h\nu}{mc^2})^2}. \qquad (35.15)$$

xxxxx

Let us now transfer the first term from the right side of (1) to the left one and then square the equation (see fig. 35.1). Furthermore let us exchange the places of the second terms in equation (2) and then square the equation. If now we add the equations squared in this way, the following relation can be obtained

$$\nu' = \nu \frac{1 - \frac{v'}{c}\cos\phi}{(1 - v'^2/c^2)^{1/2}}. \qquad (35.16)$$

Finally, if we square equations (1) and (2) and subtract the first from the second, the following relation can be obtained

$$\nu' = \nu \frac{(1 - v'^2/c^2)^{1/2}}{1 - \frac{v'}{c}\cos(\phi + \theta)}. \qquad (35.17)$$

Equations (6), (16), and (17) give the frequency of the scattered photon in dependence on the different angles involved.

xxxxx

Let us now consider the elastic collision of a photon and a particle in a frame where the particle moves with a velocity $\vec{v}$ before the collision.

In this most general case the momentum and energy conservation laws lead to the following equations

$$\frac{m\vec{v}}{(1 - v^2/c^2)^{1/2}} + \frac{h\nu}{c}\vec{n} = \frac{m\vec{v}'}{(1 - v'^2/c^2)^{1/2}} + \frac{h\nu'}{c}\vec{n}', \qquad (35.18)$$

$$\frac{mc^2}{(1 - v^2/c^2)^{1/2}} + h\nu = \frac{mc^2}{(1 - v'^2/c^2)^{1/2}} + h\nu'. \qquad (35.19)$$

where $\vec{v}$ and $\vec{v}'$ are the velocities of the particle before and after the collision, ν and ν' are the frequencies, and $\vec{n}$ and $\vec{n}'$ are the directions of propagation of the photon respectively before and after the collision.

Squaring both these equations and subtracting the first from the second, we obtain

$$\nu' = \nu\left(\frac{1 - v'^2/c^2}{1 - v^2/c^2}\right)^{1/2} \frac{1 - \frac{v}{c}\cos(\vec{v},\vec{n})}{1 - \frac{v'}{c}\cos(\vec{v}',\vec{n}')}. \qquad (35.20)$$

This is the dependence between the characteristics of the photon and of the particle before and after the collision in which only 3-dimensional invariants are involved.

§36. THE MÖSSBAUER EFFECT

Consider a spontaneous emission of a photon from a particle (see p. 101). Let us denote the corresponding binding energy by (see p. 80 and (IIB,28.5))

$$\varepsilon_0 = \Delta M c^2 , \tag{36.1}$$

where ΔM is the mass deffect of the particle and the photon when considered first together as one particle, and then as two particles isolated from one another. After the radiation this binding energy transforms into kinetic energy of the daughter particles, i.e., into kinetic energy of the radiating particle and the photon.

When a photon is radiated from an atom we say that the latter passes from an EXCITED STATE to another less excited or GROUND STATE. In quantum mechanics it is shown that for a specific atom the energetic transitions between states of different excitation are strictly defined. Thus the energy can be radiated in the form of definite "portions" or "quanta" only. Here the binding energy is also called EXCITATION ENERGY.

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In case where a particle disintegrates spontaneously into two others, we can write (1) in the form

$$\varepsilon_0 = (M - m_1 - m_2)c^2, \tag{36.2}$$

where m_1 and m_2 are the masses of the daughter particles and M is the mass of the initial particle. This formula can also be obtained using formulas (32.7) in the expression of the binding energy as kinetic energy of the daughter particles, i.e., in the expression

$$\varepsilon_0 = (e_{01})_c + (e_{02})_c - m_1c^2 - m_2c^2, \tag{36.3}$$

where $(e_{01})_c$ and $(e_{02})_c$ are the proper energies of the daughter particles in the c-frame.

In case of a photon emission, we have, on the one hand,

$$\varepsilon_0 = (M - m)c^2, \tag{36.4}$$

and, on the other hand,

$$\varepsilon_0 = e_0 + h\nu - mc^2, \tag{36.5}$$

where M is the mass of the particle before the emission, m is the mass of the particle after the emission, e_0 is the energy of the particle after the emission, and ν is the frequency of the emitted photon.

Substituting (32.23) into (5) and using (4), we get

$$h\nu = \varepsilon_0(1 - \varepsilon_0/2Mc^2), \tag{36.6}$$

thus we expressed the photon energy (and frequency) through the excitation energy ε_0 and the mass M of the atom before the radiation.

From this formula we see that the energy of the photon $h\nu$ is less than the excitation energy ε_0 by an amount

$$T_0 = e_0 - e = \varepsilon_0^2/2Mc^2, \tag{36.7}$$

representing the high-velocity kinetic energy, e_{ok}, of the radiating particle (see (5) an (23.16)). This energy determines the recoil of the particle after the radiation.

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Let a photon collide with an atom in the ground state, and let the photon be of the correct energy to excite the atom to the excited state of energy ε_0 above the ground state. As the momentum must be conserved in the absorption process, the momentum of the recoiling atom after the absorption will be equal to the momentum of the incident photon. The kinetic energy of recoil will now be given again by (7), so that, in order to have a high probability of being absorbed, the incident photon must have an energy

$$h\nu = \varepsilon_0 + T_0 = \varepsilon_0(1 + \varepsilon_0/2Mc^2). \tag{36.8}$$

Thus the energy of a photon emitted by a free excited atom is normally too low by an amount $2T_0$ to re-excite a similar atom from the ground state to the excited state.

But if the emission and absorption can be realized without recoil of the atom, such a re-excitation will be possible. The phenomenon when an atom can be excited by the same photons which it radiates, is called RESONANCE ABSORPTION (or RESONANCE FLUORESCENCE).

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So far it has been assumed that the excited level ε_0 above the ground state has always the same energy. However, the released energy is not always equal to ε_0; it is given rather with a whole spectrum about this value, and we know only the probabilities with which a certain energetic transition may occur (see V, §58). If we plot the ratio of the probability $P(\varepsilon)$ for the energetic transition ε and the probability $P(\varepsilon_0)$ for the most probable energetic transition ε_0 as function of the energy ε (see fig. 36.1), we shall obtain a curve with the form of a resonance curve (see fig. IIB,45.1 and fig. V,58.1). This curve is called the NATURAL EMISSION LINE for the energetic transition from the level ε_0 to the ground state.

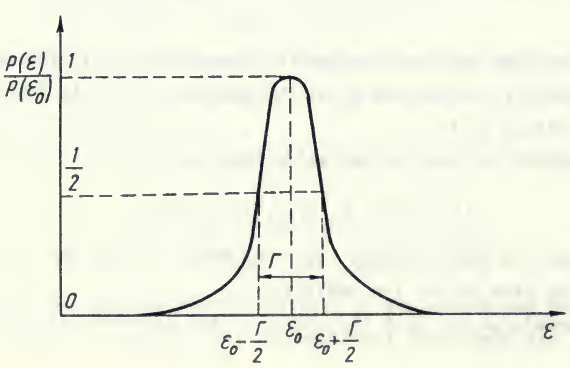

Fig. 36.1

The energetic interval

$$\Gamma = \Delta\varepsilon \tag{36.9}$$

from both sides of the most probable transition ε_0 on whose ends the probability for energetic transition (i.e., for transition with energy $\varepsilon_0 + \Gamma/2$ or $\varepsilon_0 - \Gamma/2$) is twice smaller than the probability for transition with energy ε_0, is called the NATURAL LINE WIDTH.

The energetic transition of an atom from an excited state to another less excited or ground state can be of different magnitude. These transitions which lead, in general, to emission of photons of visible light, correspond to different energetic states of the atomic electrons, while those transitions which lead to emission of γ-rays photons correspond to different energetic states of the atomic nucleus. The photons of visible light are of low energy, while the γ-rays photons are of high energy. On the other hand, the natural widths of the visible emission lines are large in comparison with the natural widths of the γ-rays emission lines.

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In case of emission of a photon from a nucleus, the kinetic energy T_0 of the nucleus (see (7)) is much larger than the natural line width of the corresponding γ-ray emission line. If we plot the probabilities for emission or absorption of a photon by a nucleus with or without recoil of the latter, we shall obtain a picture as shown in fig. 36.2 (compare it with fig. 36.1). The first line from the left corresponds to emission of photons with recoil; these photons carry off energy $h\nu$ given by (6). The first line from the right corresponds to absorption of photons with recoil; these photons must have energy $h\nu$ given by (8). The middle line (which is shown only in fig. 36.1a) corresponds to either emission or absorption of photons without recoil; in this case the emitted photons carry off, respectively the absorbed photons must have, energy

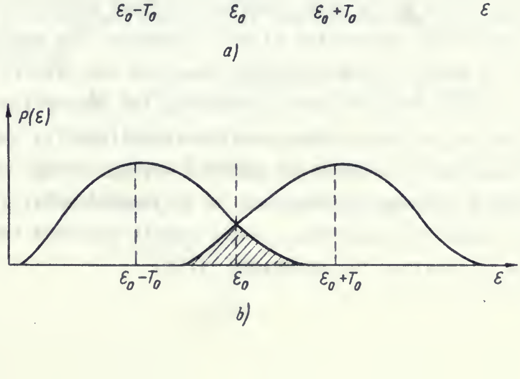

Fig. 36.2

$$h\nu = \varepsilon_0. \quad (36.10$$

Thus (see fig. 36.2a) the natural line widths are not sufficient to make up the deficit of $2T_0$, so that at first sight it seems that, in general, it would be impossible to observe nuclear resonance absorption with free atoms unless it were possible, by some method or other, to make up for the deficit of $2T_0$ in the energy of the γ-photons.

But if the nucleus which emits γ-photons is moving relative to the laboratory when it decays, then the energy of the photon relative to the laboratory is changed, being increased if the photon is emitted in the same direction as the direction in which the decaying nucleus is moving, or decreased if the photon is emitted in the opposite direction. (This is the so-called Doppler effect which will be investigated in §38.) Such a motion leads to a spread in the energy of the emitted γ-photons (the so-called Doppler broadening) as illustrated in fig. 36.2b. We assume that the nuclei have all possible velocities in some range, so that the Doppler broadening is symmetric from both sides of the emission line, obtained when such a Doppler broadening is not assumed (see the left curve in fig. 36.2b In the same way, if the absorbing nucleus is moving, then the absorption line is also Doppler broadened (see the right curve in fig. 36.2b), so that the emission and absorpti line profiles may sometimes overlap, as illustrated in the graph. If the radioactive source (i.e., the nucleus emitting γ-photons) is warmed up sufficiently, the thermal Doppler broadening may be sufficient for nuclear resonance fluorescence to be observed in some cases.

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If an excited nucleus forms part of the crystal lattice of a solid when it decays, then, under certain conditions, the γ-photons may sometimes be emitted with no momentum and kinetic energy transfer to the nucleus after decay. In the case of recoilless emission, the whole of the crystal effectively takes up the recoil momentum, so that practically all the excitation energy goes into the emitted γ-photons. In these cases not all the decays are recoilless but under certain conditions for certain radioactive crystalline metals the number of recoilles decays amounts to a few per cent or more.

If the γ-photons pass through an absorber, which contains the same type of nuclei in the ground state as those which emit γ-photons without recoil (see fig. 36.3), then it is possible to have recoilless absorption of the γ-photons. The recoilles absorption line is also very sharp. Its width is approximately equal to the natural line width. This is much sharper than the thermal Doppler widths necessary for absorption by free atoms. If the recoilless absorption and emission lines overlap significantly, then there is a strong probability that those γ-photons which are emitted without recoil will be absorbed without recoil when they pass through the absorber in the experimental arrangement shown in fig. 36.3. The nuclear resonance absorption, under conditions when the emission and absorption are both recoilles, is called the MÖSSBAUER EFFECT.

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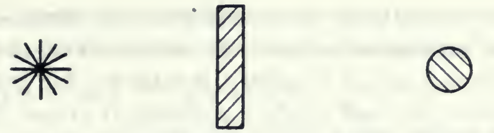

Fig. 36.3

If the source (or the absorber) in fig. 36.3 is moved, then the energy of the recoilless emission line (or recoilless absorption line) is shifted due to the Doppler effect, and if the velocity of the source (or absorber) is high enough, the recoilless emission and absorption lines will no longer overlap significantly, and the absorption of the γ-photons decreases since the Mössbauer effect is no longer present, so that the detector in fig. 36.3 of the γ-photons passed through the absorber records a higher counting rate.

Since the emission and absorption lines are very sharp, small shifts in the energy (or frequency) of either the recoilles emission or absorption lines change the probability for the Mössbauer effect substantially and produce measurable effects. The Mössbauer effect represents an acurate method of measuring differences or changes in frequency (or energy); with this method changes in frequency (or energy) of the order of 1 part in 10^{12} can be detected. Such a precision cannot be reached with other methods.

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Although the recoilless emission of the γ-photons proceeds without any transfer of momentum to the crystal, the availability of thermal motion leads to a small (of second order in v/c) change in the emitted frequency.

One can calculate this change in the frequency in the following way:

If the frequency of the γ-photon at recoilless emission is ν_0, then (see (4) and (10))

$$h\nu_0 = (M - m)c^2, \tag{36.11}$$

and the mass defect of the nucleus is

$$\Delta M = h\nu_0/c^2. \tag{36.12}$$

Supposing that there is a thermal motion, let the high-velocity kinetic energy of the nucleus before the emission be

$$T_0 = (P_0^2c^2 + M^2c^4)^{1/2} - Mc^2 \cong P^2/2M, \tag{36.13}$$

since the thermal velocity is very small with respect to c.

After the emission the nuclear mass diminishes with ΔM at a constant value of the momentum. This is possible if the velocity V of the nucleus changes so that

$$P \equiv MV = mV', \tag{36.14}$$

where V' is the recoil velocity of the nucleus after the emission.

This change in the mass and velocity of the nucleus will lead to a new kinetic energy

$$T'_0 = (P_0^2c^2 + m^2c^4)^{1/2} - mc^2 \cong P^2/2m. \qquad (36.15)$$

The change in the kinetic energy before and after the emission will be

$$\Delta T = T'_0 - T_0. \qquad (36.16)$$

Substituting here (13), (15), and then (12), we obtain in the assumed approximation

$$\Delta T \cong \frac{P^2}{2m} - \frac{P^2}{2M} = \frac{P^2\Delta M}{2\,mM} \cong \frac{P^2}{2M^2}\,\frac{h\nu_0}{c^2}. \qquad (36.17)$$

Since the nuclei have different thermal velocities, we have to take in (17) the square of the average momentum

$$P^2 = M^2\bar{V}^2, \qquad (36.18)$$

where $\bar{V}$ is the average thermal velocity of the nuclei. Thus for the change in the kinetic energy we obtain

$$\Delta T = h\nu_0\,\frac{\bar{V}^2}{2c^2}. \qquad (36.19)$$

The increment of the kinetic energy of the emitting nucleus during this recoilless emission leads to the result that the γ-photon carries off energy

$$h\nu = h\nu_0 - \Delta T = h\nu_0(1 - \bar{V}^2/2c^2). \qquad (36.20)$$

This energy is less than the energy of the γ-photon emitted by a nucleus at rest with the value (19). Thus the thermal motion of the emitting nucleus leads to a shift of the emitted frequency from ν_0 to ν, and the relation between them is

$$\nu = \nu_0(1 - \bar{V}^2/2c^2). \qquad (36.21)$$

Similar analysis must be performed also in the case of recoilless absorption at the presence of thermal motion.

§37. THE VELOCITY MASS INCREASE IS A NEWTONIAN PHENOMENON

We said (see p. 4) that the differences between the Newton and Marinov mechanics are not drastic and are caused by small differences existing between the Newton and Marinov models of light propagation, postulated in our axioms and summarized on p. 4. Physicists, unfamiliar with the specificity of the space-time problems, often claim that there is an undisputable proof for the "non-Newtonian character" of our world furnished by the accelerators of elementary particles and by the whole equipment of high-velocity physics which "works according to Einstein's theory", so that the latter is to be considered not only as a "physical theory" but rather as an "engineering science". We shall show that these claims are inconsistent.

The primordial cause impelling the engineers to project the accelerators and the high-energy equipment not with the traditional low-velocity mathematical apparatus is the discovery of Kaufmann (1901) that the mass of a particle increases with its velocity according to formula (23.20). And it is generally accepted that this formula is a result of the Lorentz transformation and of the theory of relativity. Now we shall demonstrate that formula (23.20) is a result of Newtonian mechanics, i.e., it can be obtained without the introduction of our tenth axiom and consequently of the Lorentz and Marinov transformations.

We shall obtain formula (23.20) for the dependence of the proper mass on the velocity of the particle considering the absorption and emission of a photon by a moving particle, i.e., from the Compton effect, taking into account only the first-order in v/c Doppler-effect formulas (see §38) which appartain to Newtonian mechanics.

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Let us have an atom (a compound particle) which can absorb or emit a photon with a frequency ν. If the atom moves with a velocity v, the photon which can be absorbed or emitted must have a frequency determined by the Doppler-effect formulas (38.5) or (38.12)

$$\nu' = \nu\frac{(1 - v^2/c^2)^{1/2}}{1 - v\cos\theta/c} \cong \nu(1 + v\cos\theta/c), \qquad (37.1)$$

where θ is the angle between the velocity of the atom and the direction of propagation of the photon. The result on the right side of (1) is written within an accuracy of first order in v/c, where Newton and Marinov mechanics are identical, as can be seen comparing the Galilean and Marinov transformations.

If this atom absorbs a photon proceeding parallel to the velocity of the atom ($\theta' = 0$) and then emits this photon in a backward directions ($\theta'' = \pi$), the energy and momentum acquired by the atom during the absorption are (see (31.29))

$$e_0' = h\nu' = h\nu(1 + v/c), \qquad p_0' = h\nu'/c = h\nu(1 + v/c)/c, \qquad (37.2)$$

while the energy lost and the momentum acquired during the emission are

$$e_0'' = h\nu'' = h\nu(1 - v/c), \qquad p_0'' = h\nu''/c = h\nu(1 - v/c)/c. \qquad (37.3)$$

These equations are written proceeding from the laws of energy and momentum conservation, and we attach the subscript "o" to the energy and momentum aiming to have a conformity with the designation of the proper quantities in Marinov mechanics; as now we are working in the frame of Newton's mechanics, these subscripts, of course, may be omitted.

As a result of this cycle of "absorption - emission", the energy and momentum of the atom will increase by

$$de_0 = e_0' - e_0'' = 2h\nu v/c, \qquad dp_0 = p_0' + p_0'' = 2h\nu/c. \qquad (37.4)$$

Thus

$$de_0/dp_0 = v. \qquad (37.5)$$

The momentum of the particle can be written (see the remark in IIA, p. 12)

$$p_o = m_o v = (e_o/c^2)v, \tag{37.6}$$

and for its differential we obtain

$$dp_o = (e_o dv + v\, de_o)/c^2. \tag{37.7}$$

Substituting (7) into (5), we obtain the differential equation

$$\frac{de_o}{e_o} = \frac{1}{2}\frac{d(v^2/c^2)}{1 - v^2/c^2}, \tag{37.8}$$

whose solution is (see (I,25.29))

$$e_o = \frac{e}{(1 - v^2/c^2)^{1/2}}, \tag{37.9}$$

where e is the energy of the atom when v = 0. Substituting here the definition equations for mass (IIA,2.3) and (IIA,2.4), we obtain formula (23.20).

xxxxx

We said (see IIA, p.20) that the Newton and Marinov mechanics are identical only for effects of first order in v/c (this becomes evident comparing the Marinov transformation (3.32) with the Galilean transformation (3.5)). Meanwhile the difference between the traditional Newtonian mass m and the proper mass m_o is of second order in v/c. It is logical to pose the question, how have we obtained formula (23.20) working in the domain of Newtonian mechanics and why the "Newton mass increase" $mv^2/2c^2$ figures in Newton's mechanics when, according to the mentioned restriction, it must be neglected with respect to the universal mass m. The answer is as follows:

The gravitational world potential (see p. 69), $-c^2$, is much bigger than the gravitational potentials of local systems (say, that of the Earth). A change in the velocity of the particle leads to a considerable change in its world gravitational energy, while such a change does not affect its local gravitational energy. For this reason in all physical formulas the Newtonian apparatus retains only terms of first order in v/c, however, for the world gravitational energy, i.e., for time energy, it retains also terms of second order in v/c. Thus the velocity increase of the mass according to formula

$$m_f = m + mv^2/2c^2, \tag{37.10}$$

where m_f is to be called the FACTUAL MASS, is a legitimate Newtonain phenomenon. Here, of course, we have to resend the reader to the remark on p. 66 and remind that the mass does not change but the time momentum does, because the proper time velocity of the particle, c_o, changes with the change of the absolute velocity v.

Let us compare formula (23.13) for the factual energy with formula (10) for the factual mass. Conventional physics has led to an extremely confusing situation. It asserts, on the one hand, that in e_f the term mc^2 is non-Newtonian and the term $mv^2/2$ is Newtonian, while, On the other hand, in m_f the term m is Newtonian and the term $mv^2/2c^2$ is non-Newtonian. Meanwhile e_f can be obtained by multiplying m_f by the constant c^2, i.e., we have not two different quantities but one and the same. Thus if $mv^2/2$ is a Newtonian quantity, then $mv^2/2c^2$ must be a Newtonian quantity, too, and if m is a Newtonian quantity, then mc^2 must be a Newtonian quantity, too.

Chapter VII

KINEMATICS OF LIGHT

§38. KINEMATIC FREQUENCY AND WAVELENGTH SHIFTS (THE DOPPLER EFFECT)

The LIGHT DOPPLER EFFECT is the difference between the frequency and wavelength with which a photon is emitted from a source and those measured by an observer, due to the motion of source or/and observer with respect to absolute space; we call this effect also the KINEMATIC FREQUENCY AND WAVELENGTH SHIFTS OF LIGHT. This effect was analysed first for the propagation of sound by Doppler (1842).

A. Source and observer both at rest

Let us suppose that there is a source (emitter) of photons which is at rest in absolute space. The frequency ν registered by an observer (receiver) who is also at rest in absolute space and the wavelength λ, which he measures, are called EMITTED FREQUENCY and EMITTED WAVELENGTH. The relation between them is given by formula (31.30).

B. Source moving, observer at rest

Let us now suppose (see fig. 38.1) that the observer is at rest in absolute space at the point O' and the light source moves with velocity v from the position S' where a photon is emitted to the position S where the source is at the moment when the photon is received by the observer. We shall suppose that the wavelength of the photon is much less than the distance between source and observer and, thus, the EMISSION and RECEPTION POSITIONS of the source can be considered as points.

The source is at the MIDDLE POSITION S_m at the MIDDLE MOMENT between the EMISSION and RECEPTION MOMENTS, where the EMISSION ANGLE is θ', the RECEPTION ANGLE is θ, and the MIDDLE ANGLE is θ_m. As in §6A, we define: The emission, reception, and middle angles are subtended by the velocity of the moving object and the line connecting the object at rest with the moving object at the emission, reception, and middle moments, respectively. We attach the subscript "o" to the received (observed) frequency and wavelength, while the emitted will be written without any subscript. The prime will be attached to the emission distance, position, and angle, while the reception distance, position, and angle will be written without any superscript.

When the source is moving, the observer at rest will not register the frequency ν and will not measure the wavelength λ which are to be registered and measured if the source were at rest and which we have called the emitted frequency and wavelength, but some other in general, different quantities ν_o, λ_o, which we call the OBSERVED (or RECEIVED) FREQUENCY and WAVELENGTH.

If in fig. 38.1 the emitted wavelength is represented by the segment S'Q, the observed

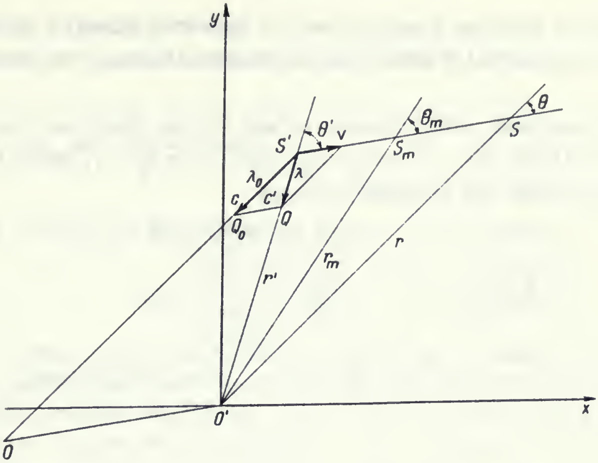

Fig. 38.1

is represented
avelength by the segment $S'Q_0$ (as a matter of fact, by that segment equal and parallel
$S'Q_0$ whose final point is Q). We repeat that we consider the case where the distance etween source and observer is much greater than the wavelength of the photon, and we raw the wavelength so large only for the sake of clarity.

Since the photon moves in absolute space with velocity c, we have

$$\nu_0 \lambda_0 = c. \tag{38.1}$$

From (31.30) and (1) we obtain

$$\nu_0/\nu = \lambda/\lambda_0. \tag{38.2}$$

The triangles $S'Q_0Q$ and $O'SS'$ are similar and thus

$$\lambda/\lambda_0 = r'/r. \tag{38.3}$$

On the other hand, if we should suppose that the source is at rest and the observer oving from the emission position O' to the reception position O, we obtain from the simiar triangles $S'Q_0Q$ and S'OO'

$$r'/r = c'/c, \tag{38.4}$$

ince the segments $S'Q_0$ and S'Q are, respectively, proportional to the absolute light veocity c (i.e., to the velocity of light with respect to absolute space) and to the relaive light velocity c' (i.e., to the velocity of light with respect to the moving observer), oth measured on an universal clock.

From the last three relations, using formulas (4.19) in which we write V = v, we obtain

$$\nu_0 = \nu \frac{1 - v\cos\theta/c}{(1 - v^2/c^2)^{1/2}} = \nu \frac{(1 - v^2/c^2)^{1/2}}{1 + v\cos\theta'/c}, \tag{38.5}$$

$$\lambda_0 = \lambda \frac{(1 - v^2/c^2)^{1/2}}{1 - v\cos\theta/c} = \lambda \frac{1 + v\cos\theta'/c}{(1 - v^2/c^2)^{1/2}}. \tag{38.6}$$

Formulas (6) can be obtained directly from (3) (and then formulas (5) from (2) and (6)) if we should use formulas (6.17) which give the relation between the EMISSION and RECEPTION DISTANCES.

Multiplying, on one hand, both formulas (5) and, on the other hand, both formulas (6), squaring them and writing $\cos\theta = \cos\theta_m + a$, $\cos\theta' = \cos\theta_m - a$, where a is an algebraic quantity, we obtain within the necessary accuracy

$$\nu_o = \nu\left(\frac{1 - v\cos\theta_m/c}{1 + v\cos\theta_m/c}\right)^{1/2}, \tag{38.7}$$

$$\lambda_o = \lambda\left(\frac{1 + v\cos\theta_m/c}{1 - v\cos\theta_m/c}\right)^{1/2}. \tag{38.8}$$

For $\theta = \theta' = \theta_m = 0$ (or π), we call the Doppler effect LONGITUDINAL.

For $\theta = \pi/2$, $\theta' = \pi/2 + v/c$, $\theta_m = \pi/2 + v/2c$, we call the Doppler effect ANTE-TRAVERSE.

For $\theta' = \pi/2$, $\theta = \pi/2 - v/c$, $\theta_m = \pi/2 - v/2c$, we call the Doppler effect POST-TRAVRESE.

For $\theta_m = \pi/2$, $\theta' = \pi/2 + v/2c$, $\theta = \pi/2 - v/2c$, we call the Doppler effect TRAVERSE.

The ante-traverse, post-traverse, and traverse Doppler effects are designated collectively by the common term TRANSVERSE Doppler effect.

C. Source at rest, observer moving

Let us now suppose (see again fig. 38.1) that the source is at rest in absolute space at the point S' and the observer moves with velocity v from the emission position O' to th reception position O.

Since the photon proceeds with respect to the moving observer with the relative velocity c', the relation between the observed frequency and wavelength will be

$$\nu_o\lambda_o = c'. \tag{38.9}$$

According to our model for the photons (and for any particle), their wavelength can change only when the source moves with respect to absolute space. The motion of the observer with respect to absolute space leads only to a change in the velocity and frequency of the observed photons but not to a change in their wavelengths. We have to emphasize that the wavelength is to be measured always with respect to absolute space, even in the case of a moving observer. The photon is a reality that exists independently of the observer, and the motion of the latter can exert no influence on the photon's wavelength, which is an immanent photon's property defined outside of time. We have further to emphasize that a direct measurement of the wavelength cannot be performed. One can measure only the wavelength of standing waves, i.e., of "to and fro" propagating photons, which interfere (see §66). All measurements of the wavelength of uni-directionally propagating photons are indirect (see §67). If one would accept that the motion of the observer leads to a change in the wavelength, then one is impelled to accept Einstein's dogma about the constancy of light velocity in any inertial frame, which, as we have experimentally shown, does not correspond to physical reality.

Thus for the case of source at rest and moving observer we have

$$\lambda_0 = \lambda. \tag{38.10}$$

From (31.30), (9), and (10) we obtain

$$\nu_0/\nu = c'/c. \tag{38.11}$$

Making use of formulas (4.19) in which we write V = v, we obtain

$$\nu_0 = \nu \frac{1 - v\cos\theta/c}{(1 - v^2/c^2)^{1/2}} = \nu \frac{(1 - v^2/c^2)^{1/2}}{1 + v\cos\theta'/c}. \tag{38.12}$$

Here again a formula analogical to formula (7) can be introduced, as well as the definitions for longitudinal and transverse Doppler effects.

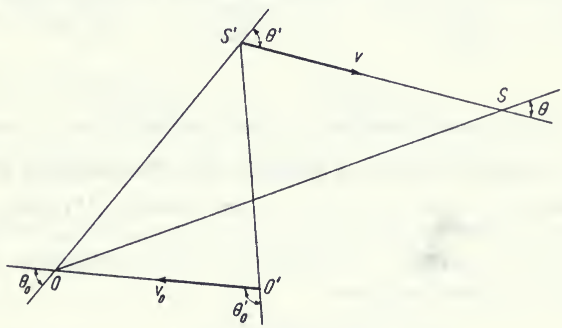

Fig. 38.2

). Source and observer both moving

Finally, suppose (see fig. 38.2) that the source moves with velocity v with respect to absolute space and the observer with velocity v_0, so that S', O' are the emission positions of source and observer and S, O are their reception position.

We introduce two pairs of emission and reception angles: θ_0' and θ_0 are the emission and reception angles if the source is at rest at its emission position, while θ' and θ are the emission and reception angles if the observer is at rest at its reception position. For certain problems it is convenient to use the angles θ', θ_0, for others the angles θ_0', θ.

To find the relation between the emitted and received frequencies and wavelengths, we proceed as follows: Suppose that the real source emits a photon and an imaginary observer is at rest at point O (the reception position of the real observer). The frequency and wavelength registered by this observer, called intermediary, will be (use formulas (5) and (6))

$$\nu_{int} = \nu \frac{1 - v\cos\theta/c}{(1 - v^2/c^2)^{1/2}} = \nu \frac{(1 - v^2/c^2)^{1/2}}{1 + v\cos\theta'/c}, \tag{38.13}$$

$$\lambda_{int} = \lambda \frac{(1 - v^2/c^2)^{1/2}}{1 - v\cos\theta/c} = \lambda \frac{1 + v\cos\theta'/c}{(1 - v^2/c^2)^{1/2}}. \tag{38.14}$$

If now an imaginary source is at rest at point S' (the emission position of the real source) and emits a photon with frequency ν_{int} and wavelength λ_{int}, then the frequency and wavelength registered by the real observer when he crosses point O will be (use formulas (12) and (10))

$$\nu_o = \nu_{int} \frac{1 - v_o\cos\theta/c}{(1 - v_o^2/c^2)^{1/2}} = \nu_{int} \frac{(1 - v_o^2/c^2)^{1/2}}{1 + v_o\cos\theta'/c}, \tag{38.15}$$

$$\lambda_o^o = \lambda_{int}. \tag{38.16}$$

Putting (13) and (14) into (15) and (16), we obtain

$$\nu_o = \nu \frac{1 - v_o\cos\theta_o/c}{1 + v\cos\theta'/c} \left(\frac{1 - v^2/c^2}{1 - v_o^2/c^2}\right)^{1/2} = \nu \frac{1 - v\cos\theta/c}{1 + v\cos\theta_o'/c} \left(\frac{1 - v_o^2/c^2}{1 - v^2/c^2}\right)^{1/2}, \tag{38.17}$$

$$\lambda_o^o = \lambda \frac{(1 - v^2/c^2)^{1/2}}{1 - v\cos\theta/c} = \lambda \frac{1 + v\cos\theta'/c}{(1 - v^2/c^2)^{1/2}}. \tag{38.18}$$

For the special case where source and observer move along together with the same velocity, $\vec{v}_o = \vec{v}$, we have $\theta_o = \pi - \theta'$, $\theta_o' = \pi - \theta$, and formulas (17) give

$$\nu_o = \nu, \tag{38.19}$$

while formulas (18) remain the same, and

$$\nu_o\lambda_o^o = c', \tag{38.20}$$

c' being the relative light velocity with respect to source and observer. Now θ is the angle between the direction opposite to that of light propagation and the velocity of source and observer registered with respect to both of them, while θ' is the same angle registered with respect to absolute space.

In formula (19), ν is the frequency of the photons emitted by the source moving at velocity v and, thus, λ in the corresponding formulas (18) is the emitted wavelength of such photons. However, if the <u>same</u> source should be at rest in absolute space (remember that only when the source is at rest in absolute space can an observer, also at rest, measure the emitted frequency and wavelength), the period of the emitted photons will become shorter (and the frequency higher) because of the absolute time dilation; thus the emitted wavelength λ of such photons will become shorter (by the factor $(1 - v^2/c^2)^{1/2}$), so that instead of (18) we have to write

$$\lambda_o = \frac{\lambda}{1 - v\cos\theta/c} = \lambda \frac{1 + v\cos\theta'/c}{1 - v^2/c^2}, \tag{38.21}$$

and now

$$\nu_o\lambda_o = c_o', \tag{38.22}$$

c_0' being the proper relative light velocity with respect to source and observer, and λ the wavelength of the photons if the source would be at rest in absolute space.

Equation (19) shows that if an observer moves with the same velocity as the light source, then the measurement of the received frequency can never give information about their absolute velocity. However, equation (21) shows that the measurement of the wavelength can give such information. These conclusions are of extreme importance. Let us note that according to contemporary physics, which proceeds from the principle of relativity, a Doppler effect appears only when source and observer move with respect to one another. By contrast, we have shown that a Doppler effect appears also when source and observer move with the same velocity, to wit, the received wavelength is different from that which would be measured if source and observer were at rest in absolute space. In §66 and §67 we analyse which are the real possibilities for the measurement of the wavelength in a moving frame.

. 4-dimensional treatment

All Doppler effect formulas in this section can be obtained by the help of the 4-dimensional mathematical apparatus. We shall deduce the general formula for source and observer both moving.

The scalar product $\overleftrightarrow{k}_{em}.(\overleftrightarrow{v}_0)_{rec}$, where $\overleftrightarrow{k}_{em}$ is the 4-wave-vector of the emitted photon and $(\overleftrightarrow{v}_0)_{rec}$ is the 4-velocity of the receiver is a 4-dimensional invariant and thus we can write

$$\overleftrightarrow{k}_{em}.(\overleftrightarrow{v}_0)_{rec} = \overleftrightarrow{k}_{rec}.(\overleftrightarrow{v}_0)_{em}, \qquad (38.23)$$

where $\overleftrightarrow{k}_{rec}$ is the 4-wave-vector of the received photon and $(\overleftrightarrow{v}_0)_{em}$ is the 4-velocity of the emitter.

Using (19.1) and (31.31) in (23), we obtain

$$\frac{\frac{\nu}{c} v_0 \cos(\vec{n},\vec{v}_0) - \nu}{(1 - v_0^2/c^2)^{1/2}} = \frac{\frac{\nu_0}{c} v \cos(\vec{n},\vec{v}) - \nu_0}{(1 - v^2/c^2)^{1/2}}, \qquad (38.24)$$

where we have used the same type of notations as in the previous subsections (thus the subscript "o" signifies "observer" and not "proper").

Taking into account that (see fig. 38.2)

$$\cos(\vec{n},\vec{v}_0) = \cos(\theta_0), \qquad \cos(\vec{n},\vec{v}) = \cos(\pi - \theta') = -\cos\theta', \qquad (38.25)$$

because $\vec{n}$ is the direction of propagation of the photon in absolute space (i.e., along the line S'O in fig. 38.2), $\vec{v}$ is the velocity of the emitter (source), and $\vec{v}_0$ is the velocity of the receiver (observer), from (24) and (25) we obtain the first formula (17).

The 4-dimensional invariants in (23) have the same value in any inertial frame and thus we conclude that the relation (24) is independent of the absolute velocity of the inertially moving laboratory. This, however, is not true if instead of the frequencies we should write in (24) the wavelengths (see §33D), as $\vec{v}_o$ has no influence on λ.

F. The equivalence of Compton and Doppler effects

Now we shall show that the Compton and Doppler effects represent two faces of the same coin, i.e., that they are equivalent.

The elastic collision of a photon and a particle can be considered as "absorption" of the photon by the particle followed by an immediate "re-emission", i.e., as a reflection of the photon by the particle. The "absorbed" frequency of the photon will be denoted by ν_{int} (ν intermediary) and the "re-emitted" frequency, which can be received by an observer at rest with respect to the primordial emitter of the photon, by ν'.

According to the Doppler effect formula (12), for the case of a source at rest (the primordial emitter) and a moving observer (the reflecting particle), which involves the reception angle θ, letting $\nu_o = \nu_{int}$, we have

$$\nu_{int} = \nu \frac{1 - \frac{v}{c}\cos(\vec{n},\vec{v})}{(1 - v^2/c^2)^{1/2}}, \tag{38.26}$$

since $\theta = (\vec{n},\vec{v})$ is the angle between the wave-vector of the received light and the velocity of the observer.

On the other hand, according to the Doppler effect formula (5), for the case of a moving source (the reflecting particle) and receiver at rest (the observer), which involves the emission angle θ', letting $\nu_o = \nu'$, $\nu = \nu_{int}$, $v = v'$, we have

$$\nu' = \nu_{int} \frac{(1 - v'^2/c^2)^{1/2}}{1 - \frac{v'}{c}\cos(\vec{n}',\vec{v}')}, \tag{38.27}$$

since $\theta' = (-\vec{n}',\vec{v}')$ is the angle between the opposite wave-vector of the emitted light and the velocity of the source.

From formulas(26) and (27) we obtain the Compton formula (35.20).

We must emphasize that when the particle is elementary (for example,an electron), its mass cannot change and the "re-emission" must follow immediately after the "absorption", i.e., the photon will only be reflected by the particle. If the particle is compound (for example, an atom), its mass can change and the re-emission can follow a certain time after the absorption.

The Doppler effect formulas give the relation between the frequencies of the emitted and received light when source and observer move with respect to one another. In the Compton effect source and observer are at rest. However, between them there is a moving "mirror" (the particle) which, moreover, changes its velocity under the hit of the photon. Hence it is obvious that the Doppler-effect formulas (where we are interested only in the mirror's velocity before and after the reflection of the photon) must yield the same relations as obtained from the momentum and energy conservation laws.

Formulas (35.18) and (35.19) represent four relations for six unknown quantities: $\vec{v}'$, ν', $\vec{n}'$. Thus two of these quantities must be taken arbitrarily and they are determined by the unit vector $\vec{n}_r$ which is perpendicular to the "reflecting plane" of the moving "mirror" (Doppler treatment), or by the unit vector $\vec{n}'$ along the direction of propagation of the "re-emitted" photon (Compton treatment). Using the law of light reflection (the incident and reflected rays lie in the same plane with the perpendicular to the reflecting plane and make equal angles with it (see §41A), we can find $\vec{n}_r$ when $\vec{n}$ and $\vec{n}'$ are given, or $\vec{n}'$ when $\vec{n}$ and $\vec{n}_r$ are given.

Thus the Compton scattering represents a Doppler effect where one observes reflection of light from a "mirror" which changes its velocity under the action of any single photon.

The experiments of Lebedev (1901) on light pressure represent a Compton effect for a macroscopic body where the mirror changes its velocity under the action of many incident photons.

§39. ABERRATION (THE BRADLEY EFFECT)

The ABERRATION OF LIGHT is the difference in the angles between the directions of propagation of emitted and received light due to the motion of source or/and observer with respect to absolute space. This effect was observed first by Bradley (1727).

It is clear that the aberration of light and the light Doppler effect are closely connected and always when there is a Doppler effect one has also aberration of light. At a longitudinal Doppler effect, when the difference between emitted and received frequencies is maximum, the aberration is equal to zero, and at a traverse Doppler effect, when this difference is equal to zero, the aberration is maximum.

The angle of aberration S'O'S or OS'O' (see fig. 38.1) is designated by α and we have

$$\alpha = \theta' - \theta. \tag{39.1}$$

If t is the time between emission and reception, we have

$$S'S = vt, \qquad O'S_m = ct. \tag{39.2}$$

Let us raise a perpendicular from the point S_m to the line S_mO'. As α is a small angle, we can write

$$S'S \sin\theta_m = O'S_m \alpha. \tag{39.3}$$

Substituting here (2), we obtain the so-called ABERRATION FORMULA

$$\alpha = \frac{v}{c} \sin\theta_m. \tag{39.4}$$

This formula can be obtained also from the relations (6.16) between the emission and reception angles.

Indeed, writing in terms of first order in v/c

$$\theta' \cong \theta \cong \theta_m, \tag{39.5}$$

we obtain from formulas (6.16)

$$\cos\theta + \frac{v}{c}\cos^2\theta_m = \cos\theta' + \frac{v}{c}. \tag{39.6}$$

Taking into account the third formula (I,9.9), we can write (6) in the form

$$2\sin\theta_m \sin\frac{\theta' - \theta}{2} = \frac{v}{c}\sin^2\theta_m, \tag{39.7}$$

from where we obtain the aberration formula (4).

The aberration of light in astronomy is briefly discussed in IV, §22B. The relation

$$K = v/c \tag{39.8}$$

is called the ABERRATION CONSTANT if we take for v the Earth's velocity on its trajector around the Sun. Since v = 29.77 km/sec, we obtain

$$K = 20''.47. \tag{39.9}$$

During a year any star describes an ellipse on the celestial sphere whose major semi-axis is equal to K and is parallel to the ecliptic. The minor semi-axis is equal to Ksin where β is the celestial latitude of the star (see §46).

§40. PROPAGATION OF LIGHT IN A MEDIUM

MEDIUM is a space domain filled with particles the distances between which remain constant. When the medium is set in motion, all particles move with the same velocity.

A photon propagating through a medium can be absorbed, reflected, or absorbed and re-emitted by particles of the medium. In the first case we call the medium OPAQUE, in the second DISPERSIVE, and in the third TRANSPARENT. As a rule, the re-emission of a photon by a particle occurs a certain time after its absorption and proceeds along th direction of incidence. This is our "model" for the propagation of light in a medium. We have to add, however, that the mechanism of propagation of light in a transparent medium is not clear, because one cannot say whether there is an absorption and re-emission of the photons by the particles, or the velocity of the photons diminishes because of a certain other cause. As we show in §44H, the convection of light by a medium is a purely kinematical phenomenon and can be analysed without taking into account the mechanism of propagation of light in a medium by which the light velocity in a transparent medium becomes lower than c. One can suppose that this effect may be due to the diminution of light velocity in a stronger gravitational field (see IV, §65), namely, in the gravitational field of the medium's particles. Although the particles' masses are very small and their gravitational action very feeble, nevertheless, when the distances between the photons and the particles become very small (when the photon "crosses" the particle) the particle's gravitational potential can become very strong. Future experimental and theoretical investigations may show whether this is the cause for the diminution of light velocity in a medium.

We shall consider the problem of how to determine the velocity of light in a transparent medium when the medium, or the observer, or both move with respect to absolute space.

A. Medium and observer both at rest.

If the photon travels a distance c/n through a medium in a unit of time, then it may be concluded that the photon propagates (1/n)th of the time as a "free" photon and (1 - 1/n)th of the time it rests "absorbed" (or "hitched") by the particles. Thus, with respect to the observer, who is also at rest in absolute space, it moves with the mean velocity

$$c_m = c/n.$$

The factor n is called the REFRACTIVE INDEX of the medium.

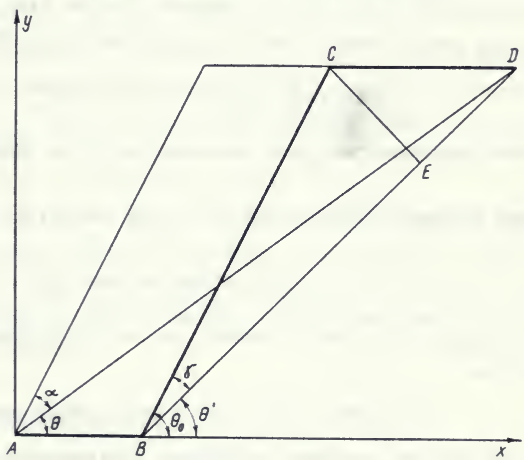

Fig. 40.1

B. Medium moving, observer at rest (the Fizeau effect)

Let us now calculate the velocity of light in a medium moving with velocity v in absolute space with respect to an observer at rest.

Suppose first (see fig. 40.1) that the medium moves with velocity v along the x-axis of the rest frame K only during this time when the photon is absorbed by some particle (molecule) of the medium and suppose that during the time between the re-emission and next absorption the medium is at rest. If we consider the path of the photon between two successive absorptions, this path can be represented by the broken line ABC in fig. 40.1. Letting the time between two successive absorptions be unity, i.e.,

$$AB/v + BC/c = 1, \tag{40.2}$$

we obtain

$$AB = v(1 - 1/n), \qquad BC = c/n. \tag{40.3}$$

If now we suppose that the medium moves with velocity v during the whole time, then the next molecule will be met not at point C but at point D, where the distance CD is covered by this molecule in the time in which the photon covers distance BD, i.e.,

$$CD = v/n. \tag{40.4}$$

Thus now the distance covered by the photon between two successive re-emission and absorption will be not BC but

$$BD = BE + ED = (\frac{c^2}{n^2} - \frac{v^2}{n^2} \sin^2\theta')^{1/2} + \frac{v}{n} \cos\theta', \tag{40.5}$$

where

$$\theta' = \theta_0 - \gamma \tag{40.6}$$

is the angle between the "free path" of the photon and the velocity of the medium with respect to the observer (i.e., to frame K) while θ_0 is the same angle with respect to the medium (i.e., to the moving frame K' which is attached to the medium). The angle

$$\gamma = \arcsin\frac{CE}{BC} = \frac{v/n}{c/n} \sin\theta' = \frac{v}{c} \sin\theta_0 \tag{40.7}$$

represents the difference between these two angles which is small, being of the order of v/c.

Within the same accuracy of first order in v/c we can write, having in mind (6) and (7),

$$\cos\theta' = \cos\theta_0 + \frac{v}{c} \sin^2\theta_0. \tag{40.8}$$

The distance covered by the photon between two successive absorptions with respect to the observer will be

$$AD^2 = (AB + CD)^2 + BC^2 + 2(AB + CD)\, BC \cos\theta_0. \tag{40.9}$$

Substituting (3) and (4) into (9), and working with an accuracy of second order in v/c we obtain

$$AD = (\frac{c^2}{n^2} + 2\,\frac{vc}{n} \cos\theta_0 + v^2)^{1/2} = \frac{c}{n} + v\cos\theta_0 + \frac{1}{2}\,\frac{v^2}{c} n \sin^2\theta_0. \tag{40.10}$$

To obtain the mean velocity of the photon with respect to the observer, we have to divide the distance AD by the time the photon travels the broken line ABD. This time, to within an accuracy of second order in v/c, is

$$t_m = \frac{AB}{v} + \frac{BD}{c} = 1 + \frac{v}{cn} \cos\theta' - \frac{1}{2}\,\frac{v^2}{c^2 n} \sin^2\theta' = 1 + \frac{v}{cn} \cos\theta_0 + \frac{1}{2}\,\frac{v^2}{c^2 n} \sin^2\theta_0, \tag{40.11}$$

where (3), (5), and (8) have been used.

Thus, the mean velocity of the photon in the moving medium measured by the observer at
est, to within an accuracy of second order in v/c, becomes

$$_m = \frac{AD}{t_m} = \frac{c}{n} + v(1 - \frac{1}{n^2})\cos\theta_o - \frac{v^2}{cn}(1 - \frac{1}{n^2})\cos^2\theta_o + \frac{1}{2}\frac{v^2 n}{c}(1 - \frac{1}{n^3})\sin^2\theta_o. \quad (40.12)$$

The angle θ between the velocity of the medium and the mean velocity of the photon,
hat is measured by the observer at rest, is given by

$$\theta = \theta_o - \alpha, \quad (40.13)$$

here

$$\alpha \cong \frac{v\sin\theta}{c/n} = \frac{vn}{c}\sin\theta \quad (40.14)$$

s small, being of the order of v/c.

To within the accuracy of first order in v/c we can write, using (13) and (14),

$$\cos\theta_o = \cos\theta - \frac{vn}{c}\sin^2\theta. \quad (40.15)$$

Substituting this into (12) yields

$$_n = \frac{c}{n} + v(1 - \frac{1}{n^2})\cos\theta - \frac{v^2}{cn}(1 - \frac{1}{n^2})\cos^2\theta - \frac{1}{2}\frac{v^2}{c}(1 - \frac{2}{n^2} + \frac{1}{n^3})\sin^2\theta. \quad (40.16)$$

This "model" for the propagation of photons in a moving medium is called by us the
HITCH-HIKER" MODEL. Let us mention that in our youth, when crossing countries by "hitch-
iking", we never waited for the next car at the same point where have been dropped by
he previous one, but always tried to "gain" more distance by walking. Of course, our pe-
estrian velocity could never be higher than that of the cars.

The change of light velocity in a moving medium is called by us the DRAG OF LIGHT or
he FIZEAU EFFECT, as Fizeau (1851) has observed it for the first time. The factor
$1 - 1/n^2$) is called the FRESNEL DRAG COEFFICIENT, as Fresnel predicted the effect procee-
ing from a model, according to which in media with different index of refraction the
aether" has a different density.

. Medium at rest, observer moving (the Dufour effect)

Let us now assume that a medium with refractive index n is at rest in absolute space.
ight is assumed to propagate along a direction that makes an angle θ with respect to the
-axis of a frame K attached to absolute space, as shown in fig. 40.2. An observer attach-
d to a frame K' is assumed to move at velocity v along the positive x-direction of frame
, the axes of both frames being colinear.

We choose again the time between two successive absorptions of a photon by the mole-
ules of the medium as unity. Thus the fraction of the time a photon is "hitched" to a
olecule at point A is 1 - 1/n. The fraction of the time it moves along the line AF until
ecomes "hitched" again to another molecule at point F is 1/n.

In the moving frame K' we have the following picture: During the time in which the pho-

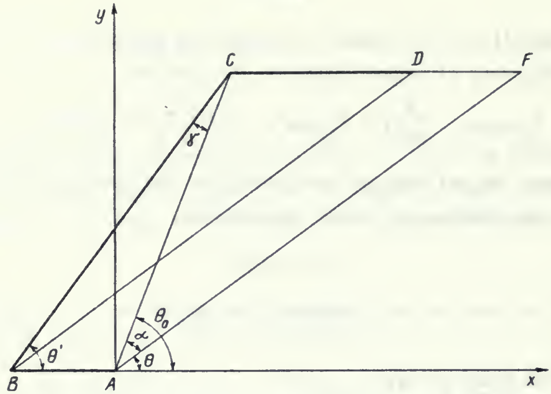

Fig. 40.2

ton is "hitched" it will cover distance AB with velocity v and during the time in which the photon propagates with velocity c in absolute space it will cover distance BC in K' (with an angle θ' to the x-axis) with the proper relative velocity (see (4.28))

$$c'_o = \frac{c}{1 + v\cos\theta'/c}, \tag{40.17}$$

since during the time in which the photon has covered the broken line ABC in frame K' the molecule that rests at point F in absolute space has covered distance FC in K' with velocity v. The mean proper relative light velocity in frame K' (i.e., the average light velocity measured in K' by the help of a clock which rests there) will make an angle θ_o with the x'-axis and have magnitude

$$c'_m = AC = (AB^2 + BC^2 - 2\,AB\,BC\cos\theta')^{1/2}, \tag{40.18}$$

since the time between two successive absorptions of the photon is taken equal to unity. We do not attach the index "o" for "proper" to this velocity with the aim to make a distinction between it and the mean velocity of light from §40D.

Putting into (18)

$$AB = v(1 - \frac{1}{n}), \qquad BC = \frac{c}{1 + v\cos\theta'/c}\,\frac{1}{n} \tag{40.19}$$

and working within an accuracy of second order in v/c, we obtain

$$c'_m = \frac{c}{n} - v\cos\theta' + \frac{v^2}{cn}\cos^2\theta' + \frac{1}{2}\frac{v^2}{c}\,n(1 - \frac{1}{n})^2\sin^2\theta'. \tag{40.20}$$

The angle which the observer in frame K' will measure between the direction of light propagation and his own velocity is θ_o. Thus, substituting into (20)

$$\theta' = \theta_o - \gamma, \tag{40.21}$$

where γ is a small angle and taking

$$\sin\gamma = \frac{AB\sin\theta'}{AC} \cong \frac{v}{c}(n-1)\sin\theta' \cong \frac{v}{c}(n-1)\sin\theta_o, \tag{40.22}$$

we obtain

$$c'_m = \frac{c}{n} - v\cos\theta_o + \frac{v^2}{cn}\cos^2\theta_o - \frac{1}{2}\frac{v^2}{c}n\left(1 - \frac{1}{n^2}\right)\sin^2\theta_o. \tag{40.23}$$

The angle between the direction of propagation of light and the velocity of the observer which should be measured in frame K is θ. Thus, putting into (23)

$$\theta_o = \theta + \alpha, \tag{40.24}$$

where α is a small angle and taking

$$\sin\alpha = \frac{CF\sin\theta}{AC} \cong \frac{v}{c}n\sin\theta, \tag{40.25}$$

we obtain

$$c'_m = \frac{c}{n} - v\cos\theta + \frac{v^2}{cn}\cos^2\theta + \frac{1}{2}\frac{v^2}{c}n\left(1 + \frac{1}{n^2}\right)\sin^2\theta. \tag{40.26}$$

The change of the light velocity for an observer moving with respect to the medium is called by us the DUFOUR EFFECT, as Dufour and Prunier (1942) observed it for the first time on the rotating disk (see §59B).

D. Medium and observer both moving (the Marinov effect)

Let us now find the velocity of light in a medium moving at velocity v with respect to absolute space, as measured by an observer attached to the medium.

Since in such a case for $(1-1/n)$th part of the time the photon is "hitched" and does not move with respect to the frame K' which is attached to the moving medium, the "effective" velocity of the frame with respect to the free photon will be v/n. Thus, according to formula (4.28), the proper velocity of the free photon with respect to K' will be (letting $V = v/n$ in (4.28))

$$c'_o = \frac{c}{1 + \frac{v}{cn}\cos\theta'} = c\,\frac{1 - \frac{v}{cn}\cos\theta}{1 - v^2/c^2n^2}. \tag{40.27}$$

The photon moves with this velocity only $(1/n)$th part of the time, so that the mean proper velocity of light with respect to K' is

$$c'_{om} = \frac{1}{n}c'_o = \frac{c}{n}\,\frac{1}{1 + \frac{v}{cn}\cos\theta'} = \frac{c}{n}\,\frac{1 - \frac{v}{cn}\cos\theta}{1 - v^2/c^2n^2}, \tag{40.28}$$

where θ' and θ are the angles between the direction of light propagation and the velocity of the medium measured in the moving and rest frames, respectively.

The change of the light velocity when medium and observer are both moving is called by us the MARINOV EFFECT, as Marinov (1974) observed it for a first time, taking as a medium air, practically vacuum (see §51 and §52). On the rotating disk this effect was observed for the first time by Harress (1912), whose medium was glass, and by Sagnac (1913), whose medium was air, practically vacuum. The Marinov effect for closed paths of the photons is called by us the SAGNAC EFFECT. The difference between the Marinov and Sagnac effect is the following: The Marinov effect is proportional to the straight paths covered by the photons in the moving medium and to the absolute linear velocity of the medium, while the Sagnac effect is proportional to the area encircled by the closed paths of the photons and to the absolute angular velocity of the medium (abundant information on these two extremely important effects can be found in many sections of ch. VIII).

E. Application of the Lorentz transformation

For the case B (medium moving, observer at rest), the use of the Lorentz transformation leads to the same results. Indeed, putting into the first two formulas (4.20), according to fig. 40.1,

$$v'_x = (c/n)\cos\theta_o, \qquad v'_y = (c/n)\sin\theta_o, \qquad V = v, \tag{40.29}$$

and working within an accuracy of second order in v/c, we obtain for $(v_x^2 + v_y^2)^{1/2}$ formul (12).

For the components of the velocity the identity is only within terms of first order i v/c. Indeed, if we use in the equations

$$c_m\cos\theta = v_x, \qquad c_m\sin\theta = v_y \tag{40.30}$$

formulas (12) and (15) on the left sides, and formulas (4.20) and (29) on the right side we see that only the terms of zeroth and first order in v/c are identical on both sides of these equations.

For the case C (medium at rest, observer moving), the use of the Lorentz transformati in the context of the relativity theory must give the same result as for the case B, because these two cases are identical for the theory of relativity. As we have seen, however, and as it was experimentally established, the velocities of light for these two cases are substantially different. The case D, according to the theory of relativity, must be identical to the case A, meanwhile we have seen and it is experimentally established that these two cases, again, are substantially different.

§41. PHENOMENA ON THE BOUNDARY OF TWO MEDIA

A. Reflection

If photons meet a boundary surface between two media on which they are reflected, the corpuscular model of light leads immediately to the following LAW OF REFLECTION (see fig 41.1):

The INCIDENT ANGLE ϕ (i.e., the angle between the line of propagation of the incident photons and the perpendicular raised to the boundary) is equal to the REFLECTION ANGLE ϕ

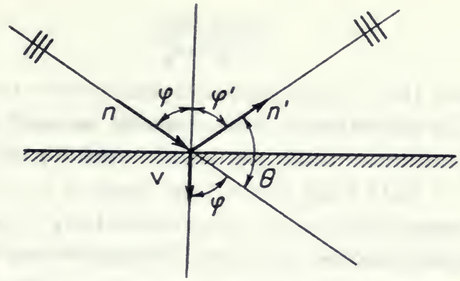

Fig. 41.1

(i.e., the angle between the line of propagation of the reflected photons and the same perpendicular), and the three mentioned lines lie in the same plane.

This law can be deduced from the formulas obtained in §35. Indeed, if we assume $h\nu \ll mc^2$, i.e., if the energy of the incident photons is very small with respect to the universal energy of the reflecting medium, then from formula (35.5) we obtain $\nu = \nu'$, i.e., the energies of the incident and reflected photons are equal. Then from formula (35.1) we obtain

$$\vec{v}' = \frac{h\nu}{mc}(\vec{n} - \vec{n}'), \qquad (41.1)$$

i.e., the velocity $\vec{v}'$ of the reflecting medium is infinitely small and its direction, which defines the direction of the perpendicular raised to the boundary at the point of incidence, divides the angle between $-\vec{n}$ and $\vec{n}'$ into two equal parts, so that we obtain the law of reflection

$$\phi = \phi'. \qquad (41.2)$$

The same law can be obtained also from formula (35.10) under the same condition $h\nu \ll mc^2$, if we take into account that (cf. fig. 35.1 with fig. 41.1)

$$\phi' = \pi/2 - \theta/2, \qquad (41.3)$$

where θ is the angle between $\vec{n}$ and $\vec{n}'$.

. Refraction

Let us now consider the case where the photons are not reflected by the boundary surface but enter the second medium propagating through it. Our "hitch-hiker" model of light propagation leads immediately to the following LAW OF REFRACTION (called also SNELL'S LAW) (see fig. 41.2):

The relation between the INCIDENT ANGLE ϕ (i.e., the angle between the line of propagation of the photons in the firts medium, with index of refraction n_A, and the perpendicular raised to the boundary) and the REFRACTION ANGLE ψ (i.e. the angle between the line of propagation of the photons in the second medium, with index of refraction n_B, and the same perpendicular) is the following

$$\frac{\sin\psi}{\sin\phi} = \frac{n_A}{n_B}, \tag{41.4}$$

and the three mentioned lines lie in the same plane.

Before demonstrating the validity of this formula, we shall give a simple analogy.

Let us consider a squadron of marching soldiers (see fig. 41.2). If the soldiers march with a given speed in a field A and with a lower speed in a field B, and if the squadron meets the boundary between both fields not perpendicularly, then the soldiers on the one flank of a given row which reaches the boundary between fields A and B should begin to march with a lower speed, while the soldiers on the other flank will continue to march a certain time with the initial speed, until they also reach the boundary. The squadron must change its direction after crossing the boundary; as we shall show billow, the "refraction" must be exactly the same to that prescribed by Snell's law, if c/n_A is the marching speed of the soldiers in the field A and c/n_B in the field B.

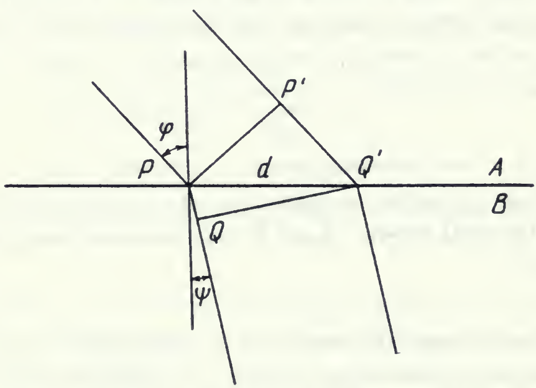

Fig. 41.2

Let us now replace the squadron of soldiers by a bundle of photons (a narrow light beam) which is incident under an angle ϕ on the boundary between the media A and B with indices of refraction n_A and n_B ($n_B > n_A$).

Consider two photons which lie on the extremities of the bundle and on a line which is perpendicular to the bundle. Let the distance between the points at which these two flank photons cross the boundary be d. After the instant at which the first flank-photon crosses the boundary, the second flank-photon has to move a certain time t with the velocity c/n_A in the medium A and cover a distance $d\sin\phi$, until it also reaches the boundary. Thus we can write

$$t = \frac{d\sin\phi}{c/n_A}. \tag{41.5}$$

During this time t, the first photon will move with velocity c/n_B in the medium B and will cover the distance $d\sin\psi$, where ψ is the refraction angle (note that when the second flank-photon has reached the boundary the first and second flank-photons must lie on a line which is perpendicular to the bundle). Thus we can write

$$t = \frac{d\,\sin}{c/n_B}. \qquad (41.6)$$

From the last two formulas we obtain Snell's law (4).

It is important to note that one can speak about a reflection of a single photon, while one can only speak about a refraction of a bundle of photons.

§42. DRAG ABERRATION (THE JONES EFFECT)

The DRAG ABERRATION OF LIGHT is the lateral displacement of a light ray due to the motion of the medium crossed by light. It is clear that the drag aberration and the Fizeau effect are closely connected and always when there is a Fizeau effect one has also a drag aberration of light. The drag aberration was observed for the first time by Jones (1975), and for this reason we call it the JONES EFFECT (see §58).

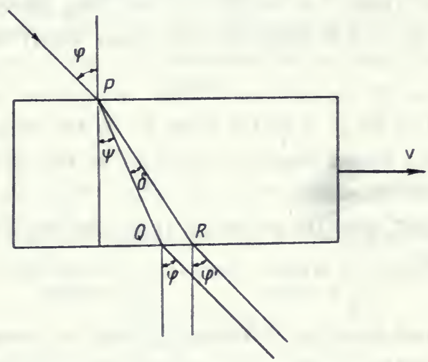

Fig. 42.1

Let us have (see fig. 42.1) a transparent medium with parallel surface planes on which a light beam is incident at an angle ϕ at a point P. If the angle of refraction is ψ, the beam will leave the medium at a point Q at the angle of incidence ϕ.

Let the medium be set in motion with velocity v from left to right. Now the light beam will be displaced laterally to the point R. We call the angle δ = QPR the DRAG ANGLE (or the DRAG ABERRATION).

We shall find the drag angle, using our "hitch-hiker" model for light propagation, and performing all calculations within an accuracy of first order in v/c.

Formula (40.12) shows that, to within an accuracy of first order in v/c, the velocity of light in a moving medium represents a vector sum of the velocities $\vec{c}/n$ and $\vec{v}(1 - 1/n^2)$, θ_0 being the angle between them.

Taking into account that the photons cross the medium for a time

$$t = \frac{PQ}{c/n} = \frac{d\,n}{c\,\cos\psi}, \qquad (42.1)$$

where d is the thickness of the medium, we obtain for the resultant distance QR along which the photons will be drgged

$$QR = v(1 - \frac{1}{n^2})t = \frac{d\,v}{c\cos\psi}(n - \frac{1}{n}). \qquad (42.2)$$

Thus the drag angle will be

$$\delta = \frac{QR\cos\psi}{PQ} = \frac{v}{c}(n - \frac{1}{n})\cos\psi. \qquad (42.3)$$

The theory of relativity solves the problem about the drag of light applying automatically the Lorentz transformation without referring to the physical model of light propagation in a medium (see §40E). Now we shall show that such an automatic application of the Lorentz transformation leads to a physically incorrect result. Let us note that Rogers (1975) and Players (1975) made a theoretical analysis of the drag aberration, using the Lorentz transformation for velocities, for the case $\phi = 0$ only, where the results coincide with those of our theory. An analysis of the drag aberration by the help of the Lorentz transformation for $\phi \neq 0$ (which will be given below) has been made by nobody.

xxxxx

Let us attach (see fig. 42.1) a moving frame K' to the medium and a rest frame K to the laboratory, so that their x-axes should be parallel to the velocity V of the medium and their y-axes should point downwards.

The components of light velocity in vacuum (i.e., before the entrance in the medium), measured in frame K, are

$$c_x = c\sin\phi, \qquad c_y = c\cos\phi. \qquad (42.4)$$

The components of light velocity in vacuum measured in frame K', on the grounds of the direct Lorentz transformation (4.20), are (we omit the subscript "L" to the Lorentz velocities - see the remark on p. 59)

$$c'_x = \frac{c\sin\phi - V}{1 - V\sin\phi/c}, \qquad c'_y = \frac{c\cos\phi(1 - V^2/c^2)^{1/2}}{1 - V\sin\phi/c}. \qquad (42.5)$$

Thus the angle of incidence of the photons in frame K' will be

$$\sin\phi' = \frac{c'_x}{c} = \frac{\sin\phi - V/c}{1 - V\sin\phi/c} \cong \sin\phi - \frac{V}{c}\cos^2\phi, \qquad (42.6)$$

and the angle of refraction in frame K', according to Snell's law (see formula (41.4), putting there $n_A = 1$, $n_B = n$) will be

$$\sin\psi' = \frac{\sin\phi'}{n} = \frac{1}{n}(\sin\phi - \frac{V}{c}\cos^2\phi) = \sin\psi + \frac{V}{c}(n\sin^2\psi - \frac{1}{n}). \qquad (42.7)$$

The components of the velocity of the photons in the medium with respect to frame K' will be

$$c'_{mx} = \frac{c}{n}\sin\psi', \qquad c'_{my} = \frac{c}{n}\cos\psi', \qquad (42.8)$$

nd the components of this velocity with respect to frame K, according to the inverse Lorentz transformation (4.21), will be

$$c_{mx} = \frac{c'_{mx} + V}{1 + c'_{mx}V/c^2}, \qquad c_{my} = \frac{c'_{my}(1 - V^2/c^2)^{1/2}}{1 + c'_{mx}V/c^2}. \qquad (42.9)$$

Thus, when the medium is moving, the refraction angle with respect to frame K will be

$$\sin(\psi + \delta) = \frac{c_{mx}}{c_m} = \sin\psi + \frac{V}{c}(n - \frac{1}{n}). \qquad (42.10)$$

nd for the drag angle we obtain

$$\delta \cong \sin\delta = \frac{V}{c}(n - \frac{1}{n})\frac{1}{\cos\psi}. \qquad (42.11)$$

This result is unsound, because for $\psi \to \pi/2$ it gives $\delta \to \infty$, while, obviously, the drag ngle must go to zero, $\delta \to 0$, as this can be obtained from our formula (3). Thus, it is lear that the application of the Lorentz transformation to the drag aberration leads to n unsound result. This unsound result has been obtained because we used in the same probem both the direct and inverse Lorentz transformations for velocities, but, as we know ell, the Lorentz velocities are not the right velocities of the particles.

§43. RELATION BETWEEN REFRACTIVE INDEX AND DENSITY

The relation between the index of refraction n of a transparent medium and its density is usually assumed to be given by the well-known formula of Lorentz-Lorenz

$$\frac{n^2 - 1}{n^2 + 2}\frac{1}{\mu} = K_L, \qquad (43.1)$$

here K_L is a constant which we call the LORENTZ-LORENZ CONSTANT.

However, our model for light propagation in a transparent medium leads to the relation

$$\frac{n - 1}{\mu} = K_M, \qquad (43.2)$$

here K_M is a constant which we call the MARINOV CONSTANT.

We derive formula (2) in the following extremely simple way:

As stated in §40, at a density μ of the transparent medium, $1/n$ is the fraction of ime in which, on the average, a photon travels with velocity c in vacuum and $1 - 1/n$ is he fraction of the time during which, on the average, the photon remains "hitched" to the olecules of the medium. Now suppose that the density of the medium changes from μ to μ'. he refractive index will change respectively from n to n'. For that distance for which he photon was "hitched" $1 - 1/n$ seconds and has traveled $1/n$ seconds it will now be hitched" $(\mu'/\mu)(1 - 1/n)$ seconds and, as before, will travel $1/n$, since the sum of the free flight" distances remains the same and only the numbers of the "hitch-points" has hanged. Thus we can write

$$1 - \frac{1}{n'} = \frac{(\mu'/\mu)(1 - 1/n)}{(\mu'/\mu)(1 - 1/n) + 1/n}. \qquad (43.3)$$

From here formula (2) can immediately be obtained.

Our formula (2) finds a better fit with the experimental results than the Lorentz-Lorenz formula (1). In fig. 43.1 the index of refraction n is shown as a function of the density μ according to (1) and (2). The experimental points are taken from Michels et al (PHYSICA, 13, 343 (1947)). Michels et al. have measured the dependence $n = f(\mu)$ for ethylene. The density μ is given in Amagat units. Let us recall that a substance has density one Amagat unit if one mole of it takes up a volume of 22415 cm^3. Thus any gas at standard conditions has density 1 Am, and 1 Am = $\frac{1}{22415}$ mole/cm^3. We have taken for K_M the average experimental value which, according to our estimation, is $K_M = 7.60\times10^{-4}\ Am^{-1}$, and $K_L = \frac{2}{3}K_M$, as for low densities, when $n \cong 1$, formula (1) turns into

$$\frac{n - 1}{\mu} = \frac{3}{2}K_L . \qquad (43.4)$$

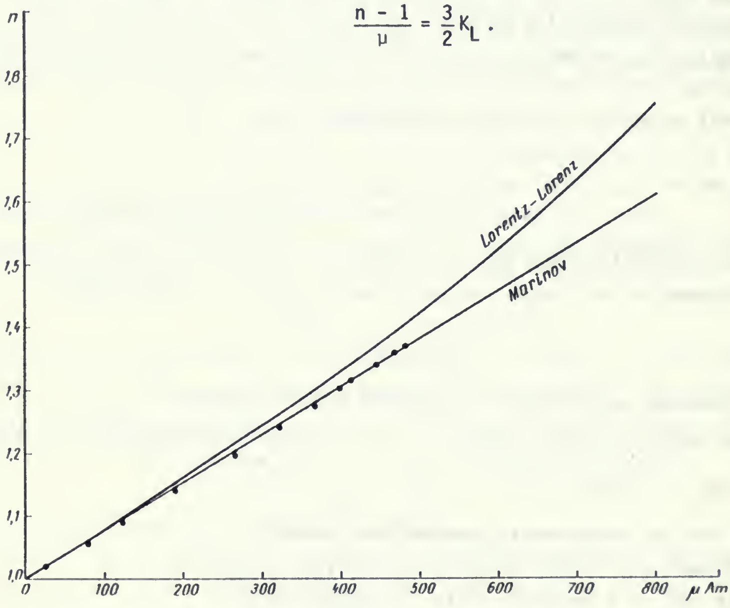

Fig. 43.1

Quite apart from these experimental results, formula (1) appears unsound from the point of view of simple logic. Indeed, writing formula (1) in the form

$$n = \left(\frac{1 + 2K_L\mu}{1 - K_L\mu}\right)^{1/2}, \qquad (43.5)$$

it may be seen that the refractive index becomes infinite at a certain critical density $\mu_{cr} = 1/K_L$. The Lorentz theory cannot offer a sound explanation of this peculiarity. According to our formula (2), no such peculiarity arises, and n becomes equal to infinity only for μ going to infinity.

§44. THE ABSOLUTE CHARACTER OF MOTION

A. The Marinov and Einstein forms of the proper quantities

In this section we shall present the fundamental difference in the treatment of motion between the theory of relativity and our absolute space-time theory. We consider this section as one of the most important in part III.

The proper energy (the proper time momentum multiplied by c) of a mass m depends on its velocity according to formula (23.9). According to the relativity theory v is the velocity of the particle with respect to this inertial frame of reference in which one observes the motion. Thus, according to relativity the time energy of a particle is a frame dependent quantity, and it must be the lowest in this inertial frame in which the particle is at rest; in any other frame in which the particle moves with a relative velocity v', its energy is larger, equal to

$$e_0' = \frac{mc^2}{(1 - v'^2/c^2)^{1/2}}. \qquad (44.1)$$

According to our absolute theory, v in formula (23.9) is the absolute velocity of the particle, i.e., its velocity with respect to absolute space. Thus the energy is a frame independent quantity, and in any frame it has the same value, as the absolute velocity of the particle does not depend on the frame in which one observes the motion. The energy is the lowest when the particle is at rest in absolute space. As according to our theory the energy of the particle is its gravitational energy with the mass of the whole world, and as the gravitational energy of mass m depends on the velocity of the latter, it is fully justified to consider v in formula (23.9) as the absolute velocity of the particle. We called (see p. 77) the frame independent quantities Marinov invariant quantities, and the proper time energy is such a Marinov invariant quantity.

With the aim to make a clear difference between the treatment of the time energy by the theory of relativity and by our absolute space-time theory, we call (1) EINSTEIN PROPER ENERGY (in general, EINSTEIN FORM OF THE PROPER QUANTITIES) and (23.9) MARINOV PROPER ENERGY (in general, MARINOV FORM OF THE PROPER QUANTITIES).

Let us consider now the space momentum). The proper momentum of mass m which covers distance $d\vec{r}$ in a frame K attached to absolute space during the proper time dt_0 (i.e., the time measured on a clock attached to the particle), according to formula (26.21), is

$$\vec{p}_0 = m\frac{d\vec{r}}{dt_0} = \frac{m\,d\vec{r}}{dt(1 - v^2/c^2)^{1/2}} = \frac{m\vec{v}}{(1 - v^2/c^2)^{1/2}} = m\vec{v}_0 = m_0\vec{v}, \qquad (44.2)$$

where dt is the corresponding universal time (i.e., the time measured on a clock attached to absolute space). Let us observe the motion of this mass in a frame K' moving with a velocity $\vec{V}$ in absolute space. If in this frame the mass covers a distance $d\vec{r}'$ during a proper time dt_0, its proper momentum with respect to K' will be the following

$$\vec{p}'_o = m\frac{d\vec{r}'}{dt_o} = \frac{m\,d\vec{r}'}{dt(1-v^2/c^2)^{1/2}} = \frac{m\vec{v}'}{(1-v^2/c^2)^{1/2}} = m\vec{v}'_o = m_o\vec{v}', \tag{44.3}$$

as we have always to reduce the proper time interval to universal time.

In the theory of relativity the proper momentum of the particle with respect to frame K' will be written

$$\vec{p}'_o = m\frac{d\vec{r}'}{dt_o} = \frac{m\,d\vec{r}'}{dt'(1-v'^2/c^2)^{1/2}} = \frac{m\vec{v}'}{(1-v'^2/c^2)^{1/2}}, \tag{44.4}$$

where the relative velocity $\vec{v}'$ is measured in the laboratory time (i.e., on a clock attached to the laboratory). Formula (3) gives the MARINOV PROPER MOMENTUM, while formula (4) gives the EINSTEIN PROPER MOMENTUM.

According to our theory, absolute space is the scene of all physical phenomena, and when performing precise calculations we have to always use the Marinov form of the proper quantities. According to the theory of relativity an absolute space does not exist, and any inertial frame is good enough, so that the Einstein forms of the proper quantities must be sufficient. The experimental evidence shows that our approach is this one which is adequate to physical reality.

The dependence of the proper energies and momenta of the particles on their absolute velocities shows that any particle is "related" to absolute space. Thus the motion of any particle has an absolute character. According to Einstein the motion of any particle is relative, even the propagation of light. The supporters of the aether conception affir that the motion of the massive particles is relative, but light moves in a hypothetical aether, as sound in the air, and, consequently, the propagation of light is absolute. According to our theory, the motion of any particle (massive or massles, microscopic or macroscopic) is absolute. As representatives of the massless particles one can taken photons, as representatives of the massive particles neutrons and electrons, but also bullets, birds, and tanks. According to our theory, even the motion of PHONONS (hypothetical particles attached to sound waves) is absolute, since the propagation of sound is propagation of energy and, thus, represents a transfer of mass. The phonons must show the same effects of attachment to absolute space as photons and neutrons. In §54 we consider an experiment which permits to show that the motion of sound is absolute.

B. The first-order in V/c effects

Let us consider two particles with masses M and m ($M \gg m$) which move, respectively, with the velocities $\vec{V}$ and

$$\vec{v}_a = \vec{v} + \vec{V} \tag{44.5}$$

in absolute space. Thus $\vec{v}$ is the relative velocity of m with respect to M (in other subsections the absolute velocity will be denoted by $\vec{v}$ and the relative velocity by $\vec{v}'$). In §44B we are interested into effects of first order in V/c, and for this reason we shall use the Galilean formula for velocity addition (5).

If the particles m and M will collide perfectly elastically, the momentum and energy conservation laws give (see (34.1))

$$m_o\vec{v}_a + M_o\vec{V} = m'_o\vec{v}'_a + M'_o\vec{V}', \tag{44.6}$$

$$m_oc^2 + M_oc^2 = m'_oc^2 + M'_oc^2, \tag{44.7}$$

where m_o, M_o, $\vec{v}_a$, $\vec{V}$ are the proper masses and the velocities before the collision, and m'_o, M'_o, $\vec{v}'_a$, $\vec{V}'$ are the same quantities after the collision. We can perform all calculations in this subsection by replacing the proper masses in (6) and (7) by their low-velocity expressions, i.e., by the factual masses (see (37.10)) but, for methodological purposes, we shall use the high-velocity expressions, i.e., the proper masses(23.20).

Remark. The elastic collision of two particles was considered in §34 by using the 4-dimensional formalism. In §34 we used the law of 4-momentum conservation operating with 4-dimensional scalars and for this reason the theory was Lorentz invariant. In §44 we consider the laws of momentum and energy conservation separately, and because of the Marinov invariance of the proper mass, absolute effect will come to light.

Multiplying equation (7) by $\vec{V}/c^2$ and subtracting it from equation (4), we obtain, taking into account (5),

$$\frac{m\vec{v}}{(1 - v_a^2/c^2)^{1/2}} - \frac{m\vec{v}'}{(1 - v_a'^2/c^2)^{1/2}} = \frac{M(\vec{V}' - \vec{V})}{(1 - V^2/c^2)^{1/2}}. \tag{44.8}$$

For $M \gg m$, this equation gives $\vec{V} = \vec{V}'$. Thus the heavy particle receives the whole relative momentum of the light particle but, as there is no registrable motion of the heavy particle and the collision is perfectly elastic, the impringing particle must receive back the same quantity of momentum. Of course, the direction of the momentum returned back to m can make an arbitrary angle $0 < \theta' \leq \pi$ with $\vec{V}$, independent of the angle θ between $\vec{v}$ and $\vec{V}$. Thus the magnitudes of the relative proper momenta of m before and after the collision must be equal (cf. (34.18))

$$\frac{m^2v^2}{1 - v_a^2/c^2} = \frac{m^2v'^2}{1 - v_a'^2/c^2}, \tag{44.9}$$

and M changes its momentum with a quantity equal to their geometrical difference. Using (5) in equation (9) and solving it with respect to v', we obtain

$$v' = v\,\frac{\{v^2V^2\cos^2\theta' + (c^2 - V^2)(c^2 - V^2 - 2vV\cos\theta)\}^{1/2} - vV\cos\theta'}{c^2 - V^2 - 2vV\cos\theta}. \tag{44.10}$$

Putting $v = c/n$, where $n \geq 1$, we can write the last formula within the relevant accuracy of first order in V/c as follows

$$v' = \frac{c}{n} + \frac{V}{n^2}(\cos\theta - \cos\theta'). \tag{44.11}$$

Of course, equation (8) can also be solved with respect to v' for the case where M is comparable with m, taking into account that the angles between $\vec{v}'$ and $\vec{v}$ as well as between $\vec{V}'$ and $\vec{V}$ can be expressed by θ and θ'.

It is to wonder that the relation (11) between v' and v (= c/n) which is of first order in V/c and follows directly from the laws of momentum and energy conservation has bee noticed by nobody. The cause for this overseeing is the following: In the traditional Newtonian theory one works with the low-velocity relation (23.18) but neglects the low-velocity relation (37.10) (see §37). Thus in the frame of the traditional Newtonian theory one obtains from the momentum and energy conservation laws (6) and (7) the following relation between v' and v

$$v^2 - v'^2 = \frac{m}{M}(\vec{v} - \vec{v}')^2, \tag{44.12}$$

where the absolute velocity V does not take part. Nevertheless, note that, for $M \gg m$, one obtains in the traditional Newtonian theory that the momentum and energy received by the heavy particle are

$$\vec{P}_{ao} = \vec{p}_{ao} - \vec{p}'_{ao} = m(\vec{v} - \vec{v}'), \quad E_{ao} = e_{ao} - e'_{ao} = m\vec{V}(\vec{v} - \vec{v}'). \tag{44.13}$$

According to the theory of relativity, a low-energy light particle can transfer to a very heavy particle only momentum, as the heavy particle can always be considered at rest According to formula (13), however, the light particle transfers to the heavy particle also energy which is proportional to the absolute velocity of the latter. And we should like to emphasize once more that formulas (13) are Newtonian in the most traditional sense.

C. The second-order in V/c effects

If we wish to calculate also the effects of second order in V/c in the elastic collision of particles, we have to use in (9) the Marinov formula for velocity addition (4.27 putting there $v = v_a$, $v' = v$. Now instead of the relation (10) we obtain the following r lation, after solving (9) with respect to v',

$$v' = cv(c^2 - V^2)^{1/2}\,\frac{\{c^4 - V^2(c^2 - v^2\cos^2\theta) - 2cvV\cos\theta(c^2 - V^2)^{1/2}\}^{1/2} - vV\cos\theta'}{c^4 - V^2\{c^2 - v^2(\cos^2\theta - \cos^2\theta')\} - 2cvV\cos\theta(c^2 - V^2)^{1/2}}. \tag{44.14}$$

Easily can be shown that using the vector formula of the Marinov velocity transformation (4.25), in order to express the absolute velocities $\vec{v}_a$ and $\vec{v}'_a$ in (6) through $\vec{V}$ and the relative velocities $\vec{v}$ and $\vec{v}'$, one will come again to formula (9), if working within an accuracy of second order in V/c.When working within an accuracy of second order in V/ it is of importance to note that the angles θ and θ' are measured by an observer who is at rest with respect to M, as θ in formula (4.27) is the angle between $\vec{v}$ and $\vec{V}$.

Putting v = c/n, where $n \geq 1$, we can write the last formula within the relevant accuracy of second order in V/c as follows

$$v' = \frac{c}{n} + \frac{V}{n^2}(\cos\theta - \cos\theta') + \frac{V^2}{cn^3}(\cos\theta - \cos\theta')^2. \qquad (44.15)$$

. The relativity treatment

The theory of relativity, for which an absolute space does not exist and which does ot consider the proper energies of the particles as their gravitational energies with he mass of the whole world, considers the heavy particle at rest. Thus relativity will rite $V = V' = 0$, and instead of (9) will come to the following relation between the re- ative velocities of m before and after the collision

$$\frac{m^2v^2}{1 - v^2/c^2} = \frac{m v'^2}{1 - v'^2/c^2}, \qquad (44.16)$$

hich leads to the result $v = v'$.

If working in an inertial frame in which M moves with a velocity V, and writing for- ulas (6) and (7), relativity will use neither the Galilean nor the Marinov but the Lo- entz formula for velocity addition (4.17) - let us consider for simplicity's sake a one- imensional case, supposing that $\vec{v}$ and $\vec{V}$ are parallel. Putting into (4.17) $v = v_a$, $v'_L = v$, although knowing well that v'_L is the Lorentz relative velocity but not the real elative velocity, and thus $v'_L \neq v$, we obtain

$$v_a = \frac{v + V}{1 + vV/c^2}. \qquad (44.17)$$

Now one will come to the following relation between the velocities of m before and af- er the collision (the equation is written within an accuracy of second order in V/c)

$$m^2v^2(1 + \frac{v^2}{c^2} - \frac{V^2}{c^2}) = m^2v'^2(1 + \frac{v'^2}{c^2} - \frac{V^2}{c^2}), \qquad (44.18)$$

hich, again, leads to the result $v = v'$.

The experiment supports the anisotropic formulas (11) and (15), as we shall show in the ollowing subsection and then in ch. VIII. Thus the relativistic treatment of the elastic ollision of particles (and of whole high-velocity physics) is not adequate to physical eality.

. The "rotating disk" experiment

Let us have a disk (representing the heavy particle M) rotating with respect to abso- ute space with an angular velocity Ω. Light particles with masses m, propagating along he radius of the disk, are split by a "semi-transparent mirror" attached to the disk into wo beams: the first ones, which will be called "direct", propagate along the direction f rotation, and the second ones, which will be called "opposite", propagate against.

Thus, if the velocity of the particles along the radius of the disk is equal to c/n, after the splitting, according to formula (11), the velocity of the "direct" particles will become $c/n - V/n^2$ and of the "opposite" particles $c/n + V/n^2$. During a couple of reflections on mirrors attached to the disk, for which $\theta = \theta'$, the magnitudes of the relative velocities will not change. Covering the same closed path d, the "direct" and "opposite" particles reach again the "semi-transparent mirror" where they are unified in a single beam.

The time delay with which the "direct" particles cover the path d respectively to the "opposite" particles is

$$\Delta t = \frac{d}{v'_{dir}} - \frac{d}{v'_{opp}} = \frac{d}{c/n - V/n^2} - \frac{d}{c/n + V/n^2} = \frac{2dV}{c^2} = \frac{4\Omega S}{c^2}. \qquad (44.19)$$

The last transition in this formula can be obtained in an elementary way if one supposes that the closed path represents a polygone with area S circumscribed around a circle with readius R and if one makes the substitutions $d = 2S/R$, $V = \Omega R$.

The time difference Δt is extremely small. It can be registered by taking elementary particles (photons or neutrons) and by observing the interference of their de Broglie waves, thus making use of the relations (see (31.2) and (31.4))

$$h/\lambda_0 = mv_0, \qquad h(\nu_0/c) = mc_0, \qquad (44.20)$$

where λ_0 is the proper wavelength of the particles and ν_0 is their proper frequency.

When the "opposite" particle returns at the starting point, after having covered the closed path, the "direct" particle which was its neighbour in the initial beam has still to cover a distance

$$\Delta l = v_0 \Delta t = (h/\lambda_0 m)\Delta t \cong h\,\Delta t/\lambda m. \qquad (44.21)$$

We write the expression on the right side for low-velocity particles, so that we can put $\lambda_0 = \lambda$.

The phase shift, or the difference in the eikonals (see V, §44, §59) of the de Broglie waves of the "opposite" and "direct" particles is

$$\Delta\psi = \psi' - \psi = \oint \vec{k}'_0 . d\vec{r}' - \omega'_0 t + \alpha' - \{\oint \vec{k}_0 . d\vec{r} - \omega_0(t + \Delta t) + \alpha\} = \omega_0 \Delta t = 2\pi\nu_0 \Delta t, \qquad (44.22)$$

where ψ, ψ' are the eikonals (the phases), $\vec{k}_0$, $\vec{k}'_0$ are the circular wave-vectors ($k_0 = k'_0$), ω_0, ω'_0 are the circular frequencies ($\omega_0 = \omega'_0 = 2\pi\nu_0$), and α, α' are the initial phases ($\alpha = \alpha'$) of the "direct" and "opposite" particles, respectively; t is the time in which the "opposite" particle returns to the "semi-transparent mirror" and the integrations are performed over the closed paths of the "opposite" and "direct" particles which are equal.

Squaring equations (20) and subtracting the first from the second, we obtain

$$\nu_0 = c\left(\frac{m^2c^2}{h^2} + \frac{1}{\lambda_0^2}\right)^{1/2} \cong \frac{mc^2}{h} + \frac{h}{2\lambda^2 m}, \qquad (44.23)$$

since, for low-velocity particles, $1/\lambda_o = mv/h \ll mc/h$. Thus, using (19) and (23) in (22), we obtain

$$\Delta\psi = \frac{8\pi m}{h}\Omega S + \frac{4\pi h}{\lambda^2 mc^2}\Omega S. \qquad (44.24)$$

The second term on the right side is small with respect to the first one (in the relation v^2/c^2) and for low-velocity particles can be neglected.

If the particles are massless ($m = 0$), the de Broglie relations must be written in the form (see (31.29))

$$h/\lambda = m_o c, \qquad h(\nu/c) = m_o c, \qquad (44.25)$$

and the phase shift consists of one term only

$$\Delta\psi = 2\pi\nu\Delta t = 2\pi c\Delta t/\lambda. \qquad (44.26)$$

The first experiment in the history which has confirmed formula (19) was the "rotating disk" experiment of Harress, performed independently of him also by Sagnac, where the particles were photons, and the formula used was (26) (see §59). The "rotating disk" experiment performed with massive particles (neutrons) was carried out by Werner et al., and the formula used was (24) (see §60).

f. The "rotating axle" experiments

The time difference with which light particles (massless or massive) cover the <u>same</u> distance d in a direction coinciding with the absolute velocity of the laboratory $\vec{V}$ and in the opposite direction is (see formula (19)) $\Delta t = 2dV/c^2$. We called this the Marinov effect (see p. 137), meanwhile the effect $\Delta t = 4\pi\Omega S/c^2$ was called the Sagnac effect (see p. 138). As a matter of fact, the physical essence of these two effects is the same. The difference, as already stated, is the following: the Marinov effect appears when light particles cover the same <u>straight path</u> in mutually opposite directions parallel to the absolute velocity V of the laboratory, while the Sagnac effect appears when light particles cover the same <u>closed path</u> in a plane perpendicular to the absolute anglular velocity of the laboratory. There is a certain subtlety which must be well understood: The resultant absolute <u>angular</u> velocity Ω of the laboratory may be zero, while the resultant absolute <u>linear</u> rotational velocity V may be different from zero. So if the Moon <u>would</u> perform during a month one revolution about its axis in a direction opposite to its rotation around the Earth, the Sagnac effect for an apparatus on the Moon's surface will be zero (and the Foucault pendulum (see IV, §21) will swing all the time in its initial plane), but the Marinov effect will be different from zero. Indeed, in this case the Moon's "day" and "year" will have the same duration, equal to one month, but as the radius of the "yearly" rotation is <u>much bigger</u> than the radius of the "dayly" rotation, the geometrical sum of the products "angular velocity by radius" will be not zero. The absolute velocity of a laboratory is the resultant geometrical sum of all such products for the different rotations of the laboratory around the corresponding centers in the universe (daily rotation, yearly rotation, galactic rotation, super-galactic rotation).

The Marinov effect was registered for the first time by the help of our "coupled mirrors" experiments (see §52) which represent a variation of the large class of "rotating axle" experiments (see §50).

If a "rotating axle" experiment would be performed with massive particles (neutrons), a phantastical accuracy in the measurement of the laboratory's absolute velocity can be achieved (see §52D).

G. The Michelson experiment

The essence of the historicla Michelson experiment (considered in detail in §49) is as follows:

A beam of particles is split by a semi-transparent mirror into two mutually perpendicular beams which cover equal distances, d, to two mirrors "to and fro"; after their unification at the semi-transparent mirror, one observes the produced interference picture. If there is a difference in the times in which the beams propagate parallel and perpendicular with respect to the absolute velocity of the laboratory, then, after rotating the apparatus in a plane parallel to the absolute velocity, one should observe a change in the interference picture.

Let us suppose that before being separated the particles move with a velocity c/n with respect to the laboratory in a direction perpendicular to the absolute velocity of the latter. Let one half of the particles go through the semi-transparent mirror and continue to move with the same velocity c/n perpendicular to the absolute velocity V of the laboratory, and let the other half collide with the semi-transparent mirror changing their direction parallel to V. The velocity of the "perpendicular" particles "to" is thus $v_\perp = c/n$, and "fro" will remain the same, as can be seen putting into (15) $\theta = \pi/2$, $\theta' = \pi/2$. The velocity of the "parallel" particles "to" can be obtained putting in (15) $\theta = \pi/2$, $\theta' = 0$ and "fro" putting $\theta = \pi/2$, $\theta' = \pi$, thus obtaining $v_{\uparrow\uparrow} = c/n - V/n^2 + V^2/cn^3$, $v_{\uparrow\downarrow} = c/n + V/n^2 + V^2/cn^3$. The velocity $v_{\uparrow\downarrow}$ can also be obtained from $v_{\uparrow\uparrow}$, by writing in (15) instead of the factor 1/n the factor $1/n - V/cn^2 + V^2/c^2n^3$ and taking $\theta = 0$, $\theta' = \pi$, i.e., the velocity of the "anti-parallel" particles can be calculated from the collision of the "parallel" particles with the reflecting mirror. It can be immediately seen that the time in which the "perpendicular" particles cover the distance d to their reflecting mirror "to and fro" $\Delta t_\perp = 2d/v_\perp = 2dn/c$ will be exactly equal to the time spent by the "parallel" particles $\Delta t_\parallel = d/v_{\uparrow\uparrow} + d/v_{\uparrow\downarrow} = 2dn/c$.

If instead of the Marinov formula (15) one would use the Newton formula (11) which is true only within effects of first order in V/c, one will obtain $\Delta t_\parallel - \Delta t_\perp = dV^2/nc^3$. This result (for n = 1) was expected by Michelson to be observed in his experiment.

The Michelson experiment performed with photons is called the Michelson-Morley experiment. If the Michelson experiment would be performed with neutrons, we call it the Michelson-Marinov experiment (see §49B). As we showed, the Michelson-Marinov experiment will give the same null result as the Michelson-Morley experiment, and there is no sense for spending time, efforts, and money in its performance.

H. The convection of light as a kinematic phenomenon

Putting into formula (15) $\theta = \pi/2$, we obtain the following result

$$v' = \frac{c}{n} - \frac{V}{n^2}\cos\theta' + \frac{V^2}{cn^3}\cos^2\theta'. \qquad (44.27)$$

Comparing formula (27) with the first formula (40.28), we see that they are identical within effects of second order in V/c. Formula (40.28) was obtained proceeding from our "hitch-hiker" model for light propagation in a medium, while formula (22) was obtained from the law of conservation of the proper momentum of a particle in a moving laboratory. Let us write this common formula within an accuracy of first order in V/c, in order to neglect the difference between universal time and proper time,

$$v' = \frac{c}{n} - \frac{V}{n^2}\cos\theta', \qquad (44.28)$$

where c/n is the velocity of a particle (massive or massless) in a frame (medium) when propagating in a direction perpendicular to the absolute velocity $\vec{V}$ of the frame, or the "to and fro" velocity along any other direction.

If v' in (28) is the velocity of light in the moving medium measured by an observer attached to the medium, then the velocity of light measured by an observer attached to absolute space will be obtained by adding to v' the projection of the medium's velocity along the direction of light propagation

$$v = v' + V\cos\theta' = \frac{c}{n} + V(1 - \frac{1}{n^2})\cos\theta'. \qquad (44.29)$$

This is the Fresnel formula (40.16) for the convection of light by a moving medium.

Proceeding from our "hitch-hiker" model of light propagation, we obtained that the velocity of light in a medium at rest, measured by an observer moving with velocity V is (see formula (40.20))

$$v^o = \frac{c}{n} - V\cos\theta'. \qquad (44.30)$$

This formula can be obtained directly subtracting from the velocity of light in a medium at rest, c/n, the projection of the observer's velocity along the direction of light propagation.

Let us make a recapitulation. If there is a light pulse (a moving particle) and a medium (a reference frame), one can realize the following four combinations:

1. Medium and observer both at rest. The velocity is c/n.
2. Medium at rest, observer moving. The velocity v^o is given by (30).
3. Observer at rest, medium moving. The velocity v' is given by (29).

4. Medium and observer both moving. The velocity v' is given by (28).

The first case can be considered as a definition combination. If the medium (the frame) is not at rest, c/n can be considered as a velocity of the particles in a direction perpendicular to V, or the "to and fro" velocity along any other direction.

The second and third combinations are clear: first we have medium and observer both at rest, and then we move either the observer (case 2) or the medium (case 3). The velocity V which appears in the velocity of the particles measured by the observer is the relative velocity between medium and observer. The formulas are not symmetric, as it must be when proceeding from the principle of relativity, because in case 2 we move the observer with respect to distant matter, while in case 3 we move the medium. These two formulas are valid also in the case when the initial medium and observer, which are at rest with respect to one another, are moving together with a certain velocity in absolute space, but the velocities v^0 and v are measured not by an absolute method but by a relative method with respect to the case 1 (or to the case 4) in which one measures the "to and fro" velocity of the particles. Such were the experiments of Fizeau and Dufour.

The fourth case is the most important. The velocity v' is the one-way velocity of the particles propagating in the medium which is at rest with respect to the observer. The velocity V which appears in velocity v', causing its anisotropy, is the absolute velocity of source and observer.

As the reader sees, formula (28) is not the result of a certain "drag" exerted by the moving medium on the particles propagating through it, but is due to the attachment of the particles to absolute space, as their proper energies and momenta are functions of their absolute velocities. Thus the "drag" of light is a purely kinematic phenomenon and has the same character for massive and massless particles. Thus the same purely kinematic character has the "drag" described by the Fresnel formula (29). Concerning the Dufour formula (30) there is no room at all to speak about "drag".

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With the help of the Lorentz transformation one can write only formula (29). The adepts of the theory of relativity assert that formula (29) is obtained from the Lorentz transformation formulas for velocities (use the simple formulas (4.17) for the case $\theta' = 0$) and consider the Fizeau effect as an experimental confirmation of the "strange" Lorentz velocity transformation. As a matter of fact, formula (29) results from the Lorentz transformation formula for the 4-wave-vector of a particle. Indeed, using the special Lorentz transformation (13.5) for the 4-wave-vector (31.9) of a particle, we obtain (assume the motion of the particle parallel to the x-axes of the frames K and K')

$$\nu_0 v = \frac{\nu_0' v' + V\nu_0'}{(1 - V^2/c^2)^{1/2}}, \qquad \nu_0 = \frac{\nu_0' + V\nu_0' v'/c^2}{(1 - V^2/c^2)^{1/2}}, \qquad (44.31)$$

where ν_0 (ν_0') is the proper frquency of the particle in the rest (moving) frame, v (v') is the velocity (Lorentz velocity) of the particle in the rest (moving) frame, and V is the velocity of the moving frame.Putting in this formula $v' = c/n$ and dividing the first formula by the second one, we obtain, if working within an accuracy of first order in V/c, formula (29) for the case $\theta' = 0$.

This deduction as well as our deduction of formula (27) show that in the phenomenon "drag of light by a medium" there is no "drag" at all, as the same "drag" is exerted also for massive particles by "vacuum". Thus the factor n in all our formulas is not a mysterious optical "index of refraction" of a medium, but is simply a number showing how many times the velocity of the particle (of the photon) is less than the velocity of light in vacuum.

I. The interaction of particles with fields considered as elastic collision

If a particle changes the direction of its velocity because being deviated by a certain field originated by a very heavy system, it must change the velocity's magnitude exactly in the same manner as when the deviation is caused by an elastic collision with a very heavy particle. As a matter of fact, when the particle collides, it interferes with a certain short range field. Thus in both cases there is an interaction of a particle with a field, short range or long range, and the physical essence of the problem is the same.

We shall clear the problem considering a simple example which we shall present in the form of the following THEOREM ON THE ROTATING DISK:

Let us have a disk rotating with an angular velocity Ω with respect to absolute space. Let a particle moving with a velocity $v_\perp = c/n$ along the radius be reflected at a distance R from the center at a right angle in a direction against the rotation. According to Werner's experiment, i.e., according to formula (11), its velocity with respect to the disk will become $v_{\uparrow\downarrow} = c/n + V/n^2$, where $V = \Omega R$ is the linear rotational velocity of the disk, and we work with an accuracy of first order in V/c. Let then the particle, which has an electric charge, enter a magnetic field where it describes half a circle with a radius $r \ll R$ and, leaving the field, let it proceed in a direction along the rotation with a velocity $v_{\uparrow\uparrow}$. If $v_{\uparrow\uparrow}$ would be the same as $v_{\uparrow\downarrow}$, we can reflect the particle at a right angle and letting it cover a distance 2r along the radius, where its velocity, according to Werner's experiment, will become $v'_\perp = v_{\uparrow\uparrow} + V/n^2 = c/n + 2V/n^2$, we can reflect it once more at a right angle again against the rotation, where its velocity, according the Werner's experiment will become $v'_{\uparrow\downarrow} = v'_\perp + V/n^2 = c/n + 3V/n^2$. Thus we should increase the velocity of the particle after having performed a closed cycle where no energy was lost, creating thus a perpetuum mobile. As this is impossible, the velocity of the particle when it leaves the magnetic field must be $v_{\uparrow\uparrow} = c/n - V/n^2$.

An important application of this theorem on the rotating disk see in V, §36C.

Chapter VIII

THE EXPERIMENTAL EVIDENCE FOR THE SPACE-TIME ABSOLUTENESS

In this chapter an account is given on the most important experiments which we consider as decisive for the refutation of the principle of relativity, for the rejection of the relativistic space-time conceptions, and for the restoration of the old Newtonian absolute conceptions clothed in our absolute space-time theory.

Certain of the experiments reveal the absolute motion of the laboratory, others fail to as all of the absolute effects cancel out each other. Certain experiments can give or have given reliable results within present day experimental capabilities, others cannot. The experiments are described in terms of their essentials, without discussing the details, which the reader can find in the original papers. Although we tried to present the experiments in the possibly most simple way, the use of certain notions which are outside the domain of classical physics was inevitable. Nevertheless, the information taken from other parts of physics (predominently experimental) is so limited, that a reader with a modest general physical background can smoothly read the descriptions of the experiments and understand their essence.

The "rotating axle" experiments (see §50, §51, §52) and the "rotating disk" experiments (see §59, §60, §61, §62, §63) are decisive for the rejection of the principle of relativity and of Einstein's space-time conceptions. The analysis of the other experiments can strengthen our conceptions about the space-time absoluteness.

§45. THE QUASI-RÖMER EXPERIMENT

By observing the time of the eclipses of a Jovian satellite during the course of a year, for the first time in history, Römer (1676) measured the velocity of light. If the RÖMER EXPERIMENT should be performed with the aim of measuring the Earth's absolute velocity, we call it the QUASI-RÖMER EXPERIMENT. The quasi-Römer experiment was suggested first by Maxwell (1878).

Now we shall show that according to our absolute space-time theory the Earth's absolute motion cannot be revealed by the help of the quasi-Römer experiment.

Suppose (see fig. 45.1) that at the initial year of observation, when the Earth and Jupiter are in opposition, the absolute velocity of the Solar system v_S makes an angle θ with the opposition line.

Let the zeroth eclipse of the satellite be at the moment t^0, read on a terrestrial clock, when the Earth and Jupiter are at positions E_0, J_0, i.e., half a year before the moment when they will be in opposition.

The first eclipse will then occur at the moment

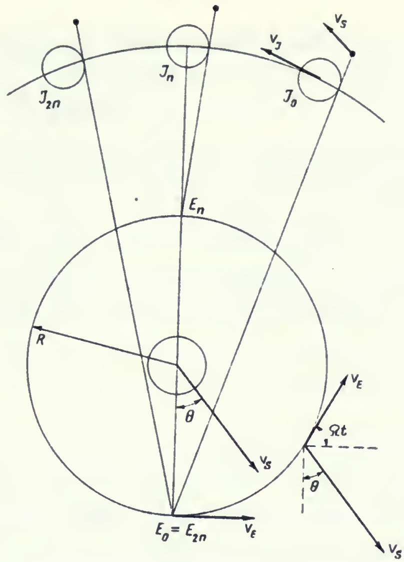

Fig. 45.1

$$t^1 = t^0 + T - \frac{J_1E_1 - J_oE_o}{c_o'}, \qquad (45.1)$$

where T is the period of revolution of the satellite, E_1, J_1 are the positions of the Earth and Jupiter at the moment t^1, and c_o' is the proper relative velocity of light coming from Jupiter with respect to the Solar system.

According to formula (4.28), with $V = v_S$, we have

$$c_o' = \frac{c}{1 + v_S\cos\theta'/c} = \frac{1 - v_S\cos\theta/c}{1 - v_S^2/c^2}, \qquad (45.2)$$

where θ' is the angle between v_S and the line of light propagation registered with respect to the moving Sun's frame and θ is the same angle registered with respect to absolute space.

Since Jupiter covers (1/12)th part of its orbit during an Earth year, it may be assumed that the positions J_o, J_n are very near to one another. Thus in (2) we can consider angle θ' ($\cong \theta$ within the necessary accuracy) to be equal to angle θ in fig. 45.1, i.e., equal to the angle between the opposition line and the Sun's absolute velocity.

Let the n-th eclipse be observed at the moment t^n when the Earth and Jupiter are in opposition. We shall have

$$t_o^n = t^n - t^o = nT - \frac{J_nE_n - J_oE_o}{c_o'} = nT - \frac{2R}{c}\left(1 + \frac{v_S}{c}\cos\theta\right), \qquad (45.3)$$

where R is the radius of the Earth's orbit.

Finally, supposing the 2n-th eclipse is observed at the moment t^{2n} after another half-year when the Earth and Jupiter are at the positions E_{2n}, J_{2n}, we have

$$\Delta t_o^{2n} = t^{2n} - t^o = 2nT - \frac{J_{2n}E_{2n} - J_oE_o}{c_o'} = 2nT. \tag{45.4}$$

From here we can determine the period T of revolution of the satellite. Using (4) in (3), we find

$$\Delta t_o^n = \frac{\Delta t_o^{2n}}{2} - \frac{2R}{c} - \frac{2Rv_S}{c^2}\cos\theta = \overline{\Delta t_o^n} - \frac{2Rv_S}{c^2}\cos\theta, \tag{45.5}$$

where $\overline{\Delta t_o^n}$ is this time interval after whose elapsing one has to observe the n-th eclipse if the absolute velocity of the Sun is equal to zero, or if the velocity of light is not direction dependent. When Römer made his observations, he compared the calculated time interval $\Delta t_o^{2n}/2$ with the really measured time interval Δt_o^n and, knowing R, he established c.

A traditional absolutist would conclude that making use of formula (5) one would be able to establish the component v_S of the Sun's absolute velocity in the plane of the ecliptic when performing observations of the eclipses of a Jovian satellite during 12 years in which the angle θ between v_S and the opposition line takes different values in the range of 360^o, so that the difference $\delta t = \overline{\Delta t_o^n} - \Delta t_o^n$ will vary in the range $-(2Rv_S/c^2) \leq \delta t \leq (2Rv_S/c^2)$.

However, if we take into account the absolute kinematic time dilation (see §7), we shall come to the conclusion that, since time is measured on a moving terrestrial clock, no positive effect can be registered.

Indeed, let us assume that the Earth covers the path E_oE_n during the absolute time interval (read on a clock which rests in absolute space) Δt^n. The time Δt_o^n read on the proper terrestrial clock will be (see (7.4))

$$\Delta t_o^n = \int_0^{\Delta t^n} \{1 - \frac{(\vec{v}_E - \vec{v}_S)^2}{c^2}\}^{1/2} dt \cong (1 - \frac{1}{2}\frac{v_E^2 + v_S^2}{c^2})\Delta t^n - \int_0^{\Delta t^n} \frac{\Omega R v_S}{c^2}\cos(\frac{\pi}{2} - \theta + \Omega t)dt =$$

$$(1 - \frac{1}{2}\frac{v_E^2 + v_S^2}{c^2})\Delta t^n + \frac{2Rv_S}{c^2}\cos\theta, \tag{45.6}$$

where v_E is the Earth's velocity with respect to the Sun, Ω is its angular velocity, and thus $v_E = \Omega R$.

Comparing formulas (5) and (6), we conclude that no positive effect can be registered in the quasi-Römer experiment because the time interval between the zeroth and n-th eclipses actually registered on a terrestrial clock will vary exactly in such a manner that the effect δt, which a traditional absolutist expects to be registered, will be compensated for by a change in the rate of the terrestrial clock.

For the case shown in fig. 45.1, the time rate of the terrestrial clock is higher on a larger part along the path between the zeroth and the n-th eclipses and lower on a larger part along the path between the n-th and 2n-th eclipses.

To explain better the annihilation of the absolute effect, suppose that in the first year of observation one has $\theta = \pi/2$. Assume that the time between the zeroth and the n-th ecplipses is $\Delta t_o^n = 100$ sec, registered on an Earth's clock, which corresponds, say, to $\Delta t^n = 110$ sec registered on an universal clock, i.e., a clock at rest in absolute space. After three years one will have $\theta = 0$. The universal time interval between the zeroth and the n-th eclipses will be, say, $\Delta t^n = 109$ sec, because of the appearance of the additional term in formula (5). However, now to 109 universal seconds will correspond not 99 sec of the proper clock but 100 sec, because of the appearance of the additional term in formula (6). Thus the time interval Δt_o^n registered on the Earth's clock will be the same for any angle θ.

§46. THE QUASI-BRADLEY EXPERIMENT

By observing the change in the angular positions of stars during the course of a year, for the second time in history, Bradley (1727) measured the velocity of light. If the BRADLEY EXPERIMENT should be performed with the aim of measuring the Earth's absolute velocity, we call it the QUASI-BRADLEY EXPERIMENT. The quasi-Bradley experiment was suggested first by Poincaré (1904).

Now we shall show that according to our absolute space-time theory the Earth's absolute motion can be revealed by the quasi-Bradley experiment.

If a star on the celestial sphere is observed from the Earth moving with an absolute velocity $\vec{v}$, then the relation between the emission angle θ', which represents the angle between the velocity $\vec{v}$ and the source-observer line at the moment of emission, and the reception angle θ, which represents the same angle at the moment of reception, will be (see formula (38.5))

$$(1 + \frac{v}{c}\cos\theta')(1 - \frac{v}{c}\cos\theta) = 1 - \frac{v^2}{c^2}. \qquad (46.1)$$

Now supposing the Earth moves with velocity $\vec{v}_E$ with respect to the Sun which moves with velocity $\vec{v}_S$ with respect to absolute space, we shall have

$$\vec{v} = \vec{v}_E + \vec{v}_S. \qquad (46.2)$$

Let us suppose that at the moment of emission of light from a star at S an Earth observer and an imaginary observer at rest with respect to the Sun (called the Sun observer) are at the point O_S (see fig. 46.1). The light emitted at this moment will be received by the Sun observer when he arrives at point O_E and by the Earth observer when he arrives at point O. The following situation pertains:

a) Observation by the Sun observer in absolute space. For this case θ'_S is the emission angle and θ_S the reception angle, distance SO_E is proportional to the absolute light velocity c and distance SO_S to the relative light velocity with respect to the Sun c'_S.

b) Observation by the Earth observer in absolute space. For this case θ' is the emission angle and θ the reception angle, distance SO is proportional to the absolute light velocity c and distance SO_S to the relative light velocity with respect to the Earth c'.

c) Observation by the Earth observer in a frame attached to the Sun. For this case θ'_E is the emission angle and θ_E the reception angle, distance SO is proportional to the relative light velocity with respect to the Sun c'_S (since this is the velocity of light which travels along the given direction with respect to a frame attached to the Sun) and distance SO_E to the relative light velocity with respect to the Earth c'.

We must emphasize that all velocities (absolute and relative) are assumed to be measured in universal time.

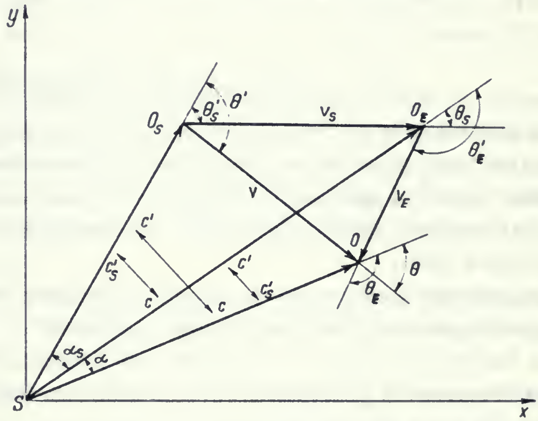

Fig. 46.2

Consider now an imaginary emission which occurs at the moment when the Earth observer is at the point O_E and a reception which occurs at the moment when this observer arrives at the point O, the velocity of light being equal to c'_S. By analogy with (1) we have

$$(1 + \frac{v_E}{c'_S} \cos\theta'_E)(1 - \frac{v_E}{c'_S} \cos\theta_E) = 1 - \frac{v_E^2}{c'^2_S}, \quad (46.3)$$

where (see formula (38.11) and the text after formula (38.12))

$$c'_S = c \left(\frac{1 - \frac{v_S}{c} \cos\theta_{Sm}}{1 + \frac{v_S}{c} \cos\theta_{Sm}}\right)^{1/2} \quad (46.4)$$

is the relative velocity of light in a frame attached to the Sun and θ_{Sm} is the middle angle between θ'_S and θ_S, i.e.,

$$\theta_{Sm} = (\theta'_S + \theta_S)/2. \quad (46.5)$$

Putting (4) into (3) and working within an accuracy of third order in "velocity/light velocity" (i.e., putting $\theta_E = \theta'_E = \theta_{Em}$ in terms of second and third order), we obtain

$$\cos\theta_E = \cos\theta_E' + \frac{v_E}{c}\sin^2\theta_{Em}(1 + \frac{v_S}{c}\cos\theta_{Sm}). \qquad (46.6)$$

Designating by $\alpha = \theta_E' - \theta_E$ the aberration angle, we find within the necessary accuracy

$$\alpha = \frac{v_E}{c}\sin\theta_{Em} + \frac{v_E v_S}{c^2}\sin\theta_{Em}\cos\theta_{Sm} = \alpha_E + \Delta\alpha, \qquad (46.7)$$

where α_E is the aberration angle caused by the motion of the Earth if the Sun were at rest in absolute space and $\Delta\alpha$ is the variation caused by the absolute velocity of the Sun, in dependence on the angle θ_{Sm} subtended by the light beam coming from the star and the velocity of the Sun.

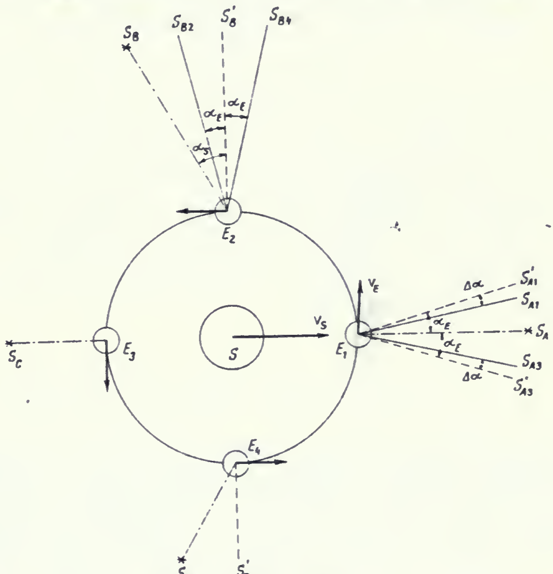

Fig. 46.2

In fig. 46.2 we show four different positions of the Earth (E_1, E_2, E_3, E_4) on its orbit around the Sun (S) at four different moments with intervals of three months when four different stars (S_A, S_B, S_C, C_D) are in range with the Earth in the plane of the ecliptic, if being observed from the Sun. The real poistions of the stars S_A and S_C and their positions observed from the Sun coincide, since the angle θ_{Sm} between the Sun's velocity v_S and the propagation direction of the light coming from these stars is equal to π or to 0. The positions of the stars S_B and S_D observed from the Sun are tilted to an angle $\alpha_S = v_S/c$ with respect to their real positions, since for these two stars the angle θ_{Sm} is equal to $\pi/2$, i.e., they will be seen along the directions to S_B' and S_D'.

The star S_A will be observed from the Earth's position E_1 tilted to an angle $\alpha_E = v_E/c$, i.e., along the direction to S_{A1}', if the Sun be at rest in absolute space and the velocity of light coming from S_A be equal to c.

However, when the Sun moves and the velocity of light coming from S_A is $c + v_S$, this star will be seen from E_1 tilted to an angle

$$\alpha_{ap} = \alpha_E - \Delta\alpha = \frac{v_E}{c + v_S} = \alpha_E - \frac{v_E v_S}{c^2}, \qquad (46.8)$$

i.e., along the direction to S_{A1}. The same star when observed from the position E_3 after six months will be tilted oppositely to the same angle α_{ap}, i.e., along the direction to S_{A3}. Thus in a year the stars which lie near the apex of the Sun should describe over the celestial sphere a small arc equal to $2\alpha_{ap} = 2\alpha_E - 2v_E v_S/c^2$. Analogically we conclude that the stars which lie near the Sun's anti-apex will describe a small arc equal to $2\alpha_{anti-ap} = 2\alpha_E + 2v_E v_S/c^2$. For the difference between these two arcs we obtain

$$\Delta = 2\alpha_{anti-ap} - 2\alpha_{ap} = 4\,\Delta\alpha = 4v_E v_S/c^2. \qquad (46.9)$$

It can be seen immediately that the star S_B will be observed from the position E_2 tilted additionally to an angle $\alpha_E = v_E/c$, i.e., along the direction to S_{B2} and the same star will be observed after six months from the position E_4 tilted oppositely to the same angle α_E, i.e., along the direction to S_{B4}, both at rest and at motion of the Sun.

Taking $v_E = 30$ km/sec, $v_S = 300$ km/sec (see §52B), we obtain $\Delta = 4\times10^{-7}$ rad = 0".08. Thus at the present state of technique, the quasi-Bradley experiment can be considered to be only a challenge to the experimenter.

§47. THE QUASI-DOPPLER EXPERIMENT

By observing the differences in the frequencies of light emitted by a given star during a year, one can measure the light velocity and we call this the DOPPLER EXPERIMENT. The Doppler experiment was performed for the first time by Hoff (Sky and Telescope, 43, 9 (1972)). Let us note that if Bradley had had a refraction prism with a resolution better than $\Delta\nu/\nu = 10^{-4}$, he could have discovered the yearly aberration in frequency which is $\Delta\nu/\nu = 2v/c = 2\times10^{-4}$, where $v = 30$ km/sec is the velocity of the Earth around the Sun. The Doppler experiment is easier than the Bradley experiment because only two photographs of the spectrum coming from a certain star (Hoff has taken the Sun) are to be taken with a time difference of six months. Of course, Bradley did not have a camera, and even if a suitable prism had been at his disposal, he would have been unable to measure the shift in the frequencies. Note that we can call the Doppler experiment the ABERRATION IN FREQUENCY, in contrast to the ABERRATION IN DIRECTION discovered by Bradley. We have already mentioned (see p. 131) that the aberration of light and the light Doppler effect are closely connected.

If the Doppler experiment should be performed with the aim of measuring the Earth's absolute velocity, we call it the QUASI-DOPPLER EXPERIMENT.

Now we shall show that the Earth's absolute motion cannot be revealed by the quasi-Dop-

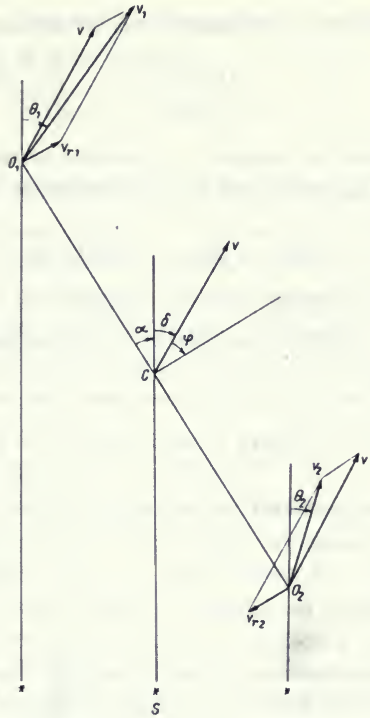

Fig. 47.1

pler experiment.

Let us consider (see fig. 47.1) a distant light source (a star) S and two observers O_1, O_2 who rotate with relative velocities $\vec{v}_{r1}$, $\vec{v}_{r2}$ ($v_{r1} = v_{r2} = v_r$) about some center C which moves with an absolute velocity $\vec{v}$. The absolute velocities of O_1 and O_2, which lie on the same line with the center of rotation, are

$$v_1^2 = v^2 + v_r^2 + 2vv_r\cos\phi,$$

$$v_2^2 = v^2 + v_r^2 - 2vv_r\cos\phi, \qquad (47.1)$$

where ϕ is the angle between the velocity $\vec{v}$ and the velocity of the first observer $\vec{v}_{r1}$. Denote by δ the angle between the source-observer line and the velocity v at the moment of reception. Obviously δ is a constant angle, while ϕ changes by 2π during the period of rotation of O_1 and O_2. All angles are taken positive clockwise and negative counter-clockwise.

Supposing that the source moves with velocity v_s and emits light with frequency ν, an observer moving with velocity v_o will register a frequency ν_o given by the first formula (38.17) in which we have to write $v = v_S$. The angles θ' and θ_o are shown in fig. 38.2. Writing in this formula first $\nu_o = \nu_1$, $v_o = v_1$, $\theta_o = \theta_1$ and then $\nu_o = \nu_2$, $v_o = v_2$, $\theta_o = \theta_2$

(see fig. 47.1) and dividing the formulas obtained, we get

$$\frac{\nu_1}{\nu_2} = \frac{1 - v_1\cos\theta_1/c}{1 + v_2\cos\theta_2/c}\left(\frac{1 - v_2^2/c^2}{1 - v_1^2/c^2}\right)^{1/2}. \tag{47.2}$$

Thus the relation between the frequency ν_1 received by the first observer and the frequency ν_2 received by the second observer does not depend on the velocity of the source.

From fig. 47.1 we have

$$v_1\cos\theta_1 = v\cos\delta + v_r\cos(\delta+\phi),$$
$$v_2\cos\theta_2 = v\cos\delta - v_r\cos(\delta+\phi). \tag{47.3}$$

Substituting (1) and (3) into (2), we find within an accuracy of second order in "velocity/light velocity"

$$\frac{\nu_1}{\nu_2} = 1 - 2\frac{v_r}{c}\cos(\delta+\phi) + 2\frac{vv_r}{c^2}\{\cos\phi - \cos\delta\cos(\delta+\phi)\} + 2\frac{v_r^2}{c^2}\cos^2(\delta+\phi). \tag{47.4}$$

This final expression is convenient for discussion. Let us measure ν_1 and ν_2 received from a given light (radio) source for which $\delta+\phi = \pi/2$. If $\delta = 0$, we will have $\nu_1 = \nu_2$ for $\phi = \pi/2$; however, if $\delta = \pi/2$, we will have $\nu_1 = \nu_2(1+2vv_r/c^2)$ for $\phi = 0$. This result leads to the conclusion that we can measure the Earth's absolute velocity in the following manner: Two receivers may be placed at the same latitude with 180° difference in longitude. Let a radio source be observed when it is seen along the line $0_1 0_2$ and let the frequencies received be sent to some middle point to be compared. Imagine for simplicity that this middle point is at the pole and that the Earth represents a flat disk. As we show in §65D when considering the so-called "rotor-rotor" experiment, if light is sent from the rim of a rotating disk to its center, then the change in frequency depends only on the rotational velocity, not on the velocity of the disk as a whole. Hence ν_1 and ν_2 when received at the pole will suffer equal changes, due to the daily rotation of the Earth. If now we compare ν_1 and ν_2 for any radio source which crosses the line $0_1 0_2$, then for $\delta = 0$ the frequencies ν_1 and ν_2 will be equal precisely at the moment when the radio source is on the line $0_1 0_2$, i.e., for $\delta+\phi = \pi/2$, however, for $\delta = \pi/2$ the frequencies ν_1 and ν_2 will be equal when (see (4) and take into account that for the case considered $\cos(\delta+\phi) \cong 0$)

$$\cos(\delta+\phi) = v/c. \tag{47.5}$$

Taking v = 300 km/sec, we obtain

$$\cos(\delta+\phi) = \sin\alpha \cong \alpha = 10^{-3} = 3.'4, \tag{47.6}$$

where α is the angle between the line $0_2 0_1$ and the source-observer line. Such an angle is large enough to be reliably registered. However, the angle α is exactly equal to the aberration angle due to the motion of the Earth with velocity v. This signifies that when line $0_2 0_1$ makes an angle α with the source-observer line, the source will be seen along the direction $0_1 0_2$. Thus, because of the occurence of the aberration, the quasi-Doppler experiments leads practically to a null result.

§48. THE EXPERIMENTS FOR MEASUREMENT OF THE TWO-WAY LIGHT VELOCITY

The first men who measured the velocity of light on the Earth were Fizeau and Foucault. The methods of Fizeau and Foucault are at the basis of all laboratory experiments for the measurement of light velocity as a quotient of distance covered by light for a definite time. In all experiments of this type one measures the two-way light velocity, and in such experiments the effects caused by the absolute motion of the laboratory are always annihilated. As we showed in §45 and §46, Römer and Bradley measured the one-way light velocity.

A. The Fizeau "rotating cog-wheel" experiment

The scheme of the FIZEAU "ROTATING COG-WHEEL" EXPERIMENT, performed in 1849, is shown in fig. 48.1.

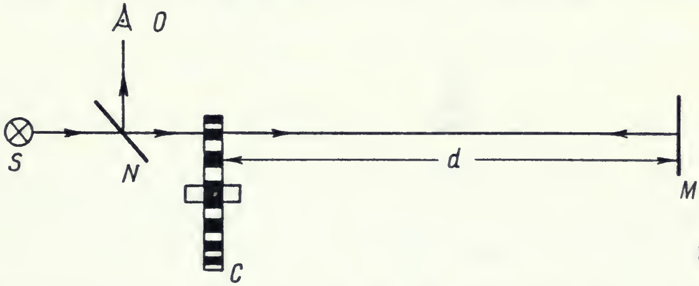

Fig. 48.1

Light emitted by the source S passes through the semi-transparent mirror N and goes through the notches of the rotating cog-wheel C to the mirror M. After a reflection by M, a second passage through the notches of C and a reflection by N, the light is observed by the observer O.

Let us suppose that for the time t in which the light going "to and fro" has covered the distances d two times, the cog-wheel has rotated over one cog, so that no light is seen by O (in the case where the cog-wheel rotates over one cog and one notch, a maximum light will be seen by O). If the cog-wheel rotates with N revolutions per second, and it has p cogs, we have

$$t = 1/2pN, \qquad (48.1)$$

so that putting here $t = 2d/c$, we obtain the following expression for the two-way light velocity

$$c = 4\,d\,p\,N. \qquad (48.2)$$

At given values of d and p Fizeau has measured this value of N when the first minimum of light was seen by O. A further increase of N leads to a second, third, and so on minima of light.

B. The Foucault "rotating mirror" experiment

In 1850 Foucault realized a second laboratory method for the measurement of light velocity which was proposed by Arago in 1838. The scheme of the FOUCAULT "ROTATING MIRROR" EXPERIMENT is shown in fig. 48.2

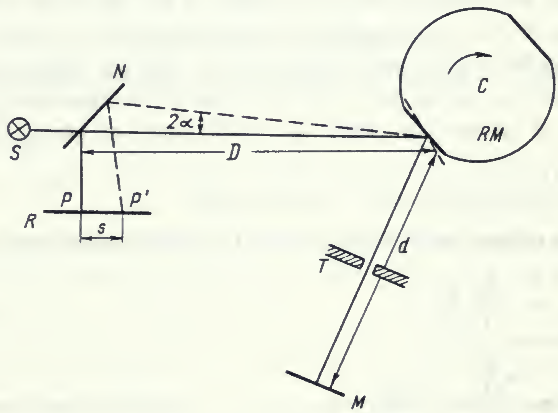

Fig. 48.2

Light emitted by the source S passes through the semi-transparent mirror N and reflects by the rotating mirror RM. After reflection by mirror M, another reflection by RM, and a reflection by N, the light illuminates the screen R.

When the mirror RM is at rest in such a position that the light reflected by it passes through the slit T, suppose that the light illuminates point P over the screen R. When the mirror RM rotates, then, for the time t during which light goes through slit T "there and back" and covers distance d two times, the mirror RM rotates over an angle α, and the light beam will illuminate point P'.

Assuming that the distance between RM and N is D and the distance between N and R is small in comparison with D, we have for the distance between points P and P'

$$s = 2\alpha D. \quad (48.3)$$

If the rotating mirror makes N revolutions per second, then for the time t = 2d/c it will rotate over an angle

$$\alpha = Nt = 2dN/c. \quad (48.4)$$

Substituting (4) into (3), we shall have for the two-way light velocity

$$c = 4dDN/s. \quad (48.5)$$

Foucault used for M a concave mirror whose center coincided with the center C of the rotating mirror, so that the whole light which reaches M always returns again to RM.

The advantage of the Foucault method with respect to that of Fizeau consists in the possibility to take distance d very short. So Foucault used a minimum basis d = 4 m and still obtained a reliable result, while Fizeau worked with a basis d = 8.6 km.

§49. THE MICHELSON EXPERIMENT

A. The Michelson-Morley experiment

The historical MICHELSON-MORLEY EXPERIMENT (1887) was proposed by Maxwell (1878) and designed by Michelson (1881) with the aim to measure the Earth's velocity with respect to absolute space. Indeed, if one treats the Michelson-Morley experiment from the view-point of the traditional Newton-aether theory, it has to give a positive result, and one comes to the conclusion that this experiment can help to reject the principle of relativity.

Thus when the experiment has given a null result, this was treated as a drastic contradiction to the aether theory of light. Fitz-gerald and Lorentz tried to explain the null result of the Michelson-Morley experiment introducing the famous length contraction dogma (see p. 32). Einstein put the null result of this experiment in the grounds of his special theory of relativity, shaking defiantly the stable columns of Newton's absolute space-time conceptions.

Humanity sacrified almost 70 years in the battle to save Newton's absolute space-time and to explain the Michelson-Morley experiment with a variety of models for the propagation of light but nobody succeeded to do this.

We think that only our absolute space-time theory, which explains the kinematics of massive and massless particles by the help of the Marinov forms of their proper momenta and energies, and which gives the right formulas for the velocity of light, has given a satisfactory explanation of this historical experiment.

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The Michelson-Morley experiment is carried out with the so-called Michelson interferometer which will be described here shortly, and for a detailed account on which the reader is referred to a textbook on optics.

The Michelson interferometer (see fig. 49.1) consists of a source of light S, a semi-transparent mirror SM, two mirrors m and M lying at the distances r and R from SM, and a screen (or observer's tube) P. Monochromatic parallel light emitted by the source S is partially transmitted and partially reflected by the semi-transparent mirror SM whose plane divides the right angle between r and R into two equal parts. These two beams, which we call the "r-beam" and the "R-beam", after being reflected by mirrors m and M, return back to SM. After a respective reflection and transmission, they proceed together in the direction to screen P. If the photons emitted by S are coherent (coherent are photons

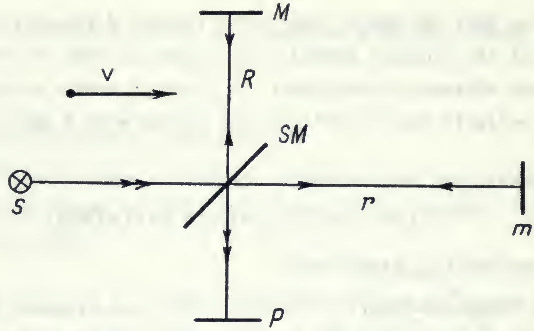

Fig. 49.1

which produce a stable interference), then over the screen P one can observe the interference pattern produced by both beams. If their paths between separation and unification have the same lengths (i.e., if the time intervals for which the photons cover the distances from SM to m and M, there and back, are the same), over P one observes a maximum illumination. If the light paths of both beams have different lengths, the illumination will be maximum when the differences in the paths is equal to a whole number of wavelengths λ of the used light, and minimum when this difference is an even number of half-wavelengths.

In the case that the reader does not wish to operate with the complicated notion "interference", he can consider the following simplified presentation of the Michelson-Morley experiment:

Let short light pulses are emitted from S. These light pulses are separated by the semi-transparent mirror SM and after going to the mirrors m and M, unify again at the semi-transparent mirror SM and proceed to the observer P. If the light pulses from both beams will come together to the observer, this will correspond to a maximum illumination in the case of interference; if the pulses will come exactly one after another, this will correspond to a minimum illumination in the case of interference. The distance between the pulses on a snap-shot, i.e. the period of "chopping" multiplied by c, will correspond to the light wavelength λ. The light pulses are "coherent" during a certain interval of time, if in this time interval all of them are chopped <u>exactly</u> with the same period. If this regularity is not maintained, the pulses are "incoherent". Obviously, "incoherent" light pulses cannot produce a stable "interference", if after the separation the separated pulses cover different paths before meeting again.Photons produced by ordinary light sources are coherent for light path differences of the order of a couple of centimeters; photons produced by lasers are coherent for light path differences of hundreds of meters and kilometers.

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The Michelson-Morley experiment can easily be analysed in the frame of the traditional Newton-aether theory if taking into account the analysis of the light clock performed in §2.

As formulas (2.3) and (2.5) show, the light path difference between the "r-beam", which we take parallel to the absolute velocity v of the interferometer, and the "R-beam", which we take perpendicular to v, is

$$\Delta = cT_{\parallel} - cT_{\perp} = \frac{2r}{1 - v^2/c^2} - \frac{2R}{(1 - v^2/c^2)^{1/2}} \cong 2(r - R) + (2r - R)\frac{v^2}{c^2}, \quad (49.1)$$

where $T_{\parallel}$ is the time for which the "parallel" beam covers the "r-arm" to and fro, and $T_{\perp}$ is the time for which the "perpendicular" beam covers the "R-arm" to and fro; the result on the right side is taken with an accuracy of second order in v/c.

If we rotate the Michelson interferometer over 90^0, so that the "parallel" beam will become "peprendicular" and vice versa, the light path difference between the "r-beam" and "R-beam" will change with

$$\delta\Delta = \Delta' + \Delta'' = 2(r - R) + (2r - R)\frac{v^2}{c^2} + 2(R - r) + (2R - r)\frac{v^2}{c^2} = (r + R)\frac{v^2}{c^2}. \quad (49.2)$$

If λ is the wavelength of the used light, the shift in the interference pattern will be of

$$z = \frac{\delta\Delta}{\lambda} = \frac{r + R}{\lambda}\frac{v^2}{c^2} \quad (49.3)$$

wavelengths.

If the interfereometer has equal "arms", i.e., r = R = d, we obtain

$$z = 2dv^2/\lambda c^2, \quad (49.4)$$

a result already obtained on p. 152.

Michelson and Morley (1887) performed the experiment with equal "arms" of the interferometer, Kennedy and Thorndike (1932) with different "arms". These experiments, as the experiments of many other investigators have given a null result. The sensitivity of the different experiments was different, but all of them could give a reliably observable effect if the absolute velocity of the Earth were only its velocity around the Sun.

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We already showed (see p. 152) that according to our theory the result in the Michelson-Morley experiment must be null.

This can also be seen if using formulas (4.28) for the velocity of light registered in the moving laboratory on a clock attached to the laboratory. Indeed, taking into account that for the "perpendicular" light beam we have $\theta = \pi/2 - v/c$, $\theta' = \pi/2$; for the "parallel" beam $\theta = \theta' = 0$, and for the "anti-parallel" beam $\theta = \theta' = \pi$, we obtain

$$T_{\perp} = \frac{R}{c'_{\perp}} + \frac{R}{c'_{\perp}} = \frac{R}{c} + \frac{R}{c} = \frac{2R}{c}, \qquad T_{\parallel} = \frac{r}{c'_{\uparrow\uparrow}} + \frac{r}{c'_{\uparrow\downarrow}} = \frac{r}{c}(1 + \frac{v}{c}) + \frac{r}{c}(1 - \frac{v}{c}) = \frac{2r}{c}. \quad (49.5)$$

Thus the light path difference between the "r-beam" and the "R-beam" does not depend on the orientation of the interferometer, as

$$\Delta = cT_{\parallel} - cT_{\perp} = 2(r - R). \quad (49.6)$$

B. The Michelson-Marinov experiment

If the Michelson experiment would be perofmred with massive particles, we call it the MICHELSON-MARINOV EXPERIMENT. We have already shown on p. 152 that the Michelson-Marinov experiment must give a null result, too.

The construction of a Michelson neutron interferometer is a difficult problem, as in neutron interferometry the points of separation and meeting of the the coherent neutrons must be different. Thus the interferometer must have an intermediary form between a Michelson-type interferometer and a square interferometer of the kind of that used for the measurement of the Sagnac effect with neutrons (see §60). Then the hypothetical Newton-aether effect will represent a $\Delta S/S$ part of the effect (4), where ΔS is the difference be tween the area S encircled by the square interferometer and the area encircled by the actual interferometer, in which the "arms" for flying "there" are the same as in the Michel son or Sagnac interferometers, but the "arms" for flying "back" are shorter, and only for a Michelson-type interferometer become equal to the "arms" for flying "there".

As our theory predicts with surety a null effect in this experiment, we think, one must not spend time, efforts, and money for its performance. Thus being deeply convinced that this difficult experiment will be never practically realized, we labeled it the Michelson-Marinov experiment.

§50. THE QUASI-FIZEAU "COUPLED SHUTTERS" EXPERIMENT

A. The "coupled shutters" experiment

In the Fizeau "rotating cog-wheel" experiment (see §48A) one measures the two-way velc city of light. If one should use a rotating axle on whose extremities there are two cog-wheels, one obtains the QUASI-FIZEAU "COUPLED SHUTTERS" EXPERIMENT (or "DOUBLE COG-WHEEL' EXPERIMENT), with the help of which the one-way velocity of light can be measured. Indeec a rotating axle allows the realization of a Newtonian time synchronization and thus, sinc the laboratory's absolute velocity figures in the one-way light velocity, by the "couplec shutters" experiment one can measure this absolute velocity.

All experiments for measurement of the laboratory's absolute velocity which we have carried out or proposed (excluding the quasi-Bradley experiment (see §46) and the "synchronous light clocks" experiment (see §56)) have as a common element a rotating axle, ar all of them appertain to the big class of "ROTATING AXLE" EXPERIMENTS.

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Now we shall give the essentials of the "coupled shutters" experiment.

Let us have (see fig. 50.1) two cog-wheels C_1 and C_2 fixed on a common shaft of lengtr d which is rotated by an electromotor EM. Intense light is emitted from the sources S_1 and S_2. Light from S_1, after passing through the notches of the cog-wheels C_1 and C_2 is is observed by the observer O_1. Similarly, light from S_2, after passing through the notches of C_2 and C_1, is observed by O_2. The direction from S_1 to O_1 will be called "direct" and from S_2 to O_2 "opposite".

First we consider the case in which the velocity of light in the "direct" and "opposite" directions is the same and equal to c. If both wheels have the same number of cogs and are placed symmetrically opposite each other (i.e., "cogs against cogs"), then, if set in rotation, the observers will register maximum photon fluxes in the case where the distance d between the wheels is covered by light in the same time that the wheels are rotated an integer number of notches. If a wheel has p notches and makes N revolutions per second, f = pN notches will pass by a point in a second. We call this number, f, chopping frequency. Obviously, the condition for observing a maximum photon flux can be written

$$n = \frac{d}{c} f, \tag{50.1}$$

where n is an integer.

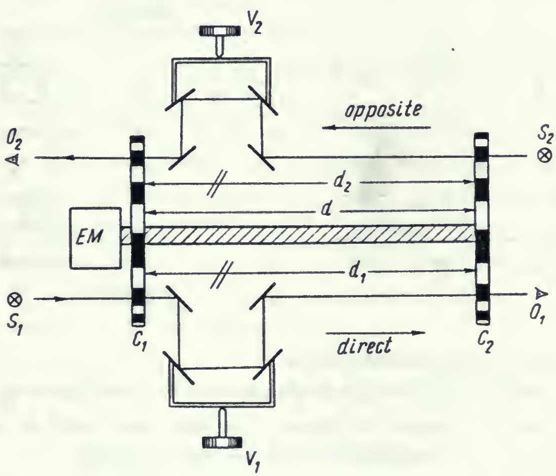

Fig. 50.1

With the help of the verniers V_1 and V_2 the optical path lengths d_1 and d_2 of the two light beams can be changed. If the velocity of light in "direct" and "opposite" directions is the same, then, obviously, O_1 and O_2 will observe maximum (or minimum) photon fluxes when $d_1 = d_2$.

If the velocity of light is c/(1 + v/c) in the "direct" and c/(1 - v/c) in the "opposite" direction (see formula (4.28)), the conditions for passing the chopped "direct" and "opposite" beams will be

$$n_1 = \frac{d_1}{c} f\left(1 + \frac{v}{c}\right), \qquad n_2 = \frac{d_2}{c} f\left(1 - \frac{v}{c}\right). \tag{50.2}$$

Assuming that, for $v \to 0$, d_1 and d_2 should become equal to d, we chall have $n_1 = n_2 = n$, and, thus, within an accuracy of first order in v/c, we can write

$$d_1 = d(1 - v/c), \qquad d_2 = d(1 + v/c), \qquad (50.3)$$

from where

$$\Delta d = d_2 - d_1 = 2\frac{d}{c}v. \qquad (50.4)$$

As the formula shows, for a given v, the effect, Δd, is proportional to the length of the axis, d. Thus the question may be posed about the feasibility of using two independent cog-wheels not fixed on a common shaft but rotating with the same angular velocity.

B. The "uncoupled shutters" experiment

For the sake of generality, we shall now speak not of two independently rotating cog-wheels but of two independently operating pairs of shutters (for instance, Kerr cells). Any pair of these shutters (see fig. 50.2) is driven by a common chopping mechanism, say, two resonators R_A and R_B. Such an experiment will be called the "UNCOUPLED SHUTTERS" EXPERIMENT.

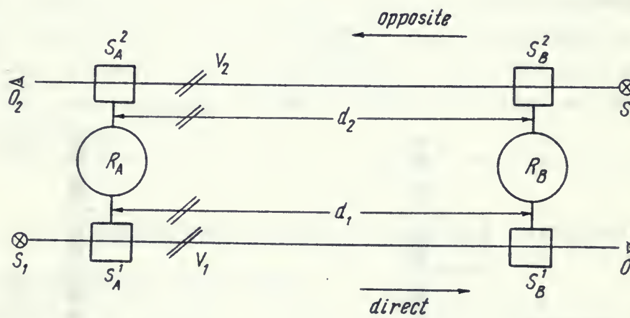

Fig. 50.2

Now the following two problems arise:

a) How to maintain <u>equal</u> chopping frequencies for both pairs of shutters.

b) How to maintain a "phase difference" between them equal to zero, i.e., how to ensure that both pairs of shutters will close and open <u>together</u>.

The first difficulty can be overcome if we use the same resonator for both pairs of shutters, which can be put near the shutter S_A, near the shutter S_B, or in the middle. However, if we transmit the signals for opening and closing the shutters by an electric line, then a "phase difference" will appear between the pairs. It can easily be shown that the "phase difference", appearing at the motion of the apparatus in absolute space, will exactly cancel the effect that we intend to observe.

Hence the resonators producing the chopping frequency must be independent. As resonators two atomic clocks can be taken which have a very exact rate. The chopping frequency f_B of the resonator R_B can be maintained equal to the chopping frequency f_A of the resonator R_A if we tune f_B in such a manner that the "beating" of the light spot observed by O_2 will always be reduced to zero.

When we use independent shutters, we cannot know the "phase difference" between them (i.e., we cannot know when the first pair od shutters is open, how far from opening is the second pair of shutters). Hence again we are unable to measure the absolute velocity v.

However, as Dart (1971) has suggested, one can rotate the apparatus with respect to absolute space. When the axis of the apparatus is perpendicular to v one can arrange the "phase difference" between both shutters so that both observers O_1 and O_2 see no light. If now the apparatus is put parallel to v, so that the "direct" direction will coincide with the direction of v, then some light will be seen by the observers; and only if we change the distances d_1 and d_2 with the help of the verniers V_1 and V_2 to give a difference Δd according to formula (4), will no light be left to pass through both pairs of the coupled shutters.

However, taking into account the absolute time dilation, we can easily see that this prediction of Dart will not hold. Indeed, during the rotation both resonators will move at different velocities with respect to absolute space. Thus their time rates will be different and exactly such that the new "phase difference" that will appear after the rotation will exactly cancel the effect to be observed if the "phase difference" after the rotation had remained the same as before the rotation.

To prove this, let us suppose that the axis of the apparatus is first perpendicular to its absolute velocity. Let us then rotate the apparatus with angular velocity ω, say, about the middle point, until the "direct" direction of its axis becomes parallel to v. Let the readings of two clocks (suppose, for simplicity, light clocks) attached to R_A and R_B be t'_A, t'_B before the rotation and t''_A, t''_B after the rotation. Let the proper times $t_A = t''_A - t'_A$, $t_B = t''_B - t'_B$ correspond to the same absolute time interval t. Because of the absolute time dilation, we have (see formula (7.4))

$$t_A = \int_0^t (1 - v_A^2/c^2)^{1/2} dt, \qquad t_B = \int_0^t (1 - v_B^2/c^2)^{1/2} dt, \qquad (50.5)$$

where

$$v_A^2 = v^2 + (d\omega/2)^2 - v\, d\omega \cos(\omega t),$$

$$v_B^2 = v^2 + (d\omega/2)^2 + v\, d\omega \cos(\omega t) \qquad (50.6)$$

are the velocities of the resonators during the rotation of the apparatus.

If we work within an accuracy of second order in v/c, we obtain, after performing the integration, putting $\omega t = \pi/2$, and subtracting the second of formulas (5) from the first,

$$\Delta t = t_A - t_B = dv/c^2. \qquad (50.7)$$

This formula shows that if before rotation the "phase difference" between both pairs of shutters is equal to zero, then after the rotation the shutter S_B^1 will open with a delay Δt relative to the shutter S_A^1, while the shutter S_A^2 will open with the same anticipation relative to the shutter S_B^2. Thus for the same light paths, $d_1 = d_2$, minimum photon fluxes will pass through both shutters.

Let us explain more clearly the difference between the independent shutters and the cog-wheels connected by a rigid shaft. The relations between the absolute time and proper times elapsed on two clocks moving with velocities v_A and v_B are given by formulas (5) only if the clocks are <u>independent</u>. If we consider both rotating cog-wheels as clocks, we do not have the right to use formulas (5) because the wheels are <u>rigidly</u> connected by a common shaft and there is a <u>unique</u> clock - the motor driving the shaft, which, if placed at the middle, does not change its velocity during the rotation. Thus, after the rotation a change in the "phase difference" between both cog-wheels cannot occur. If such a change appeared, then after the rotation the shaft <u>would be found to be twisted</u>, which, obviousl is nonsensical.

Let us note that scientists as Ives and Prokhovnik who, on the one hand, accept that the velocity of light is isotropic only with respect to absolute space and, on the other hand, accept that the principle of relativity has an absolute validity, advocate the existence of a such twist. By analogy with the fictitious Lorentz contraction (see p. 32), Marinov (1978) gave to this <u>similar fictitious phenomenon</u> the name LORENTZ TWIST.

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One can make many other different proposals (additionally to Dart's proposal) which <u>will seem</u> to lead to a positive effect in the "uncoupled shutters" experiment. It can be shown that by <u>no one</u> of such tricks can one succeed to measure the laboratory's absolute velocity, as the absolute effect due to the "aether wind" will always be annihilated by the appearance of a corresponding "phase difference" between both shutters.

Table 50.1

$n = d/\lambda$	m				$m+\frac{1}{4}$				$m+\frac{1}{2}$				$m+\frac{3}{4}$			
ψ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
O_1 sees	max	av	min	av	av	max	av	min	min	av	max	av	av	min	av	max
O_2 sees	max	av	min	av	av	min	av	max	min	av	max	av	av	max	av	min

This becomes clear if one analyses table 50.1. The table is set at the condition that the absolute velocity of the apparatus is perpendicular to its axis or that the apparatus is at rest in absolute space. In the first row is given the number of the light "waves" n along the distance d (λ is the "wavelength"), i.e., the number of light pulses which can be counted on a <u>momentary photograph</u> between both shutters; m is an integer. In the second row is given the phase difference between both shutters in radians. So if the phase difference is $\psi = 0$, both shutters are open or close together; if $\psi = \pi$, one of the shutters is open when the other is close; if $\psi = \pi/2$ (or $3\pi/2$) one of the shutters is half-open when the other is half-close. It is important to note that if the phase shift of the detecting shutter near O_1 is ψ with respect to its chopping shutter near S_1, then the phase shift of the detecting shutter near O_2 is equal to $-\psi$ with respect to its chopping shuter near S_2. In the third row is given the light intensity registered by the first observer, and in the fourth row - by the second observer.

Let us suppose, for example, $n = m$, $\psi = \pi/2$. In this case both observers see average light intensity. Let, under these conditions, an aether wind apeear: (i) by setting the apparatus in motion with a velocity v, if at the initial moment it has been at rest in absolute space or (ii) by rotating the axis of the apparatus over 90^0 if at the initial moment the axis has been perpendicular to the absolute velocity v. If the motion is in the "direct" direction and $\Delta n = 1/4$, we shall have for the number of light "waves" between both shutters in the "direct" and "opposite" directions

$$n_1 = m + 1/4, \qquad n_2 = m - 1/4 = (m-1) + 3/4. \qquad (50.8)$$

Thus, if the phase difference remains the same, $\psi = \pi/2$, both observers O_1 and O_2 will see maximum light. This change in the illumination can be achieved if, when there is no "aether wind", one changes the phase difference from $\psi = \pi/2$ to $\psi = 0$. With the help of this table one can show that always an "aether wind" effect can be annulled by a corresponding change in the phase difference. As in the "uncoupled shutters" experiment one cannot know which is the actual phase difference, one can never make conclusions about the presence and the value of the "aether wind".

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One can propose to count, at a given moment, the number of light "waves" (the number of light pulses) between both shutters in the "direct" and "opposite" directions. Unfortunately, this is not posssible as one can only count the number of standing waves where, again, the absolute effect is annulled (see §66 and §67).

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Then one can propose to count the number of light pulses during a displacement of one of the shutters with respect to the other. We shall show that this method will again lead to a null result.

Let us suppose (see fig. 50.2) that the light pulses are very narrow with respect to the time between the pulses. This signifies that the notches in the "chopping" shutter (this one which is near the light source) are very narrow. Let us then suppose that the cogs in the "detecting" shutter (this one which is near the observer) are very narrow, too. Consequently, all pulses go through the "detecting" shutter except for the case when the very narrow pulse meets the very narrow cog.

Let the velocity of the laboratory be $v = c/2$ from left to right. According to formulas (4.28), the velocity of light in "direct" and "opposite" directions measured in the laboratory on a laboratory clock will be

$$c_1 = c/(1 + v/c) = \frac{2}{3}c, \quad c_2 = c/(1 - v/c) = 2c. \tag{50.9}$$

Suppose first that the shutters S_A and S_B are very close to one another, and any of them produces 4000 pulses in a second. If now we should move the shutter S_B with a velocity $v_{tr} = c/1000$ from left to right, then, during one second of motion, 0_1 will count 3994 pulses, as over a distance 2c/3 cm, covered by light in a second, one must have 4000 pulses, and thus over a distance c/1000 cm 6 pulses will remain which the observer 0_1 will not count. Analogically 0_2 will count 3998 pulses, as over a distance 2c cm, covered by light in a second, one must have 4000 pulses, and thus over a distance c/1000 cm 2 pulses will remain which the observer 0_2 will not count. If one would not take into account that the shutter S_B has a larger time dilation with respect to the shutter S_A during the motion of the former, one should be able to calculate the laboratory's absolute velocity, v, from the equation

$$\kappa_1 c_1 = \kappa_2 c_2, \quad \text{i.e.,} \quad \kappa_1(1 - \frac{v}{c}) = \kappa_2(1 + \frac{v}{c}), \tag{50.10}$$

where κ_1 is the wave-number of the "direct" light "wave" (in our case $\kappa_1 \sim 6$), and κ_2 is the wave-number of the "opposite" light "wave" (in our case $\kappa_2 \sim 2$).

However, because of the time dilation, during this one second of motion of S_B, both observers will count the same number of pulses, namely 3996, which will be counted in the case when the absolute velocity is perpendicular to the axis of the apparatus or the apparatus is at rest in absolute space.

Indeed, if for the time t_A of transfer of S_B (equal to one second, as supposed)

$$t = t_A(1 - v^2/c^2)^{-1/2} \tag{50.11}$$

absolute second have elapsed (t_A is the time read on a clock attached to the shutter S_A whose absolute velocity is v), the time read on a clock attached to shutter S_B, which moves with an absolute velocity $v + v_{tr}$, will be (d is the distance of removal of the shutters)

$$t_B = t\{1 - (v + v_{tr})^2/c^2\}^{-1/2} \cong t_A(1 - \frac{vv_{tr}}{c^2}) = t_A - \frac{dv}{c}. \tag{50.12}$$

For our case (v = c/2) we shall have

$$t_B = t_A - d/2c. \tag{50.13}$$

Substituting here

$$t_A = d/v_{tr} = 1000d/c, \tag{50.14}$$

we obtain

$$t_B = 1000\,d/c - d/2c = \frac{1999}{2}\frac{d}{c} = 0.9995\,t_A. \tag{50.15}$$

Thus, during the time t_A (= 1 sec) the shutter S_A produces 4000 pulses, while the shutter S_B produces 3998 pulses, and 0_1 will count with two pulses more, i.e., 3994 + 2 = 3996 pulses, while 0_2 will count with two pulses less, i.e., 3998 - 2 = 3996 pulses, exactly as in the case where the absolute velocity is perpendicular to the axis of the apparatus.

Thus there is no possibility to measure the laboratory's absolute velocity by the help of the "uncoupled shutters" experiment.

C. The absolute "coupled shutters" experiment

In §50C and §50D we shall present two variants of the "coupled shutters" experiment performed by Marinov (1980). By the help of the absolute variation described in §50C for the first time in history Marinov measured the one-way light velocity in a laboratory. The differential variation described in §50D can be used with a high reliability for measurement of the Earth's absolute velocity (Marinov's accuracy was not sufficient for measurement of the absolute velocity). According to our opinion, at the present time the differential "coupled shutters" experiment represents the most simple and easily realisable apparatus for measurement of the Earth's absolute velocity. Moreover, its theoretical explanation is the most simple.

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The scheme of the ABSOLUTE "COUPLED SHUTTERS" EXPERIMENT is the following (see fig. 50.1, cancelling the verniers V_1 and V_2):

The rotating axle has two disks with holes on its extremities. The distance from the centers of the holes to the center of the axle is R = 9 cm. The distance between the extremities of the disks is d = 140 cm. The axle is driven by an electromotor EM which can rotate with a rate up to N = 200 rev/sec (measured by a digital cyclometer). A He-Ne laser is put on one side of the axle and a photodiode in front of it on the opposite side. With a galvanometer one measures either the total current generated by the photodiode or the change in the current when changing the rate of rotation of the axle.

One arranges the position of the laser beam with respect to the disk's holes so that when the axle is at rest the light of the laser which passes through the near hole illuminates only a part (say, the half) of the far hole. The axle is set in rotation and its rotational speed is increased gradually. Since the light pulses cut by the near holes have a transit time in order to reach the far holes, with the increase of the rate of rotation less and less light will pass through the far holes (when the distant holes "escape" from the light beam positions) and conversely more and more light will pass through the far holes (when the distant holes "enter" into the light beam positions).

The illumination at the different points of the light spot is not homogeneous - at the edges the illumination is lower and at the center the most intense. For the sake of simplicity, we assume that the illumination is homogeneous. The electric current I produced by the photodiode is proportional to the light flux which falls on it. Thus, under the simplified assumption about the illumination, one can assume that the current I is proportional to the breadth b of the light spot: $I \sim b$. This is so if the holes, as well as the light beams produced by the lasers, are rectangular; taking into account that they are circular, for the "breadth" b one will obtain a certain "effective" breadth. Since the breadth of the light spot cannot be measured accurately, we shall not enter deeper into the problem about the "geometrical" estimation of the "effective" breadth, moreover taking into account that the illumination is not homogeneous.

When the rotational rate of the axle changes with ΔN, a Δb part of the light spot will not pass through the far holes and the illumination over the photodiode will diminish with ΔI which is proportional to Δb: $\Delta I \sim \Delta b$. Thus one can write

$$\Delta b/\Delta I = b/I. \tag{50.16}$$

The best results are obtained if one rotates the axle first with $\Delta N/2$ counter-clockwise and then with $\Delta N/2$ clockwise, that corresponds to a change ΔN in the rate of rotation.

Since

$$\Delta b = (d/c)2\pi\,\Delta N\,R, \tag{50.17}$$

for the one-way velocity of light we obtain

$$c = \frac{2\pi\,\Delta N R d}{b}\,\frac{I}{\Delta I}. \tag{50.18}$$

The measurement of ΔI can be made by compensating the voltage produced by the photodiode by the help of a fixed voltage and then measuring the appearing current when changing the rate of rotation. We used the differential method described in §50D. We measured $\Delta I = 1.08\ \mu A$ for $\Delta N = 100$ rev/sec. The breadth of the light spot was measured, at $I = 12.1$ mA $b = 3.2 \pm 0.3$ mm (when the axle is in rotation). Thus, according to equation (18) $c = (3.0 \pm 0.3)\times 10^8$ m/sec. Let us emphasize that even if the light spot were rectangular and the intensity homogeneous, the measurement of the breadth will always include an error of about 10% (the edges of the light spot can never be sharp enough). Since the errors in the measurement of ΔN, R, d, I, and ΔI are less than 10%, the error in b is the decisive one. Thus, with this method, the absolute measurement of the one-way light velocity will always include an error of not less than 10%. If the accuracy of this experiment is high enough (higher than 0.1%), one will establish that the one-way velocity of light is direction dependent and one will be able to measure the absolute velocity of the laboratory. We doubt that the absolute velocity can be registered with the absolute method but only with the differential method (see below).

D. The differential "coupled shutters" experiment

In the DIFFERENTIAL "COUPLED SHUTTERS" EXPERIMENT instead of one laser and one photodiode two pairs are used, placed as the light sources and the observers in fig. 50.1. However, since the fluctuation of the lasers intensities may be considerable, it is better to use a single laser, dividing its beam by a semi-transparent mirror. In this case the intensities in both light beams fluctuate synchronously and since ΔI (as well as δI, see below) are measured as a difference in the currents produced by both light beams, the fluctuations in their intensities are automatically excluded.

According to equation (18), the velocity of light in the "direct" direction (i.e., along the component v of the laboratory's absolute velocity on the axis of the apparatus) and in the "opposite" direction (i.e., against v) will be

$$c - v = \frac{2\pi \Delta N Rd}{b} \frac{I}{\Delta I - \delta I}, \qquad c + v = \frac{2\pi \Delta N Rd}{b} \frac{I}{\Delta I - \delta I}, \tag{50.19}$$

where $\Delta I + \delta I$ and $\Delta I - \delta I$ are the changes in the currents generated by the corresponding photodiodes when the rate of rotation is changed by ΔN. Dividing the second of these equations by the first, we obtain

$$v = \frac{\delta I}{\Delta I} c. \tag{50.20}$$

Thus the measuring method consists in the following: One changes the rotational rate with ΔN and one measures the change in the current of any of the photodiodes which is $\Delta I \cong \Delta I \pm \delta I$; then one measures the difference of these two changes which is

$$\Delta I + \delta I - (\Delta I - \delta I) = 2\,\delta I. \tag{50.21}$$

Both these measurements can be made by a differential method with the same galvanometer, applying to it the difference of the outputs of both photodiodes. To measure $2\,\Delta I$ one makes one light spot to "escape" from the far holes and the other to "enter". To measure $2\,\delta I$ one makes both light spot to "escape" (or "enter") from the far holes. At the first case (measurement of ΔI) the phase difference between the one pair of shutters is $\psi = \pi/2$, and between the other pair is also $\psi = \pi/2$ (as a matter of fact, $\psi = -\pi/2$ - see the top of p. 175). As in our experiment the number of light "waves" between the shutters was $n = 0$, table 50.1 shows immediately that in this case the change of n (due to a change in the chopping frequency) leads to opposite changes in the observed illuminations, and the measured effect is proportional to c. At the second case (measurement of δI) the phase difference between the one pair of shutters is $\psi = \pi/2$, and between the other pair is $\psi = 3\pi/2$ (as a matter of fact, $\psi = -3\pi/2$). Table 50.1 shows that in this case the change of n leads to the same changes in the observed illuminations, and the measured effect is proportional only to v.

In our experiment the fluctuations of the galvanometer were about 1% of ΔI. We estimated the fluctuations changing the rotational rate from $\Delta N/2$ to $-\Delta N/2$ and then returning again to $\Delta N/2$. Thus we could not register the Earth's absolute velocity (which is about 0.1% of c - see §52B). Thus we can only affirm that the results obtained showed that the component of the Earth's absolute velocity along the axis of the apparatus during the different hours of the day was not bigger than 3000 km/sec.

When the axle rotates with a constant speed the fluctuations in the difference current were with about an order lower (during a time of 1 - 2 minutes). Thus if we could rotate the whole apparatus putting it on a platfrom, we could register the Earth's absolute motion. In such a case one has to rotate the platform through 360^{o} when the axle rotates with a certain rate N and register $2\,\delta I$ (at both cases of "entrance" or "escaping" of the far holes from the light beam positions). To register $2\Delta I$ one changes the rate of rotation of the axle from N clockwise to N counter-clockwise proceeding as described above. However we had not the possibility to rotate the apparatus. Let us inform the reader that it took a week to build the experimental set-up in a second-class workshop and the whole financial expenditure (excluding the cost of the laser and of the galvanometer) was about $ 500.

To obtain more accurate results, one has to increase R, d, and N, and diminish to the lower possible level the fluctuations in the difference current. An increase in the intensity of light (sending light, say, through all holes) will also enhance the accuracy. According to our estimation, with this method the Earth's absolute velocity can be measured with an accuracy as high as 1 km/sec.

§51. THE QUASI-FOUCAULT "COUPLED MIRRORS" EXPERIMENT

A. The deviative "coupled mirrors" experiment

In the Foucault "rotating mirror" experiment (see §48B) one measures the two-way velocity of light. If one should use a rotating axle on whose extremities there are two mirrors, one obtains the QUASI-FOUCAULT "COUPLED MIRRORS" EXPERIMENT (or "DOUBLE MIRROR" EXPERIMENT), with the help of which the one-way velocity of light can be measured.

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With the DEVIATIVE "COUPLED MIRRORS" EXPERIMENT, in the summer of 1973, Marinov (1974) for the first time in history, succeeded to register the Earth's absolute motion, performing measurements in a closed laboratory. Here is the description of this historical experiment (see fig. 51.1).

Let us have two disks rotated with a fixed phase difference (imagine the wheels of a bicycle). On each disk two antipodal facets are cut and one is made a mirror, while the other and the rest of the disk's rim are not light reflecting. The distance between both disks, called the rotating mirrors RM_1 and RM_2, is d. Intense light from the source S_1 (or S_2) is reflected by the semi-transparent mirror M_1 (M_2) and, after passing through the semi-transparent mirror N_1 (N_2), is incident on the mirror facet of RM_1 (RM_2). The light beam reflected then by the semi-transparent mirrors N_1 and N_2 (N_2 and N_1) whose distance from the rotating mirror is p, is incident on the mirror facet of RM_2 (RM_1). If the rotating mirrors are at rest, the light beam reflected by the cylindrical mirror CM_2 (CM_1) will illuminate screen S from the right (from the left) at a certain point. The light path from the rotating mirrors to the cylindrical mirrors is D and from the cylindrical mirrors to the screen is d/2.

If the rotating mirrors are set in motion, then, because of the slit T, only the light which is reflected by RM_1 (RM_2) when the latter is perpendicular to the incident beam will reach RM_2 (RM_1). However, for the time spent by light to cover the distance d + 2p,

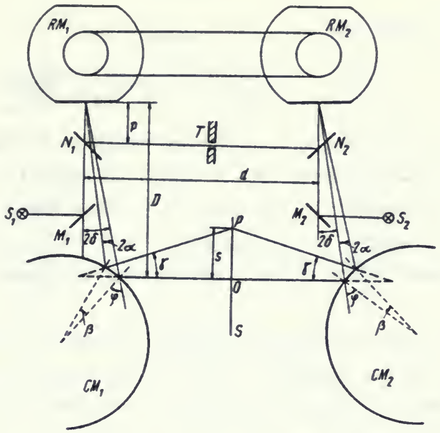

Fig. 51.1

the facet of RM_2 (RM_1) which is parallel (an exact parallelism is not necessary!) to the corresponding facet of RM_1 (RM_2) will rotate by a certain angle

$$\delta = \frac{d + 2p}{c}\Omega, \tag{51.1}$$

where Ω is the angular velocity of the rotating mirrors.

Suppose now that light velocity along the direction from RM_1 to RM_2 (which we call "direct") is (see formula (4.28), taking it within an accuracy of first order in v/c) c - v, and along the direction from RM_2 to RM_1 (which we call "opposite") is c + v. In such a case during the time in which the light pulse reflected by RM_1 will reach RM_2, the latter will rotate to an angle $\delta + \alpha$, while during the time in which the light pulse reflected by RM_2 will reach RM_1, the latter will rotate to an angle $\delta - \alpha$, and we shall have

$$\delta \pm \alpha = \left(\frac{d}{c \pm v} + \frac{2p}{c}\right)\Omega, \tag{51.2}$$

from where (assuming $v \ll c$) we get

$$\alpha = \Omega\, d\, v/c^2. \tag{51.3}$$

Our apparatus takes part in the diurnal rotation of the Earth and in 24 hours it makes all possible angles with the component of the Earth's absolute velocity in the plane determined by the different positions of the apparatus during the day; this component we shall refer to as the Earth's absolute velocity and designate by v.

Suppose first that the unit vector along the "direct" direction $\vec{n}$ is perpendicular to $\vec{v}$, and that the cylindrical mirrors are adjusted so that the chopped light beams will illuminate the same point O on the screen S. Now, if $\vec{n}$ becomes parallel to $\vec{v}$, both light beams will illuminate point P, and for the distance between O and P we shall have (suppose $\phi = \frac{\pi}{4}$)

$$s = \gamma\frac{d}{2} + 2\alpha D, \tag{51.4}$$

where $\gamma = 2(\alpha + \beta)$ and $\beta = 2\alpha(D/R)\sec\phi$; angles β, γ, and ϕ are shown in fig. 51.1, and R is the radius of the cylindrical mirrors. Thus we have

$$s = \frac{\Omega}{c^2} d^2 v\{1 + 2D(\frac{1}{d} + \frac{\sec\phi}{D})\}. \qquad (51.5)$$

The establishment of velocity v is to be performed as follows: In regular intervals of time during a whole day we maintain such a rotational velocity Ω that the chopped light beam from the left will always illuminate point O. Then the light beam from the right will illuminate point O when $\vec{n}$ is perpendicular to $\vec{v}$; it will be displaced over a distance 2s upwards when $\vec{n}$ is parallel to $\vec{v}$ and over the same distance downwards when $\vec{n}$ is antiparallel to $\vec{v}$.

In our actual set-up, both rotating disks were fixed on a common shaft because the most important requirement of the "coupled mirrors" experiment is the maintaining of an equal phase difference between both rotating mirrors during the Earth's rotation. Two He-Ne lasers were used as light sources. We used three cylindrical mirrors for each beam and such a combination of cylindrical mirrors which increase enormously the "arm" of a light beam is called by us the "cylindrical mirrors indicator". The light spots were observed over two different screens because in our actual experiment both rotating mirrors lay in two different parallel planes. According to the actual arrangement s = 0.62 mm for v = 100 km/sec. This displacement was large enough to be reliably registered. However, the inconstancy of the cylindrical mirrors radii and the trembling of the images were considerable and our experiment did not lead to an accurate quantitative measurement of v. The observed displacement was maximum 3 ± 2 hours after midnight and after noon and corresponded to a velocity v = 130 ± 100 km/sec, the "direct" direction being the one after midnight. The distance between both rotating mirrors was d = 7.2 m, the radius of the cylindrical mirrors was R = 8 cm, and the velocity of rotation of the shaft, taken from an old torpedo-boat, was $\Omega/2\pi$ = 80 rev/sec. The azimuth of the apparatus was 84^0 and the observations were performed in July-August in Sofia.

The error ± 100 km/sec was established in the following manner: An observer maintained for 2-3 minutes one of the light spots in a certain position, adjusting by hand a corresponding tension of a dc electromotor which drives the shaft. Another observer registered the diapason of trembling of the other light spot which was normally 2-3 mm. If the diapason is Δs = 2.48 mm, then the fluctuation error is ± 100 km/sec.

B. The oscilloscopic "coupled mirrors" experiment

The OSCILLOSCOPIC "COUPLED MIRRORS" EXPERIMENT was proposed by Marinov (1979). Its experimental arrangement is the following (see fig. 51.2):

EM is a motor rotating a shaft of length d. The shaft has two mirrors on its ends, RM_1 and RM_2, called the rotating mirrors. Intense light emitted by the laser L is split by the semi-transparent mirror SM into two beams. In the following description the alternative route is shown in parentheses. The "transmitted" ("reflected") beam passes through

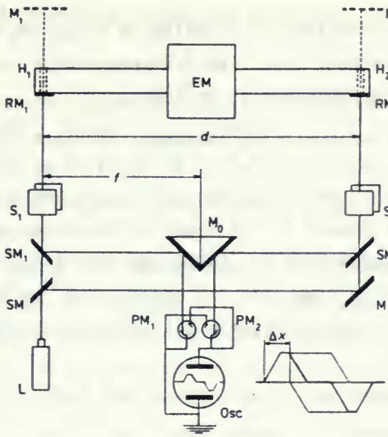

Fig. 51.2

the semi-transparent mirror SM_1 (is reflected by mirror M and passes through the semi-transparent mirror SM_2) and is reflected by the rotating mirror RM_1 (RM_2); then it is reflected by the semi-transparent mirror SM_1 (SM_2) and, reflecting on the right-angled mirror M_0, strikes the photomultiplier PM_1 (PM_2) where the light pulse is transformed into an electric pulse. The outputs of the photomultipliers (opposed to one another) are applied to the horizontal plates of the oscilloscope Osc. One is interested only in the leading edge of the light (and electric) pulses, so that the duration of the pulses is of no importance. Only the steepness of the edges is important.

The display mechanism of the oscilloscope is triggered by one of the electric pulses. We shall assume that the pulses are trapezoidal, which can be achieved by limiting the electric outputs to a certain level. If the display time is longer than the duration of the pulses, then, in the general case when M_0 is not exactly at the mid point, or RM_1 and RM_2 are not exactly parallel, we shall see two oppositely oriented pulses on the screen. If the display time is shorter than the duration of the pulses, only one pulse will be seen on the screen. Moving the system M_0-PM_1-PM_2 to the left or right, i.e., changing the distance f, we can make the leading edges of both pulses on the screen coincide in time. This signifies that the light pulses reflected from RM_1 and RM_2 reach M_0 at exactly the same moment.

The direction from SM_1 to SM_2 is called "direct" and from SM_2 to SM_1 "opposite". Let us suppose that the absolute velocity v of the laboratory is pointing in the "direct" direction. In this case the velocity of the "direct" pulse will be $c + v$ and of the "opposite" light pulse, $c + v$. Thus, if we begin to rotate the whole apparatus (which is mounted on a horizontal rotating platform), two opposite pulses will appear on the screen; after a rotation through 180^0 (when the velocity of the "direct" pulse will be $c + v$ and of the "opposite" pulse $c - v$) the pulses will be a maximum, and after a rotation through 360^0 they will again disappear. If the constant of scanning is k sec/cm, then the maximum distance Δx (see fig. 51.2) will correspond to a time difference $\Delta t = k\,\Delta x$, which can be expressed through the parameters of the apparatus as follows

$$\Delta t = \frac{f}{c-v} - \frac{d-f}{c+v} - \left(\frac{f}{c+v} - \frac{d-f}{c-v}\right) \cong \frac{2dv}{c^2}, \qquad (51.6)$$

where the result on the right side is written with an accuracy of first order in v/c.

The shaft should then be rotated in the opposite direction and Δt re-established.

Assuming v = 300 km/sec and taking d = 1.5 m, we obtain Δt = 10 ps. Thus if one uses an oscilloscope with horizontal time base 10 ps/cm, then, assuming that the inaccuracy of reading is (1/10)th part of the scale, one will be able to measure the absolute velocity with an accuracy of 10%.

Higher accuracy will be achieved if one uses a dual beam oscilloscope.

C. The oscilloscopic "coupled shutters" experiment

Instead of pulses reflected by the rotating mirrors, with the help of the holes H_1, H_2 and the mirrors M_1, M_2 one can get pulses cut by the rotating shaft (see fig. 51.2) So one obtains the OSCILLOSCOPIC "COUPLED SHUTTERS" EXPERIMENT. Making the distance between the axle and the mirrors M_1, M_2 changeable, one can reduce at pleasure the duration of the light pulses. Thus having photomultipliers with a good amplification, one can make the fronts of the pulses enough steep.

Marinov (1980) carried out the oscilloscopic "coupled shutters" experiment in a slightly different variation (because of the lack of funds). In the realized variation, the light of two lasers is cut by two holes on the extremities of a rotating axle (distance between the holes d = 140 cm). The light pulses illuminated two photodiodes and the produced electric pulses went along cables to a double beam oscilloscope. Thus not the velocity of light in air was measured but the velocity of propagation of electric pulses in a cable. Because of the low linear velocity of the "shutters" and the inevitable degradation of the pulses in the oscilloscope, the leading edge of the pulses on the screen became quasi parallel when the necessary short time of scanning was switched on. Displacement of 10 ns of the leading edge of the pulses on the screen could reliably be registered while the absolute velocity effect corresponded to about 10 ps (see §51B). Thus we remained with three orders of magnitude from the absolute effect which was to be registered.

§52. THE MARINOV "COUPLED MIRRORS" EXPERIMENT

A. General remarks

The result obtained with our deviative "coupled mirrors" experiment was very inaccurate, and the scientific community remained sceptical as to whether we had really registered the Earth's absolute motion. For this reason in the summer of 1975 we carried out the interferometric "coupled mirrors" experiment, obtaining a reliable value for the Earth's absolute velocity (Marinov, 1977).

The MARINOV (or INTERFEROMETRIC) "COUPLED MIRRORS" EXPERIMENT represents a combination of the Michelson experiment and a "rotating axle" experiment. For this reason we shall again reconsider the Michelson experiment (see §49). However, now we shall be interested not in eventual second-order in v/c effects (as we showed, they are null) but in the first-order in v/c effect. Thus we can work with the traditional Newton-aether conceptions.

To explain better the essence of the interferometric "coupled mirrors" experiment, we shall present the following simple and amusing example:

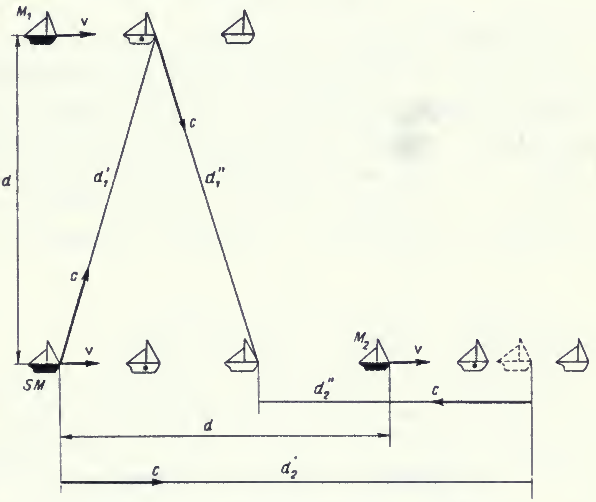

Fig. 52.1

Let us have (see fig. 52.1) a formation of three ships - SM (Santa Maria), M_1 (Maria-1), and M_2 (Maria-2) - which sail in a lake with velocity v, supposing that the distances between Santa Maria and the other two ships are equal to d and mutually perpendicular. Let two good swimmers, who can swim uninteruptedly with a velocity c ($c > v$), jump at the same moment from Santa Maria in the water. The first one swims to Maria-1 to and fro, covering a distance $d_1' + d_1''$, and the other one swims to Maria-2 to and fro, covering a distance $d_2' + d_2''$. If we imagine that SM is a semi-transparent mirror, M_1, M_2 two mirrors and the swimmers two photons, we obtain the Michelson-Morley experiment (see fig. 52.2).

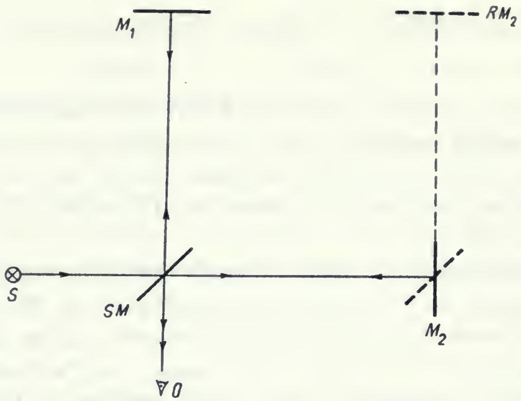

Fig. 52.2

According to Newtonian (and our) light kinematics, swimmer-2 will reach Maria-2 with the following time delay

$$\Delta t = dv/c^2 \tag{52.1}$$

after the arrival of swimmer-1 at Maria-1 (see formula (4.28)).

In fig. 52.1 we mark by spots the positions of the ships when swimmer-1 reaches Maria-1. Obviously these positions are exactly in the middle between the initial (black) and final (blank) positions. With dotted lines we mark the position of Maria-2 when swimmer-2 reaches it. When $v \ll c$, the dotted position of Maria-2 is near to the middle spot position; however, when $v \to c$ the dotted position is near to the blank position. By the help of the interferometric "coupled mirrors" experiment one measures the first-order in v/c effect (1).

B. The interferometric "coupled mirrors" experiment

To design the interferometric "coupled mirrors" experiment proceeding from the Michelson experiment (see fig. 52.2), we rotated mirror M_2 over 45^o, calling it M_1, and put the mirror RM_2 (see the dotted mirror); then we gave to mirror M_1 the name RM_1 (see now the down part of fig. 52.3) and fixed RM_1 and RM_2 on the rim of two disks set on a shaft with length d. Further on we describe together the down and upper part of the experiment.

Monochromatic parallel light emitted by the source S_1 (or S_2) was partially reflected and partially refracted by the semi-transparent mirror SM_1 (SM_2). The "transmitted" beam was then reflected successively by the mirror M_1 (M_2), by the rotating mirror RM_2 (RM_1), again by M_1, SM_1 (M_2, SM_2), and the observer O_1 (O_2) registered the interference which the "transmitted" beam makes with the "reflected" beam, the latter being reflected by the rotating mirror RM_1 (RM_2) and transmitted by SM_1 (SM_2). We call the direction from RM_1 to RM_2 "direct" and from RM_2 to RM_1 "opposite.

The shaft was set into rotation with an angular velocity Ω by an electromotor put at the middle of the axle. The shutters Sh_1 and Sh_2 allowed light to pass through only when the rotating mirrors RM_1 and RM_2 were perpendicular to the incident beams. This was accomplished by making the opening of the shutters ($\cong 10^{-6}$ sec) depend upon the rotating shaft itself. Later we realized that the shutters are even not necessary and we used simple slits placed along the light paths to the rotating mirrors. If, at very low rotational velocity, the "transmitted" light pulse reaches the second rotating mirror in the position RM_2 (RM_1) when the first rotating mirror is in the position RM_1 (RM_2), then, when the shaft is rotating with the nominal velocity, the "transmitted" pulse will reach the second rotating mirror when it is in the position RM_2' (RM_1') if the velocity of light is equal to c, and in the position RM_2'' (RM_1'') if the velocity of light is equal to c - v (correspondingly, c + v). Denoting by δ the angle between the radii of RM_2 and RM_2' (RM_1 and RM_1') and by α the angle between the radii of RM_2' and RM_2'' (RM_1' and RM_1''), we shall have

$$\delta \pm \alpha = \frac{d}{c \pm v}\Omega, \tag{52.2}$$

which for $v \ll c$, yields the result (51.3).

Let us note that, according to our theory (see formula (4.28)), the velocity of light measured on a laboratory clock in the "direct" and "opposite" directions is equal to $c - v$ and $c + v$ only within an accuracy of first order in v/c. Within an accuracy of second order in v/c the velocity of light in "direct" and "opposite" directions is $c/(1 + v/c)$ and $c/(1 - v/c)$, respectively. Thus the result (51.3) is not approximate but exact. We beg the reader very much to pay a due attention to this remark.

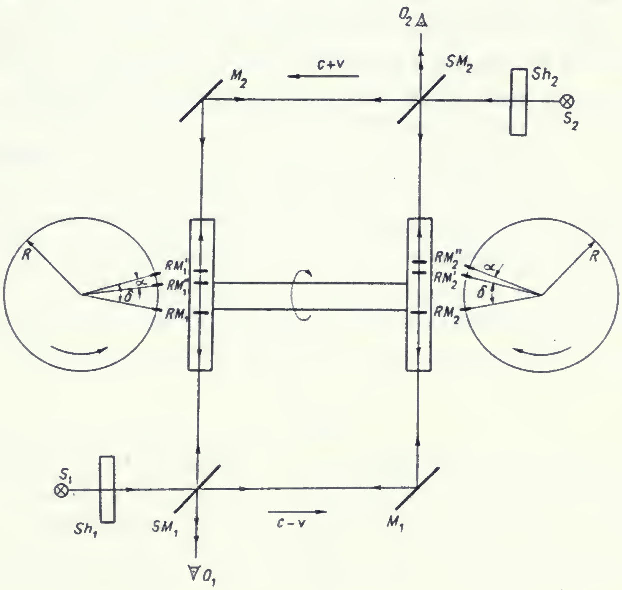

Fig. 52.3

The difference in the optical paths of the "transmitted" and "reflected" light pulses for the case when v = 0 (no "aether wind") and v ≠ 0 (presence of an "aether wind") is

$$\Delta = 2\alpha R = 2\,\frac{dR\Omega}{c^2}\,v = 2\,d\,\frac{v_r v}{c^2}, \qquad (52.3)$$

where v_r is the linear velocity of the rotating mirrors.

In our actual experiment (see fig. 52.3) the light paths of the "transmitted" and "reflected" photons are not equal. Thus the "transmitted" photons do not interfere with the "reflected" photons with which they get separated on SM_1 (SM_2) but with photons which come with a time 2d/c later to SM_1 (SM_2). As on this length the photons in a laser beam are coherent, the interference is perfect. In fig. 52.7 is presented a situation where the paths of the "transmitted" and "reflected" photons are equal.

If the wavelength of the light is λ and we maintain an angular velocity $\Omega = 2\pi N$ (N is the number of revolutions per second), then, during the rotation of the apparatus through 360^0 about an axis perpendicular to the absolute velocity v, the observers O_1 and O_2 should register changes in their interference pictures within

$$z = \frac{\Delta}{\lambda} = 4\pi\,\frac{dRN}{\lambda c^2}\,v \qquad (52.4)$$

wavelengths.

In our actual set-up, the "direct" beams were tangent to the upper parts of the rotating disks, while the "opposite" light beams were tangent to their lower parts. Thus the reflection of the beams emitted by S_1 and S_2 proceeded on the same planes of the mirrors. The "observers" in our actual set-up were two photoresistors which were put in the "arms" of a Wheatstone bridge. The changes in the interference pictures were exactly opposite. Thus in our apparatus the mirrors RM_1 and RM_2 were exactly parallel and the photoresistor were illuminated not by a pattern of interference fringes but uniformly.

A very important difference between the deviative and interferometric "coupled mirrors" experiments is that the effect registered in the latter is independent of small variations in the rotational velocity. In the interferometric variant one need not keep the illumination over one of the photoresistors constant by changing the velocity of rotation when rotating the axis of the apparatus in the horizontal plane, as this was the case in the deviative method; one need merely register the difference in the illumination over th photoresistors during the rotation. This together with the high resolution of the interferometric method are the most important advantage of the interferometric "coupled-mirrors" method.

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Since the illumination over the photoresistors changes with the change in the difference in the optical paths of the "refracted" and "transmitted" beams according to the sine law, the apparatus has the highest sensitivity when the illumination over the photoresistors is the average one (for maximum and minimum illuminations the sensitivity falls to zero). Hence a change in the velocity of rotation leads only to a change in the sensitivity. Let us consider this problem in detail.

We suppose that the electric intensities of the "reflected" and "transmitted" beams when they meet at the semi-transparent mirror SM_1 (or SM_2) are, respectively, (see V, §44)

$$E_1 = E_{max}\sin(\omega t), \qquad E_2 = E_{max}\sin(\omega t + \phi), \tag{52.5}$$

where E_{max} is the maximum electric intensity, which is equal for both beams, ω is the angular frequency of the radiation, and ϕ is the difference between the phases of the intensities in the "reflected" and "transmitted" beams.

The resultant electric intensity after the interference is

$$E = E_1 + E_2 = 2E_{max}\sin(\omega t + \frac{\phi}{2})\cos\frac{\phi}{2} = E_{ampl}\sin(\omega t + \frac{\phi}{2}), \tag{52.6}$$

where $E_{ampl} = 2\,E_{max}\cos(\phi/2)$ is the maximum electric intensity (the amplitude) of the resultant beam.

The energy flux density which falls on the photoresistors is

$$I = \frac{c}{8\pi}\,E^2_{ampl} = \frac{c}{2\pi}\,E^2_{max}\cos^2(\phi/2) = I_{max}\cos^2(\phi/2) = \frac{I_{max}}{2}(1 + \cos\phi), \tag{52.7}$$

where I_{max} is the maximum energy flux density when $\phi = 0$.

The sensitivity is

$$\frac{dI}{d\phi} = -\frac{I_{max}}{2}\sin\phi, \tag{52.8}$$

which is a maximum for $\phi = \pi/2$, $3\pi/2$, i.e., when the difference in the optical paths of the "reflected" and "transmitted" beams is $(2n+1)(\lambda/4)$, n being an integer. The sensitivity falls to zero for $\phi = 0, \pi$, i.e., when this difference is $n(\lambda/2)$.

If the resistance of the photoresistors W changes linearly with the change in the illumination (as was the case in our set-up), then to a small change dI in the energy flux density a change

$$dW = k\,dI = -\,k\,\frac{I_{max}}{2}\sin\phi\,d\phi \tag{52.9}$$

in the resistance of the photoresistors will correspond, k being a constant. For a change $\Delta\phi = \pi$ the resistance will change with $W = -\,k\,I_{max}$, as follows after the integration of (9).

Since it is $\Delta\phi = 2\pi\Delta/\lambda$, then for $\phi = \pi/2$, where the sensitivity is the highest, we shall have

$$\Delta W/W = \pi\Delta/\lambda. \tag{52.10}$$

Substituting this into equation (4), we obtain

$$v = \frac{\lambda c^2}{4\pi^2 dRN}\,\frac{\Delta W}{W}. \tag{52.11}$$

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The measuring procedure was as follws: We set such a rotational rate N_1 that the illumination over the photoresistors is minimum. Let us denote the resistance of the photoresistors under such a condition by W_1 and W_2 (it must be $W_1 = W_2$). We put the same constant resistances in the other two arms of the Wheatstone bridge, so that the same curren I_0 (called initial current) will flow through the arms of the photoresistors, as well as through the arms of the constant resistors, and no current will flow through the galvanometer in the bridge's diagonal. Then we set such a rotational rate N_2 that the illumination over the photoresistors is maximum and we connect in series with them two variable resistors, W, so that again the initial current I_0 has to flow through all arms of the bridge. After that we make the illumination average, setting a rotational rate $N = (N_1 + N_2)/2$, and we diminish correspondingly the variable resistor, W, so that again the initial current has to flow through all arms of the bridge and no current in the diagonal galvanometer. Now if we rotate the axis of the apparatus from a position perpendicular to its absolute velocity v to a position parallel to v, and if we transfer resistance ΔW from the arm where the illumination over the photoresistor has decreased to the arm where it has increased, so that again the same initial current flows through all arms and no current through the diagonal galvanometer, the absolute velocity v can be calculated from equation (11).

If over both photoresistors there is no the same illumination (i.e., the bridge is not in equilibrium) when the axis is perpendicular to the absolute velocity, we set the same illumination by changing the inclination of two glass correctors (see the correctors C' and C" in fig. 52.9). Thus disturbances (twist in the axle, temperature changes, air disturbances) can lead to disequilibration of the bridge only during the coуple of seconds when rotating the axis of the apparatus from a position perpendicular to the absolute velocity to a position parallel to the absolute velocity.

We established that the disks do not get twisted <u>differentially</u> at <u>different</u> velocitie of the shaft when setting the average illumination. We established that the illumination over the photoresistors changes from minimum to maximum (that corresponds to a change in the difference in the optical paths of the "transmitted" and "reflected" photons equal to $\lambda/2$) when the rotational rate of the shaft changed with $\Delta N = 13$ rev/sec. This change ΔN can be calculated from formula (2), putting there $\alpha = 0$, $v = 0$, multiplying it by R and setting $\delta R = \lambda/4$, i.e., from the equation

$$\Delta N = \frac{\lambda/4}{2\pi dR} c. \tag{52.12}$$

If a non-symmetric twist of the axle had led to a <u>differential</u> twist of the disks when changing the rotational velocity, the change of the illumination over the photoresistors from minimum to maximum had to appear for another change ΔN.

When the illumination over the photoresistors were average a change $\delta W = 8\times10^{-4} W$ in any of the arms of the photoresistors (positive in the one and negative in the other) could be discerned from the fluctuation of the bridge's galvanometer, and thus the resolu tion was found to be

$$\delta v = \frac{\lambda c^2}{4\pi^2 dRN} \frac{\delta W}{W} = \pm 17 \text{ km/sec.} \tag{52.13}$$

The errors that can be introduced from the imprecise values of d = 140 cm, R = 40.0 cm, N = 120 rev/sec, and λ = 632.8 nm (a He-Ne laser) are substantially smaller than the resolution and can be ignored. To guarantee sufficient certainty, we take $\delta v = \pm 20$ km/sec.

The experiment was not performed in vacuum.

The room was not temperature-controlled, but it is easy to calculate that reasonable thermal and density disturbances of the air along the different paths of the interfering light beams cannot introduce errors larger than the accepted one.

We shall calculate only the eventual thermal disturbances.The dependence of the index of refraction of air on the temperature is $dn/dT = 10^{-5}$ degree^{-1}. From formula (3) one can calculate that a fluctuation δv = 20 km/sec in the absolute velocity corresponds to a fluctuation $\delta\Delta$ = 2 Å (1 Å = 10^{-8} cm) in the difference in the optical paths of the "transmitted" and "reflected" photons. A fluctuation $\delta\Delta$ = 2 Å can be caused by a fluctuation in the temperature of the air δT which can be calculated from the equation

$$\delta\Delta = 2d \frac{dn}{dT} \delta T, \tag{52.14}$$

i.e., by a temperature fluctuation $\delta T = 7.1\times10^{-6}$ degree over the whole path of the transmitted photons. In formula (14) we take the distance covered by the "transmitted" photons equal to 2d as they go there and back, and we disregard the distances from SM_1 and M_1 (SM_2 and M_2) to the rotating mirrors RM_1 (RM_2) and RM_2 (RM_1), respectively. It is hard to believe that during the rotation of the axis of the apparatus through 90^o the temperature can change with a larger amount.

Let us note that in the variation presented in fig. 52.7 a temperature variation during the rotation cannot lead to a disequilibration of the bridge, as the illumination over both photoresistors will change by the same amount.

The whole apparatus was mounted on a platform which could be rotated in the horizontal plane. It required only a few seconds to perform a measurement.

xxxxx

The magnitude and apex of the Earth's absolute velocity were established as follows:

During a whole day we searched for the moment when the Wheatstone bridge is in equilibrium if the axis of the apparatus points east-west. At this moment the Earth's absolute velocity lies in the plane of the laboratory' meridian, and the change of the current in the diagonal galvanometer for a rotation of the platform through an angle unit is the largest. Thus turning the axis of the apparatus north-south, where the change of the current in the diagonal galvanometer for a rotation of the platform through an angle unit is the smallest, we measured the component of the absolute Earth's velocity in the horizontal plane of the laboratory. The same measurement was done 12 hours later. As can be seen from

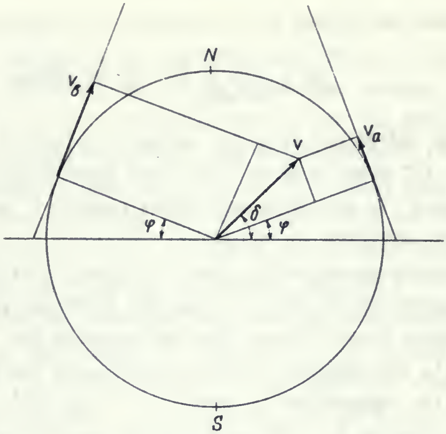

Fig. 52.4

fig. 52.4, the components of the Earth's absolute velocity in the horizontal plane of the laboratory for these two moments are

$$v_a = v\sin(\delta - \phi), \qquad v_b = v\sin(\delta + \phi), \tag{52.15}$$

where ϕ is the latitude of the laboratory and δ is the declination (see IV, §22B) of the apex. From these we obtain

$$v = \frac{v_a^2 + v_b^2 - 2v_a v_b(\cos^2\phi - \sin^2\phi)^{1/2}}{2\sin\phi\cos\phi}$$

$$\tan\delta = \frac{v_b + v_a}{v_b - v_a}\tan\phi. \tag{52.16}$$

The components v_a and v_b were taken as positive when they pointed to the north and as negative when they pointed to the south. Obviously, the apex of the absolute velocity points to the meridian of the component whose algebraic value is smaller. It was assumed $v_a < v_b$ and then the right ascension α (see IV, §22B) of the apex equaled the local sideral time of registration of v_a. We could establish this moment within a precision of abou 30 minutes. Thus it was enough to calculate (with an accuracy not larger than $\pm$ 5 min) th sideral time t_{si} for the meridian where the local time is the same as the standard time t_{st} of registration, taking into account that the sideral time at a middle midnight is as follows:

22 September	- 0^h	23 March	- 12^h
22 October	- 2^h	23 April	- 14^h
22 November	- 4^h	23 May	- 16^h
22 December	- 6^h	22 June	- 18^h
21 January	- 8^h	23 July	- 20^h
21 February	- 10^h	22 August	- 22^h.

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Our first reliable measurement of the Earth's absolute velocity with the help of the interferometric "coupled-mirrors" experiment was performed on 12 July 1975 in Sofia ($\phi = 42^0 41'$, $\lambda = 23^0 21'$). We registered

$$v_a = -\ 260 \pm 20\ \text{km/sec}, \quad (t_{st})_a = 18^h 37^m \pm 15^m,$$
$$v_b = +\ 80 \pm 20\ \text{km/sec}, \quad (t_{st})_b = 6^h 31^m \pm 15^m. \qquad (52.17)$$

Thus

$$v = 279 \pm 20\ \text{km/sec},$$
$$\delta = -\ 26^0 \pm 4^0, \qquad \alpha = (t_{si})_a = 14^h 23^m \pm 20^m. \qquad (52.18)$$

We repeated the measurement exactly six months later on the 11 January 1976 when the Earth's rotational velocity around the Sun was oppositely directed. We registered

$$v_a = -\ 293 \pm 20\ \text{km/sec}, \quad (t_{st})_a = 6^h 24^m \pm 15^m,$$
$$v_b = +\ 121 \pm 20\ \text{km/sec}, \quad (t_{st})_b = 18^h 23^m \pm 15^m . \qquad (52.19)$$

Thus

$$v = 327 \pm 20\ \text{km/sec},$$
$$\delta = -\ 21^0 \pm 4^0, \qquad \alpha = (t_{si})_a = 14^h 11^m \pm 20^m. \qquad (52.20)$$

For v and δ we took the root-mean-square error, supposing for simplicity $\phi = 45^0$. The right ascension is calculated from the moment when v_a was registered, i.e., from $(t_{st})_a$, since for this case ($|v_a| > |v_b|$) the sensitivity was better. If our experiment is accurate enough, then t_{st}, which is taken as the second, must differ with $11^h 58^m$ from t_{st}, which is taken as the first, because of the difference between solar and sideral days.

The magnitude and the equatorial coordinates of the apex of the Sun's absolute velocity are given by the arithmetical means of the figures obtained for the Earth's absolute velocity in July and January; thus

$$v = 303 \pm 20\ \text{km/sec},$$
$$\delta = -\ 23^0 \pm 4^0, \qquad \alpha = 14^h 17^m \pm 20^m. \qquad (52.21)$$

xxxxx

In fig. 52.5 we show the different rotational velocities in which our Earth takes part: v_E is the Earth's velocity about the Sun, which changes its direction with a period of one year; v_S is the Sun's velocity about the galactic center, which changes its direction with a period of 200 millions years (v_E and v_S have been established by the help of astronomical observations); v is the geometrical sum of these two and of the velocity of our Galaxy about the center of the galactic cluster, which we measure with our apparatus. If we subtract geometrically v_S ($v_S = 250$ km/sec, $\delta_S = 27^0 51'$, $\alpha_S = 19^h 28^m$) from v (see the figures in (21)), we shall obtain the rotational velocity of our Galaxy.

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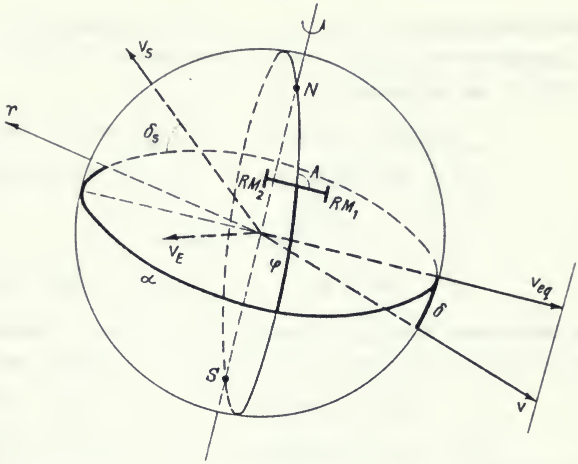

Fig. 52.5

Let us now compare the figures in (17) with those obtained in 1973 with the help of the deviative "coupled-mirrors" experiment (see p. 182). In 1972 the axis of the apparatus was fixed in the horizontal plane with an azimuth $A' = 84^o$. For the sake of simplicity (see fig. 52.5), we shall assume $A = 90^o$. In a day the axis of the deviative "coupled mirrors" implement rotated in a plane parallel to the equatorial, and thus the velocity $v'_{eq} = 130 \pm 100$ km/sec, which we measured, was the projection of the absolute velocity v in the equatorial plane. Proceeding from fig. 52.5, we obtain $v_{eq} = v\cos\delta = 251$ km/sec, if we use the figures in (17). On 12 July the maximum effect in the deviative variant must be registered 6 hours before the registration of v_a and v_b, i.e., it must be $t_{dir} = 0^h31^m$, $t_{opp} = 12^h37^m$. We established $t'_{dir} = 3^h \pm 2^h$, $t'_{opp} = 15^h \pm 2^h$ in the period between 25 July and 23 August. Thus the reduction of t_{dir}, t_{opp} to the first days of Augus (the average of our 1973 measurements) should increase the differences between t_{dir}, t_{opp} and t'_{dir}, t'_{opp}. The reduction of $A = 90^o$ to $A' = 84^o$ will, however, diminish these differences and the difference between v_{eq} and v'_{eq}. Nevertheless, despite the perceptible differences between the figures obtained in 1973 and 1975, we are even surprised that our very imperfect deviative "coupled mirrors" experiment led to such relatively good results.

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Prof. Paul Wesley (1981) proposed certain improvements in the interferometric "coupled mirrors" experiment which can make the interference picture more stable, exclude the thermal disturbances, and increase the effect to be measured. We give here these proposed improvements by the courtesy of Prof. Wesley.

In fig. 52.6 one sees a clear diagram of our interfereometric "coupled mirrors" experiment.

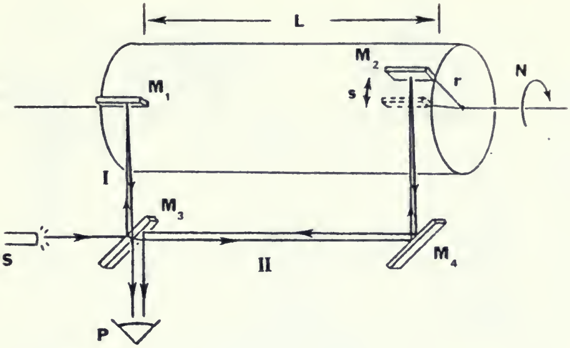

Fig. 52.6

In fig. 52.7 is presented the first improvement which makes the paths of the interfering light beams exactly equal. This will lead to a more stable interference picture and make the effect free of thermal and density air disturbances. A complete identity will be achieved if the places of S^- and P^+ will be interchanged.

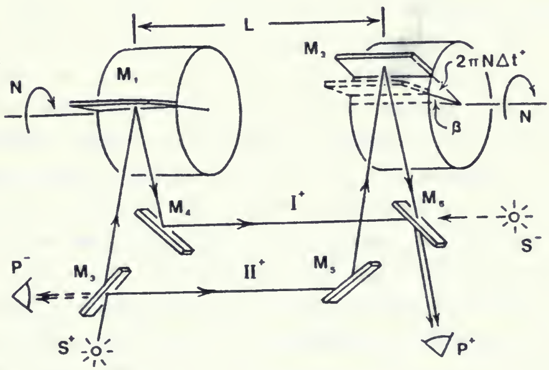

Fig. 52.7

In fig. 52.8 is presented the second improvement. If the light beams make not one but n reflections on the rotating mirrors, the effect will be increased n times. If by this trick and by amelioration of the implement's parameters one can succeed to make Δ equal to λ, then no calibration of the bridge will be needed (see §57B and §59F), and the absolute velocity of the laboratory will be measured with a much higher accuracy. In this case one should search for the rotational velocity N at which the bridge galvanometer will show 4 times the zero current when rotating the implement through 360°. The absolute velocity then is to be calculated from the formula (see (4))

$$v = n\frac{\lambda c^2}{4\pi d R N}. \qquad (52.22)$$

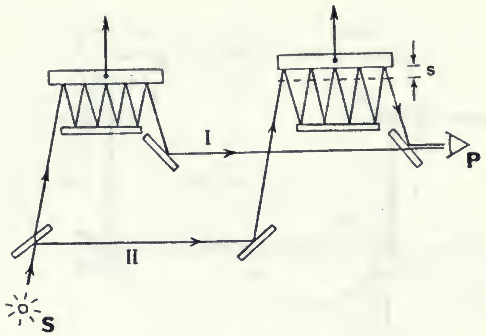

Fig. 52.8

Let us note that when $\Delta < \lambda$ the bridge galvanometer shows 2 times the zero current when rotating the implement through 360^0. On the other hand, when $\Delta > \lambda$ the bridge galvanometer shows 6 times the zero current when rotating the implement through 360^0.

C. Improved version of the interferometric "coupled mirrors" experiment

When being in Washington in 1978 Marinov presented to the National Science Foundation the following IMPROVED VERSION OF THE INTERFEROMETRIC "COUPLED MIRRORS" EXPERIMENT (see fig. 52.9), to which after six months of examination a grant was not awarded:

Light from a single light source (He-Ne laser) is split by the semi-transparent mirror SM into two beams. The one beam reflects on mirrors M_0', M_1' and is then split by semi-transparent mirror SM' into two secondary beams; the other beam reflects on mirror M_1'' and is then split by semi-transparent mirror SM" also into two secondary beams. The beam "reflected" on SM' (or SM") reflects on mirrors M_2', M_3' (M_2'', M_3''), on the rotating mirror RM' (RM"), on mirror M_4' (M_4'') and, reflecting on semi-transparent mirror SM" (SM'), illuminates the photoresistor P" (P'). The beam "transmitted" by SM' (SM") reflects on mirror M_4'' (M_4'), on rotating mirror RM" (RM'), on mirrors M_3'', M_2'' (M_3', M_2') and, passing through semi-transparent mirror SM" (SM'), illuminates the photoresistor P" (P'), where it interferes with the "reflected" beam. The light spots over the photoresistors are small and the illumination over them uniform.

The rotating axle has two disks on its extremities and the rotating mirrors are fixed on their rims. C' and C" are two thin glass plates; changing their inclination, one changes the optical paths of the "reflected" and "transmitted" beams, and so a maximum sensitivity can be adjusted (see §52B). Rotating the micrometer MM, one moves SM'-M_2' and SM"-M_2'' (together with the photoresistors) into mutually opposite directions, so that the optical paths of the "reflected" and "transmitted" beams do not change. All elements beneath the axle lie in a plane perpendicular to the figure. Thus the distances M_3'-RM' and M_3''-RM", as well as the distances M_4'-RM' and M_4''-RM", are equal. The mirrors M_3', M_4', M_3'', M_4'' are small, so that only when the light beams are nearly perpendicular to the rotating

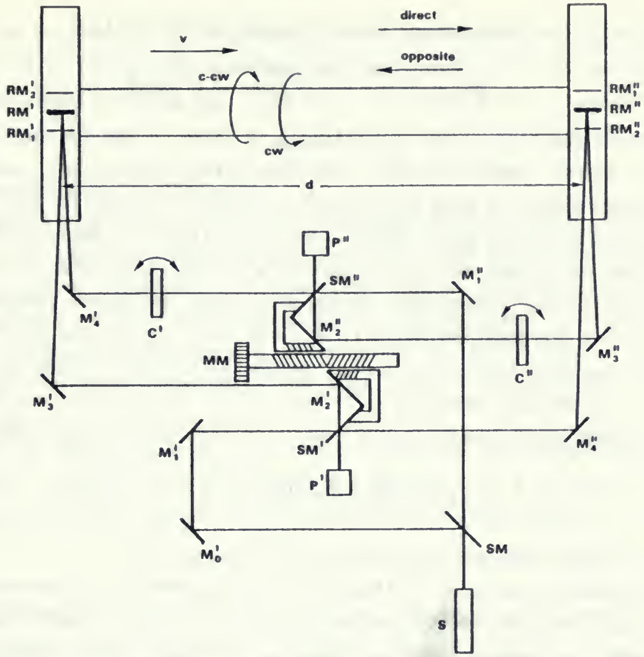

Fig. 52.9

mirrors the photoresistors are illuminated. We shall consider only the case when light goes through M_0', the beam to M_1'' being blocked, supposing that the axle is rotating clockwise (in the figure noted "cw").

Let us denote the light path SM'-M_2'-M_3'-RM' by d' and the light path SM'-M_4''-RM" by d". Suppose first that the axis of the apparatus is perpendicular to the absolute velocity of the laboratory. If d' = d", then at any rotational velocity of the axle the interference will not change, i.e., the illumination over the photoresistors will remain the same. Indeed, two photons separated by SM' at the same moment will reach the first and second rotating mirrors also at the same moment (when they are, say, in the positions RM' and RM") and thus will return to SM" also at the same moment. However, if d' > d" (denote Δd = d' - d"), then two photons separated by SM' at the same moment will reach the rotating mirrors at different moments. Consequently, if at a low rotational velocity, $v_r \cong 0$, the "reflected" photon reaches the first rotating mirror in the position RM' and the "transmitted" photon reaches the second rotating mirror in the position RM", then, at a higher rotational velocity, v_r, the "reflected" photon will reach the first rotating mirror in the position RM_1', while the "transmitted" photon will reach the second rotating mirror in the position RM_1''. If we denote by Δ_r the distances RM'-RM_1' and RM"-RM_1'', the path covered by the "reflected" photon will become shorter by $2\Delta_r$ and the path covered by the "transmitted" photon will become longer by $2\Delta_r$. The time difference with which the "transmitted" photon will arrive later than the "reflected" photon at SM" will be

$$\Delta t_r = 2\,\frac{2\Delta_r}{c} = \frac{4}{c}\,\frac{\Delta d/2}{c}\,v_r = 2\,\frac{v_r}{c^2}\,\Delta d. \qquad (52.22)$$

Suppose now that the axis of the apparatus is parallel to the absolute velocity of the laboratory which points from RM' to RM" (into the "direct" direction). Now the "reflected" photon will proceed along the path M_2'-M_3' with a velocity $c+v$, while the "transmitted" photon will proceed along the path SM'-M_4'' with a velocity $c-v$. Thus, if d' = d", the "reflected" photon will reach the first rotating mirror earlier (when it is, say, in the position RM_2'), while the "transmitted" photon will reach the second rotating mirror later (when it is, say, in the position RM_2''). If we denote by Δ the distances RM'-RM_2' and RM"-RM_2'', the path covered by the "reflected" photon will become longer by 2Δ and the path covered by the "transmitted" photon will become shorter by 2Δ. The time difference with which the "reflected" photon will arrive later than the "transmitted" photon at SM" will be

$$\Delta t = 2\,\frac{2\Delta}{c} = \frac{4}{c}\,\frac{(v/c)(d/2)}{c}\,v_r = 2\,\frac{vv_r}{c^3}\,d, \qquad (52.23)$$

where d is the distance between the beams striking the mirrors.

Thus, at an arbitrary position of the axis of the apparatus in space when the component of the laboratory's absolute velocity along the axis is v, the interference can be maintained stable at <u>any</u> rotational velocity of the axle if we put the micrometer in such a position (such a $\Delta d > 0$), that one should have $\Delta t_r = \Delta t$. From (22) and (23) we then obtain

$$v = (\Delta d/d)c. \qquad (52.24)$$

Letting light pass from SM to M_1'' (and blocking the light to M_0'), the compensation is to be made at $\Delta d < 0$, so that the difference in both positions of the micrometer will exactly give $2\Delta d$. To achieve a better accuracy, the measurement can be performed working simultaneously with both beams proceeding from SM and with both photoresistors. Then we search for such a position of the micrometer when for a certain change of the rotational velocity the illuminations over both photoresistors (and,thus, the currents produced) change with the same amount. It is expedient to change the rate of rotation so that the illumination over the photoresistors changes from minimum to minimum, i.e., the interference changes with one wavelength. One can have equal changes in the currents produced by both photoresistors only when the middle point of the micrometer is at a distance $\Delta d/2$ from the middle point between the beams striking the rotating mirrors. Thus at this method the micrometer with the mirrors SM', M_2', M_2'', SM" and the photoresistors must be removable to the left and to the right, so that the "equilibrium" positions can be found. Obviously, in both methods the <u>gap</u> of the micrometer, or the <u>shift</u> of the whole micrometer with respect to its middle position, can be expressed in km/sec. If d = 1 m, then Δd = 1 mm will correspond to v = 300 km/sec. The <u>effect</u> does <u>not</u> depend on the rotational velocity of the mirrors; only the <u>sensitivity</u> depends on this velocity. If the rotational velocity is

v_r = 3 km/sec, the interference will change with one wavelength $\lambda = \Delta t_r c$ = 633 nm for Δd = 31.6 mm. If Δd is 100 mm, the interference will change with one wavelength for a change in the rotational velocity Δv_r = 949 m/sec. More about the sensitivity see in §52B.

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The measuring method is the following:

The axle is mounted stationary pointing north/south. During a whole day one measures the component of the absolute velocity along the axis of the apparatus. There will be two extreme values v_a and v_b registered with time difference of 12 hours; v_a and v_b must be taken as positive when pointing to the north and we choose always $v_a < v_b$. The method for calculation the magnitude of the absolute velocity and the declination of its apex is described in §52B. The right ascension of the apex is equal to the local sideral time of registration of v_a.

The advantages of the improved variation with respect to the Sofia variation (see §52B) are the following: It is not necessary to put the apparatus on a rotating platform and there is no calibration. The unique quantities which are to be measured are two distances: d and Δd.

D. The interferometric "coupled mirrors" experiment with neutrons

The high sensibility of neutron interferometry allows a repetition of the interferometric "coupled mirrors" experiment using as interfering particles not photons but neutrons, so that the Earth's absolute velocity can be measured with a phantastical accuracy.

In a neutron interferometer the points of separation and meeting of the coherent neutrons cannot be made coinciding, as this can be done with a light interferometer (see p. 170). Leaving aside the problem about the geometrical form of the neutron interferometer, we shall suppose that the point of separation is at the middle of the apparatus (as is the case with the improved version - see §52C), and that the neutrons strike the rotating mirrors perpendicularly. Thus we can make use of formula (4) and for the time difference with which an "opposite" neutron will arrive at the point of interference after a "direct" neutron (see the case shown in fig. 52.9) we obtain

$$\Delta t = 4\pi dRNv/c^3. \qquad (52.25)$$

Substituting this and the first term on the right side of formula (44.23) into the formula (44.26), we obtain for the phase shift

$$\Delta\psi = 8\pi^2 mdRNv/ch. \qquad (52.26)$$

Taking into account that the mass of the neutron is $m = 1.674\times10^{-24}$ g and taking the value of h from IIA, p. 30, we obtain that the phase shift in an experiment with the same parameters as in ours (see p. 191) will be 2π for an absolute velocity of a couple of cm/sec.

The phase shift for photons calculated in the same manner is

$$\Delta\psi = 8\pi^2 m \kappa J R N v/ch, \qquad (52.27)$$

and in our interferometric "coupled mirrors" experiment (see §52B) it was 2π for an absolute velocity of tens of thousands of km/sec.

We think, however, that at the present state of technique the performance of the interferometric "coupled mirrors" experiment is only a dream. Thus, before thinking for neutrons, one has to try to repeat the interferometric "coupled mirrors" experiment with x-ray interferometry where the sensitivity is between that of visible photons and neutrons, and where, we hope, one should be able to obtain a stable interference picture from moving mirrors, as we succeeded to do this with visible photons.

§53. THE ACCELERATED "COUPLED MIRRORS" EXPERIMENT

With the help of the interferometric "coupled mirrors" experiment (see §52B) we established that the absolute velocity of our laboratory was different during the different days of the year. This was due to the fact that our Earth has a steady kinematic acceleration directed to the Sun which leads to a periodic change (with a period of one year) in its absolute velocity. Thus the kinematic character of the laboratory's acceleration can always be revealed, since it leads to a change in the absolute velocity of the laboratory. However, during the whole time our laboratory had a very big dynamic acceleration directed to the Earth's center which did not lead to a change in its absolute velocity.

Thus we can draw the following conclusion: If one measures by accelerometer a certain acceleration of a laboratory, one can immediately give the answer whether this acceleration has a kinematic or dynamic (gravitational) character: in the first case the absolute velocity of the laboratory should change, in the second case it should not. Thus if one formulates the principle of equivalence (see p. 5) in the sense that there is no possibility to make an experimental difference between a kinematic and a dynamic (gravitational) acceleration, then our "coupled mirrors" experiment can be considered as an experimentum crucis rejecting this principle.

Let us note, however, that when the laboratory is in a conservative external field (see IIB, p. 57), as is the case with our Earth in the gravitational field of the Sun, or with an artificial satellite revolving around the Earth, the accelerometer shows null effect and the absolute speedometer shows changes in the laboratory's absolute velocity. Only when the kinematic accleration is due to the action of internal forces (as is the case with a space ship moving under the action of its own thrust), both the accelerometer and the speedometer show effects.

§54. THE EXPERIMENTS WITH SOUND SYNCHRONIZATION

A. The propagation of sound

In the "coupled shutters" and "coupled mirrors" experiments a Newtonian time synchronization is realized by the help of a rotating axle and, as we said (see p. 170), those experimets appertain to the big class of "rotating axle" experiments.

Briscoe (1958) pointed out to another possibility for realization of a Newtonian time synchronization between spatially separated points: the use of sound signals. Assuming that the propagation of sound is isotropic in any inertailly moving medium, Briscoe proposed to measure the laboratory's absolute velocity by comparing the arrival times of parallel propagating sound and light (electromagnetic) signals.

We already said (see p. 146) that the propagation of sound represents a propagation of energy and thus also of mass. However, the proper energy and proper mass of massive and massless particles are attached to absolute space. Is sound energy also attached to absolute space? Are the phonons (the hypothetical particles ascribed to sound waves in a similar manner as massive and massless particles are ascribed to their de Broglie waves, and until now introduced only for mathematical convinience) really existing particles or not? The most definite answer can be given by measuring the Sagnac and Marinov effects with phonons. If the effects will exist, the phonons are to be considered as real particles; if will not, the essence of sound energy will pose terrible difficulties to theoretical physics. Indeed, if the propagation of sound is an isotropic phenomenon, i.e., if sound propagates with the same velocity along any direction in a homogeneous medium, independently of the absolute velocity of the medium, then one can realize (if not practically, at least theoretically) a transfer of energy with respect to absolute space by a velocity higher than that of light. So for certain directions of propagation the proper energy of sound will become equal to infinity, and for other it will become imaginary. Thus the phonons will represent the mysterious "TACHYONS" (hypothetical particles propagating with a velocity higher than c), to which so many scholastic papers have been dedicated in the last years. However, it is difficult to consider energy as an imaginary quantity. Thus, according to our conceptions, the Sagnac and Marinov effects for phonons must be available, and the phonons are to be considered as particles similar to neutrons and electrons.

At the present state of technique, a "rotating disk" experiment with sound cannot lead to a reliable result. Indeed, if at the rim of a rotating disk one separates a sound wave (or sound pulses), the half of which proceeds along the direction of rotation and the other half oppositely, then, on the grounds of formulas (44.22) and (44,19), the "direct" phonons will return to the separation point with the following phase difference after the "opposite" phonons

$$\Delta\psi = 2\pi\nu\Delta t = 4\pi\nu\, dV/c^2 = 8\pi\nu\Omega S/c^2. \qquad (54.1)$$

Using ultrasonic waves with a frequency $\nu = 1$ MHz, supposing that the rotational angular velocity of the disk is $\Omega = 900$ rad/sec, and that the area encircled by the phonons is $S = 1\ m^2$, we obtain $\Delta\psi = 8\pi\times10^{-8}$ rad. This is such a small phase shift which can never be measured. Let us note that the "rotating disk" experiment with visible photons and neutrons leads to reliable results because the frequency of photons and neutrons is much higher than the frequency of phonons.

Thus one must try to measure the Marinov effect with sound because (i) velocity V for the Marinov effect is much higher, and (ii) the path d of the phonons can be made much longer. So, taking in formula (1) the same frequency of ultrasonic signals as above, $V = 300$ km/sec, and $d = 10$ km, one obtains a phase difference $\Delta\psi = 0.4\pi$ rad.

For the measurement of the Marinov effect, however, one must be able to realize a Newtonian time synchronization. As the use of a rotating axle for such a long distance is not possible, one can perform a type of Briscoe's experiment in order to establish whether the velocity of sound is also anisotropic as the velocity of light, the anisotropy of which is already firmly established. Thus if a type of Briscoe's experiment will give a null result, i.e., if with this experiment the Earth's absolute velocity cannot be measured, then the velocity of sound is anisotropic, and both anisotropy effects - of sound and of light - annihilate each other. If however, the result will be positive, this will signify that the velocity of sound is isotropic in any medium moving in absolute space.

B. The ultrasonic "coupled shutters" experiment

We consider Briscoe's original proposal as practically unrealizable, because of technical difficulties. A more reliable variation of Briscoe's experiment was proposed by Yilmaz (1978). Here we shall present the variation proposed by Marinov (1977), which we consider as more relaible and easier for realization, and which is called the ULTRASONIC "COUPLEDSHUTTERS" EXPERIMENT (see fig. 54.1).We assume that sounds propagates isotropically.

Two high-frequency operating shutters Sh_A, Sh_B are set a distance d apart. The shutters lie between light source S_A, S_B (lasers) and observers O_A, O_B, as shown in the figure. A generator G produces electric pulses with a period T (peak to peak time) which: (i) govern the shutter Sh_A, (ii) are applied to the emitter of ultrasonic waves E_A, are applied to the vertical plates of the oscilloscope Osc. The ultrasonic pulses emitted by E_A, which have the same period T, propagate through water at a velocity V (where their wavelength is $\lambda_s = VT$) and are received by the receiver R_B. After being transformed into electric pulses and amplified by the amplifier A_B they: (i) are applied to the emitter of ultrasonic waves E_B, (ii) govern the shutter Sh_B. The ultrasonic pulses emitted by E_B propagate backwards in water with the same velocity V and are received by the receiver R_A. After being amplified by the amplifier A_A, they are aplied to the same vertical plates of the oscilloscope.

We assume that water is homogeneous. Then we assume that the elements of the A-part

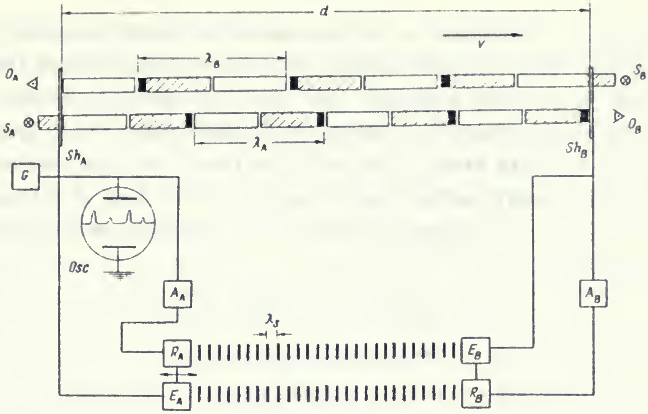

Fig. 54.1

(as well as of the B-part) lie very close to each other, so that the time it takes the electric pulses to cover the lines between shutters and ultrasonic emitter-receiver system can be ignored. In such a case we can affirm that when on the screen of the oscilloscope the emitted pulses (the high ones) coincide with the received pulses (the low ones) there is a whole number of ultrasonic pulses along the track E_A-R_B-E_B-R_A. Moving the emitter-receiver system E_A-R_A back and forth we can change the number of pulses on that track.

Suppose first that the absolute velocity of the laboratory is perpendicular to the axis d. The wavelength of light in both directions will be $\lambda = cT$, and there will be

$$n = d/\lambda = d/cT \tag{54.2}$$

light pulses between the shutters Sh_A and Sh_B. Moving the emitter-receiver system E_A-R_A, we choose such a position that O_A and O_B should see an average light intensity. In such a case a half integer number of sound waves is placed along the track E_A-R_B-E_B-R_A, and the low pulses will be exactly between the high pulses. In the real experiment, where the time lost by the pulses along the electric tracts cannot be ignored, the low pulses will have a certain position with respect to the high pulses.

Suppose now that the absolute velocity v becomes parallel to d, pointing from left to right, which direction will be called "direct". The light wavelength in the "direct" direction will become $\lambda_A = (c - v)T$ and in the "opposite" direction $\lambda_B = (c + v)T$, so that there will be

$$n_A = \frac{d}{\lambda_A} = \frac{d}{(c - v)T}, \qquad n_B = \frac{d}{\lambda_B} = \frac{d}{(c + v)T} \tag{54.3}$$

light pulses between the shutters Sh_A and Sh_B in the "direct" and "opposite" directions, respectively.

If $\Delta n = n_A - n_B$ is less than (or equal to) 1/2, the observer O_B will see the average light intensity plus a Δn-th part of the maximum light intensity, while the observer O_A will see the average light intensity minus a Δn-th part of the maximum light intensity. Thus for $\Delta n = 1/2$, O_B will see a maximum light intensity and O_B no light. If $\Delta n < 1/2$, one can reduce the time during which the shutters remain open to $\Delta n\, T$ (see the small segments shaded in black in the figure), so that O_A will see no light, while O_B will see a $(2\,\Delta n)$-th part of the normal maximum light intensity. (N.B. Such will be the case if $d/\lambda = m + 3/4$, where m is an integer; if $d/\lambda = m + 1/4$, all will be vice versa).

From (3) we obtain

$$n_A = n_B \frac{c + v}{c - v} \cong n_B + 2\,\frac{v}{c} n = n_B + \Delta n, \tag{54.4}$$

and making use of (2) we obtain for the absolute velocity

$$v = \frac{c\,\Delta n}{2n} = \frac{c^2 T\,\Delta n}{2n}. \tag{54.5}$$

Taking $f = 1/T = 1$ MHz, $d = 15$ km, we obtain, for $v = 300$ km/sec, $\Delta n = 0.1$.

In an actual experiment the water will not be homogeneous. Nevertheless, this is of no importance because the "direct" and "opposite" sound waves cross exactly the same way, and the number of sound waves in the "direct" and "opposite" directions will be the same. However, as a result of different influences (temperature, density, currents, etc.) the water can change its sound conductive properties. This signifies that during different moments different numbers of sound waves will be placed along the tract E_A-R_B-E_B-R_A, and the low pulses will "creep" with respect to the high pulses. Thus throughout the experiment, a corresponding shift of the emitter-receiver system E_A-R_A is to be performed and the low pulses are to be maintained at their initial positions. If the creep of the low pulses is conspicuous, the experiment should be performed in winter when the water is covered by ice and preserves its sound conductive properties for long enough. This experiment will be successful if the low pulses can be maintained at their initial positions a whole day with an accuracy much less than one Δn-th part of the period T. For convenience and higher accuracy the compensation of the creep is to be made not in the ultrasonic but in the electric tract.

If one can realize a stable multiplication of the frequency f and govern the shutters Sh_A, Sh_B by this enhanced frequency, then the absolute velocity v can be measured by changing the multiplication factor and by using a method similar to that used (and explained in detail) in §62, thus not waiting for the Earth's rotation.

C. Improved version of the ultrasonic "coupled shutters" experiment

The IMPROVED VERSION OF THE ULTRASONIC "COUPLED SHUTTERS" EXPERIMENT, where the highly effective method of the differential "coupled shutters" experiment (see §50D) is used, is to be performed in a laboratory with a basis, d, of a couple of meters, so that the apparatus can be mounted on a rotating paltform. One can explain the improved version, making use of fig. 54.1. As observers transducers of light into electric current, say, photodiodes will be used. The difference in the currents produced by the two photodiodes will be detected by a sensitive galvanometer (see §50D). Both light beams will be produced by a single laser sourse as to avoid differential intensity fluctuations. The material conducting the sound signals must be chosen to have a good thermal stability (if necessary a thermal stabilization is to be made), so as to exclude the creep of the low pulses during the short time of measurement when rotating the platform.

The paths of the "direct" and "opposite" light pulses must be made changeable, say, as shown in fig. 50.1, or by moving separately to and fro the "chopping" and "detecting" part of the shutters (it is sufficient to change the path only of the "direct" or only of the "opposite" light pulses but with the double amount).

The measuring procedure is as follows: During a whole day one eliminates the appearing difference current by changing the path of the "direct" or "opposite" light pulses. The best sensitivity is obtained for $n_s = d/\lambda_s$ equal to m + 1/4 or to m + 3/4, where m is an integer; for n_s equal to m or to m + 1/2 the sensitivity falls to zero. If the component of the absolute velocity of the laboratory along the axis of the apparatus changes from 0 to v, one has to change the paths of the "direct" and "opposite" light pulses by Δd, so that relation (52.24) will be valid.(The sensitivity is considered for $n = d/\lambda = m', m'+1/2$.)

The sensitivity of the method can be estimated very simply. If δd is the minimum shift which leads to an effect discernable from the fluctuations of the galvanometer, the accuracy with which the absolute velocity can be measured is

$$\delta v = (\delta d/d)c. \qquad (54.6)$$

In our differential "coupled shutters" experiment from §50D the rotating disk had 30 holes and when it rotated with a constant rate of N = 200 rev/sec, i.e., when the chopping frequency was f = 6 KHz, the current fluctuation δI (for the current fluctuation we use here the same symbol as in §50D was used for the current change caused by the absolute velocity) was about a 10^{-3} part of the current ΔI which was about a 10^{-4} part of the current I produced by the photodiodes. Thus the fluctuation represented about a $\delta I/I = 10^{-7}$ part of the total current.

Let us now consider formula (5) which says that if a δn part of the light pulses (in more in the "direct" pulses and in less in the "opposite" pulses) can be discerned from the fluctuations, than the absolute velocity can be measured with an accuracy

$$\delta v = c^2 \delta n/2fd. \qquad (54.7)$$

For the differential "coupled shutters" experiment (§50D) the inaccuray according to this formula, where we put $\delta n = \delta I/I$, is δv = 500 km/sec.

For the ultrasonic "coupled shutters" experiment, putting in formula (7) $\delta n = 10^{-7}$ and taking f = 1 MHz, we obtain, for the same basis d = 150 cm, δv = 3 km/sec.

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The description of the ultrasonic "coupled shutters" experiments in §54B and §54C was performed at the assumption that the propagation of sound is isotropic in any inertially moving medium. However, as we state in §54A, this cannot be true: the velocity of sound must be direction dependent exactly in the same manner as the velocity of massive and massless particles is direction dependent. Thus the effect in the ultrasonic "coupled shutters" experiment must be null.

§55. THE KINEMATIC TIME DILATION EXPERIMENTS

A. The Rossi-Hall "meson" experiment

The first observations which verified the kinematic time dilation were made by Rossi and Hall (1941) in the so-called "MESON" EXPERIMENT. To explain its essence, we have to introduce certain brief notions from radioactivity.

According to the law of radioactive decay (see a corresponding book on atomic physics), if there is a large enough number N_o of radioactive atoms, then after a definite time a certain strictly defined part of them (N atoms) will decay. If we take $N = N_o/e$ ($e = 2.71$), then the corresponding time is called the MEAN LIFETIME (or MEAN-LIFE). When the radioactive atoms are at rest with respect to absolute space, their mean-life is called the universal mean-life and is denoted by T. If the atoms move with a velocity v in absolute space, their mean-life (called proper mean-life), because of the time dilation, becomes (see (3.25))

$$T_o = \frac{T}{(1 - v^2/c^2)^{1/2}}, \tag{55.1}$$

where T and T_o are measured in the same time units.

Indeed, the mean-life (as the period of a light clock) can be considered as a unit of time. For this reason the same relation which exists between the periods of two identical light clocks moving with different velocities in absolute space exists also between the mean-lives of two identical amounts of radioactive atoms moving with the same different velocities in absolute space.

The same law is valid also for the decay of μ-mesons (elementary particles with masses between the electron and the proton). Since their universal mean-life (measured in an Earth laboratory) is $T = 2.2\ 10^{-6}$ sec, and μ-mesons with velocities near to v can be observed, then measuring the distance d covered by mesons until (1/e)th part of them will decay, and knowing v, one can calculate the proper mean life for the velocity v from the relation

$$T_o = d/v. \tag{55.2}$$

The proper mean-life T_o measured in this manner is to be compared with the mean-life T measured previously, and relation (1) is to be proved right.

The "meson" experiment was carried out in the following way: The rate of μ-mesons in the cosmic rays (produced near the top of the atmosphere as a result of nuclear interactions caused by the primary cosmic radiation) having velocities near to c was measured on the top of a mountain and on the sea level. The change in the rate showed that formula (1) is valid.

Thus the "meson clock" whose period is the mean-life of μ-mesons was the first natural clock which has shown that the time dilation is a real physical phenomenon.

A similar experiment with positive and negative mesons moving along a circular orbit in a storage ring of a circular accelerator was recently performed by Bailey et al. (1977), and relation (1) was verified very reliably.

B. Relativistic and absolute treatment of the "meson" experiment

The absolute character of the time dilation cannot be established with the help of the "meson" experiment.

Now we shall propose a thought experiment representing a modification of the "meson" experiment, where the different treatments of the time dilation in the framework of the theory of relativity and of our absolute space-time theory can clearly by presented.

Let us have (see fig. 55.1) two identical space ships passing clear alongside of each other with relative velocity v somewhere in the cosmos. Let us suppose that in any of these ships mesons are produced at a given moment, and their mean-life is measured, using as a clock the other space ship which is proceeding with a constant velocity along the windows of the first one. For this purpose along the boards of both ships equidistant strips are painted; counting the number of the passing strips, one can count the number of equal intervals of time.

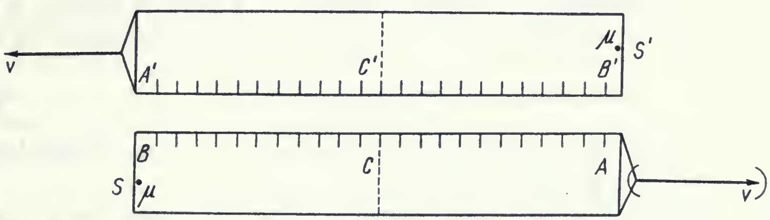

Fig. 55.1

Let us suppose that ship S is at rest with respect to absolute space and ship S' moving with velocity v. We assume that the mesons in S' are produced when points B' and A (the stern of ship S' and the prow of ship S) are in front of each other, and the mesons in S are produced when points A' and B (the prow of ship S' and the stern of ship S) are in front of each other. Further we assume for simplicity's sake that all mesons live the same time, i.e., that the lifetime of all mesons are equal. If

$$v = \sqrt{3}c/2, \tag{55.3}$$

and the lifetime of the mesons in S is T, say, the time in which point C' (the midship of S') will come in front of point B, then, according to formula (1), the lifetime of the mesons in S' will be

$$T' = \frac{T}{(1 - v^2/c^2)^{1/2}}. \tag{55.4}$$

Thus, taking into account (3), we have to conclude that the same mesons will live in S' two times longer, i.e., T' = 2T, and if the mesons in S' are produced when point A is in front of point B', then they must decay when point B will be in front of point B'.

Hence, according to our absolute space-time theory, the pictures in both ships will be different, and this ship, from which a smaller part of the other ship (a smaller number of strips) would be seen to pass during the time in which the mesons should decay, will be at rest with respect to absolute space (or will move at lower speed with respect to absolute space). If the crews can communicate by radio, they can easily establish which ship is in motion and which at rest (or which ship moves with a lower velocity in absolute space). For this purpose they have only to compare the numbers of equidistant strips which any of the crews counts for the time of decay of the mesons carried in their own ship.

Special relativity denies the existence of absolute space and asserts that the pictures in both ships must be the same, i.e., both crews have to count exactly equal numbers of equidistant strips for the time of decay of the mesons. Hence, according to special relativity, if the mesons are produced when points A and B' (respectively, A' and B) are in front of each other, they must decay when, say, points B and B' will be in front of each other. Special relativity can explain this result (preserving the time dilation dogma) only with the help of the famous Lorentz length contraction. Indeed, if ship S' is considered in motion with respect to S, then its length L' = A'B', when measured in the inertial frame K of ship S, according to the formula for the length contraction (6.42), will be

$$L' = L(1 - v^2/c^2)^{1/2}, \tag{55.5}$$

where L = AB is the length of ship S measured in frame K (i.e., L is the "proper length" of both ships). Taking into account (4), we get L' = L/2.

Hence, ship S', when moving, must become shorter (see fig. 55.2), and now it is easily seen that when the mesons are produced in S' (points A and B' in front of each other), then the prow of ship S' (point A') will be in front of the midship of S (point C). With the help of this speculation one can conclude that, indeed, the mesons is ship S' will live two times longer than the mesons in ship S, since the moment when A and B' are in front of each other will not coincide with the moment when points A' and B will be in front of each other.

However both ships are entirely identical in their "inertial rights". Thus we can consider ship S in motion and ship S' at rest; now all must be conversely, i.e., ship S will

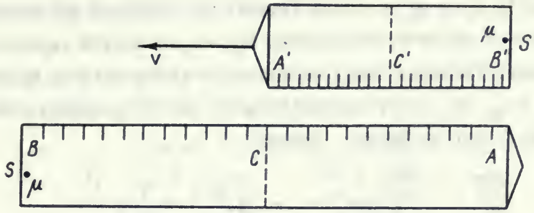

Fig. 55.2

become two times shorter than ship S', and the mesons in S will live two times longer than in S'. We shall not pose to specail relativity the provoking question, which of both ships is really the contracted one. We shall only mention that if any of two men is firmly persuaded that the other one is the more unintelligent, and that both these assertions may be true together, we have to conclude that the intelligence of these men is on the same (relatively low) level.

C. The Hafele-Keating "clocks-round-the-world" experiment

The first experiment where the Lorentz time dilation was verified using macroscopic clocks was the "CLOCKS-ROUND-THE-WORLD" EXPERIMENT of Hafele and Keating (1972). It consists of the following:

Two jet planes carrying atomic clocks left Washington in eastern and western directions, flew round the world and returned to the starting point. The readings Δt_E, Δt_W of the clocks carried by the eastern and western planes were compared with the reading Δt of a third atomic clock left in Washington.

To analyse this experiment it may be assumed that both planes flew exactly along the parallel of Washington at the same height above sea level, at which height a stationary clock in Washington was also placed. The corrections which are to be made when the planes fly at different heights can be performed by taking into account the gravitational time dilation (see IV, §65). In the actual experiment the planes made many landings during the trip, thus changing their gravitational potentials, and the Hafele-Keating experiment proved <u>both</u> the kinematic and dynamic (gravitational) time dilations.

The motion of the Earth around the Sun and their combined motion relative to absolute space may be ignored because the <u>average</u> influence of all these motions on the clocks was the <u>same</u>. Only the rotational velocity of the Earth v_r need be considered, as it leads to an asymmetry in the motion of the different clocks. We have

$$v_r = \frac{2\pi}{T} R \cos\phi, \qquad (55.6)$$

where R is the Earth's radius, ϕ is the latitude of Washington's parallel, and T is the length of the sideral day.

If v is the velocity of the planes with respect to the Earth, then the east-bound clock which moves with velocity $v_E = v_r + v$ with respect to absolute space will be slow, while the west-bound clock which moves with velocity $v_W = v_r - v$ with respect to absolute space will be fast. The time differences compared with the clock left in Washington $\delta t_E = \Delta t_E - \Delta t$, $\delta t_W = \Delta t_W - \Delta t$, according to formula (8.7), to within an accuracy of second order in "velocity/light velocity", become

$$\frac{\delta t_E}{\Delta t} = - \frac{2vv_r + v^2}{c^2}, \qquad \frac{\delta t_W}{\Delta t} = \frac{2vv_r - v^2}{c^2}. \tag{55.7}$$

If we choose $\phi = 43^o$ ($\cos 43^o = 0.730$) - that is the latitude of Washington - and if we take T = 86,200 sec, R = 6370 km, we obtain v_r = 340 m/sec. Of the same order is the commercial speed of the jet planes, so that we can suppose $v = v_r$ = 300 m/sec. Hence, under such simplified conditions, it is

$$\frac{\delta t_E}{\Delta t} = - \frac{3}{2}\frac{v^2}{c^2} = - 15\times10^{-13}, \quad \frac{\delta t_W}{\Delta t} = \frac{1}{2}\frac{v^2}{c^2} = 5\times10^{-13}. \tag{55.8}$$

Hafele and Keating's planes did not fly strictly along the parallel of Washington, nor at the same height and they made many landings, the number of which was different for each plane. After taking into account the actual routes of the planes and the influence of the different gravitational potentials at different heights of the flights Hafele and Keating calculated $\delta t_E = - 59 \pm 10$ ns, $\delta t_W = 273 \pm 7$ ns, where the theoretical incertitudes come from the inprecise knowledge of the routes. The experimentally measured differecnes were $\delta t_E = - 40 \pm 23$ ns, $\delta t_W = 275 \pm 21$ ns.

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Recently Briatore and Leschiutta (1976) claimed to have experimentally verified the kinematic and dynamic time dilations, by comparing for a couple of years the readings of atomic clocks placed at different points on the Earth which have different absolute velocities (because of the different latitudes) and different gravitational potentials (because of the different heights above sea level of the stations and not exactly spherical form of the Earth).

D. The "antipodal clocks" experiment

The "ANTIPODAL CLOCKS" EXPERIMENT, with whose help one can measure the absolute velocity of the laboratory, was proposed in 1972 by Marinov and considered in more detail in 1977. Its essence is the following (see fig. 55.3, where we have shown the Earth as seen from the north celestial pole):

Two atomic clocks are placed at two antipodal points on the Earth's equator. Let the Earth's absolute velocity be v and the linear velocity of daily rotation of the equator v_r. Suppose for the sake of simplicity that the Earth's axis is perpendicular to the plane of the ecliptic and the day of the year is such that the absolute velocity of the Sun (or at least its component in the plane of the ecliptic) v_S is parallel to the velocity of the Earth around the Sun v_E. Taking the initial zero time when it is sunrise for the first atomic clock (clock A) and sunset for the second atomic clock (clock B), the absolute velocities of these two clocks will be, respectively,

$$v_A^2 = v^2 + v_r^2 - 2\,v\,v_r \sin\frac{2\pi t}{T},$$

$$v_B^2 = v^2 + v_r^2 + 2\,v\,v_r \sin\frac{2\pi t}{T}, \qquad (55.9)$$

where $v = v_S + v_E$, and T is the length of the sideral day.

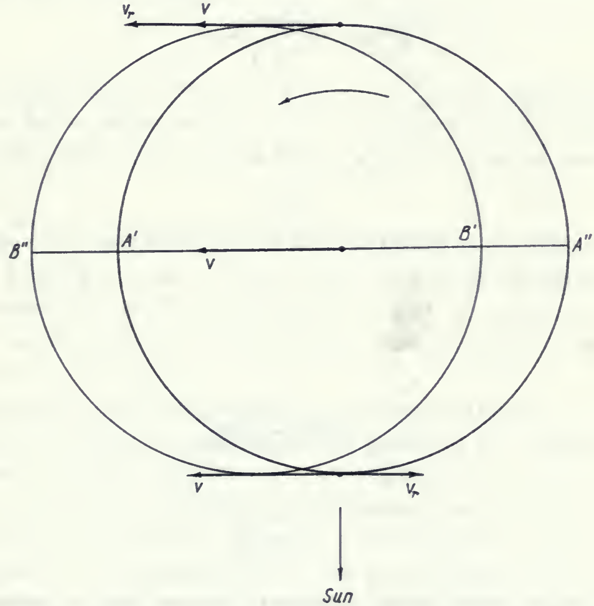

Fig. 55.3

Let us write formula (8.7) with our present notations in the form

$$\frac{\Delta t_A}{(1 - v_A^2/c^2)^{1/2}} = \frac{\Delta t_B}{(1 - v_B^2/c^2)^{1/2}} = \Delta t, \qquad (55.10)$$

where Δt_A and Δt_B are the readings of the A- and B-clock which correspond to the reading Δt of a universal clock and all readings are taken in the same time units.

If we substitute formulas (9) into (10), we see that the rates of the clocks will be different at different hours of the day. The greatest difference will be when it is noon for one clock and midnight for the other. Only during sunset and sunrise hours will the clock rates be equal. By comparing the readings of the clocks for equal universal time intervals between sunset and sunrise, we should be able to establish the absolute character of the kinematic time dilation and we should be able to measure the Earth's absolute velocity.

However, a realization of a Newtonian time synchronization between two antipodal points on the Earth is problematic. On the other hand, a time synchronization by exchanging electromagnetic signals, i.e., an Einsteinian time synchronization, leads to an annihilation of the appearing absolute effects, as described below. To get around the synchronization problem, the "antipodal clocks" experiment can be performed on a turnabaout, as shown in fig. 55.4.

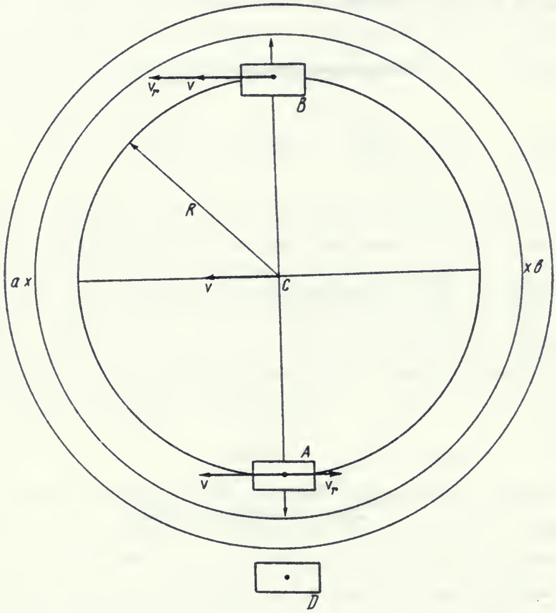

Fig. 55.4

A stationary ring ab encircles the turnabout. Suppose that the points a and b lie on a diameter which is parallel to the absolute velocity v. Let the readings t'_A, t''_A be registered on clock A when it touches, respectively, points a, b, and let the readings t'_B, t''_B be registered on clock B when it touches, respectively, points b, a.

The times $\Delta t_A = t''_A - t'_A$ and $\Delta t_B = t''_B - t'_B$ obviously correspond to the same absolute time interval Δt. It is

$$\Delta t = \Delta t_D(1 - v^2/c^2)^{-1/2}, \qquad (55.11)$$

where Δt_D is the time read on a clock D which does not rotate.

Using formula (10), we can write

$$\Delta t = \int_0^{\Delta t_A} (1 - v_A^2/c^2)^{-1/2} dt, \qquad \Delta t = \int_0^{\Delta t_B} (1 - v_B^2/c^2)^{-1/2} dt. \qquad (55.12)$$

Substituting here v_A, v_B from (9) and working within an accuracy of second order in "velocity/c", we obtain

$$\Delta t_A\left(1 + \frac{1}{2}\frac{v^2 + v_r^2}{c^2} - \frac{2}{\pi}\frac{vv_r}{c^2}\right) = \Delta t_B\left(1 + \frac{1}{2}\frac{v^2 + v_r^2}{c^2} + \frac{2}{\pi}\frac{vv_r}{c^2}\right), \quad (55.13)$$

having taken into account that

$$\Delta t_A \cong \Delta t_B \cong \Delta t = T/2, \quad (55.14)$$

where T is the period of rotation.

Denoting $\delta t = \Delta t_A - \Delta t_B$ and taking into account (14) only in the terms of second order in "velocity/c", we obtain from (13)

$$\delta t = \frac{4}{\pi}\frac{vv_r}{c^2}\Delta t = 4\frac{vR}{c^2}. \quad (55.15)$$

Taking v = 300 km/sec and R = 3 m, we find $\delta t = 4\times10^{-11}$ sec. Supposing that the revolutions per second of the turnabout are 5 (i.e., Δt = 0.1 sec), we shall have $\delta t/\Delta t = 4\times10^{-10}$, while portable cesium beam clocks can show time to within a relative error $\pm\ 10^{-13}$.

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Now we shall show that the realization of an Einsteinian time synchronization in the "antipodal clocks" experiment leads to a null result because of the mutual annihilation of the absolute effects, and thus the absolute velocity of the laboratory cannot be measured in this way. We shall consider the "antipodal clocks" experiment on the rotating Earth (see fig. 55.3).

Let clock A send the initial electromagnetic signal being at point A' (i.e., when it is sunrise for this clock) and let its reading at this moment by t'_A. Since the light signal takes a certain time to travel, clock B will move with the Earth and will receive the signal at point B'. Here we are concerned with the translational motion of the Earth only. The rotation of the Earth is neglected because for the particular case considered the linear rotational velocity v_r is perpendicular to the translational velocity v, and, as it can be shown, the effects, calculated if taking into account the velocity v_r, cancel out in the final result.

For the sake of simplicity it is assumed that the electromagnetic signal propagates along the Earth's diameter of length d. It can be shown that if the signal covers a trajectory over the Earth's surface (a radio-relay tract), then the additional effects will cancel out in the final result.

With respect to absolute space the initial signal covers the distance

$$d' = d(1 - v/c), \quad (55.16)$$

which we calculate to within an accuracy of first order in v/c because, as we shall see further (formula (19)), a higher accuracy is not necessary.

Clock A sends a final signal at point A" (i.e., when it is sunset for this clock), and let its reading at this moment be t''_A. Clock B will move with the Earth and will receive the signal at point B". With respect to absolute space the final signal covers the distance

$$d'' = d(1 + v/c). \qquad (55.17)$$

Let t'_B be the reading of clock B when clock A has sent the initial signal and t''_B the reading when clock A has sent the final signal. Making use of formulas (12), (13), and (14) yields

$$t''_A - t'_A = (t''_B - t'_B)(1 + \frac{4}{\pi}\frac{vv_r}{c^2}) = t''_B - t'_B + 2\,\frac{dv}{c^2}. \qquad (55.18)$$

If T'_B and T''_B are the readings of clock B when the initial and final signals, respectively, have been received, then, using (16) and (17) yields

$$t'_B = T'_B - \frac{d}{c}(1 - \frac{v}{c}), \qquad t''_B = T''_B - \frac{d}{c}(1 + \frac{v}{c}). \qquad (55.19)$$

Putting (19) into (18) yields

$$t''_A - t'_A = T''_B - T'_B, \qquad (55.20)$$

and, obviously, from this relation between the readings of clock A, when the initial and final signals are sent, and the readings of clock B, when these signals are received, we cannot establish the absolute velocity v.

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Now we shall propose a variant of the "antipodal clocks" experiment with whose help the equatorial components of the Earth's absolute velocity can be measured.

Consider two telescopes at a point on the Earth's equator whose fixed axes lie in a vertical plane parallel to the east-west direction, the angle between them being θ. Suppose that an equatorial star A, which lies about 90^o from the projection on the celestial equator of the apex of the Earth's absolute velocity, crosses the line of sight of the first telescope at the moment t'_A, and the line of sight of the second telescope at the moment t''_A. Let us further suppose that a star B which is antipodal to A crosses the line of sight of the first telescope at the moment t'_B and the line of sight of the second telescope at the moment t''_B.

Since the Earth rotates uniformly, the times $\Delta t_A = t''_A - t'_A$ and $\Delta t_B = t''_B - t'_B$, obviously, correspond to the same absolute time interval Δt, and we can use formulas (12), where v_A and v_B are given by formulas (9). Thus assuming for simplicity $\theta = \pi$, introducing the notation $\delta t = \Delta t_A - \Delta t_B$, and performing an analysis as above, we shall obtain the result (15). Taking v = 300 km/sec, R = 6370 km gives $\delta t = 8.5\times10^{-5}$ sec.

If a similar experiment be performed throughout a year, using as a "rotating disk" not the diurnal rotation of the Earth about its axis but the yearly revolution around the Sun, then we have to take for v in formula (15) the component of the Sun's absolute velocity in the plane of the ecliptic and for R the radius of the Earth's orbit. Taking v = 300 km/sec and R = 150×10^5 km gives δt = 2 sec. It is very instructive to compare the analysis of this experiment with the analysis of the quasi-Römer experiment (see §45).

E. The time dilation of a spring clock

We shall show that the most simple spring clock also slows its rate when set in motion with respect to absolute space according to the time dilation formula.

The equation of motion of a spring clock is given by (see (IIB,42.10))

$$m\,d^2x/dt^2 + kx = 0, \tag{55.21}$$

where x is the displacement of the oscillating part of the clock from its state of equilibrium, m is the mass of the oscillating part, and k is a constant which depends on the elasticity of the spring (k/2 is the energy which must be applied to make the displacement x equal to one length unit - see formula (IIB,42.6)).

Time t is measured on a clock attached to the spring clock. Thus, if the spring clock is at rest in absolute space, its universal period will be (see formulas (IIB,42.12) ahd (IIB,42.23))

$$T = 2\pi(m/k)^{1/2}. \tag{55.22}$$

If the spring clock moves with velocity v in absolute space, the time in (21) is to be measured on a proper clock, i.e., we have to write in (21) dt_0 instead of dt. Expressing the proper time in absolute time, we obtain

$$m\,d^2x/dt^2(1 - v^2/c^2) + kx = 0. \tag{55.23}$$

Thus the proper period of the spring clock, if expressed in absolute time, will be

$$T_0 = 2\pi\left(\frac{m/k}{1 - v^2/c^2}\right)^{1/2} = \frac{T}{(1 - v^2/c^2)^{1/2}}. \tag{55.24}$$

§56. THE "SYNCHRONOUS LIGHT CLOCKS" EXPERIMENT

As we showed in §50A and §55D, by the help of light signals one cannot realize a Newtonian time synchronization, as such a synchronization, inevitably, becomes always Einsteinian. In §§50, 51, 52 we showed that a Newtonian time synchronization can be realized only by a rotating axle. As a matter of fact, the cog-wheels (or the disks of the rotating mirrors) attached to the extremities of a rotating axle represent two clocks. However, these clocks are not independent, as the whole rotating axle represents a single clock. Thus in all "rotating axle" experiments there is a unique clock with a large spatial extention (see p. 174).

Now we shall show that with the help of two light clocks, with a space extension equal to the distance between them, one can realize a Newtonian time synchronization. This represents the "SYNCHRONOUS LIGHT CLOCKS" EXPERIMENT.

Consider two light clocks such that the points at which one counts the period of any of the clocks serves as a point of reflection for the light pulses of the other clock. Thus the "arms" of the light clocks are equal and their periods are equal too. If we rotate these two clocks about an axis perpendicular to their mutual "arm", their periods will remain equal, because the velocities of two clocks with the same spatial extension are equal. Thus sending light signals from the first to the second clock, and vice versa, and rotating the apparatus over 360^0, we shall be able to measure the absolute velocity of the laboratory. Let us note that if the "arms" of the light clocks were much shorter than the distance between them, then during the rotation their time dilation would be different. As we showed (see p. 173), in such a case their periods will change so that the absolute effect which would be registered in the case of synchronously going clocks will be annihilated. However, when their "arms" are equal, i.e., when their spatial extension is the same, their velocities remain equal for any motion of the clocks. If the clocks rotate about a point at the middle, the absolute velocity of the clocks does not change during the rotation. If the clocks rotate about another point, their velocities are equal to the velocity of the middle point, and the changes in their rates due to the absolute time dilation will be equal. Thus the "phase difference" between the readings of both clocks will not change and the clocks will remain synchronous. With the help of such synchronous clocks the one-way light velocity can be measured.

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The scheme of the "synchronous light clocks" experiment is as follows (see fig. 56.1):

At the opposite ends, A and B, of an "arm" of length d there are two high-frequency shutters (say, Kerr cells). We shall consider the situation at point A. The situation at point B is exactly the same. By applying a "starting" electric pulse of duration $T_0 < d/c$, one opens the shutter Sh_A, and a light pulse, emitted by the laser L_A, goes to the opposite end B where there is a semi-transparent mirror SM_B and a photo-receiver (a rapid photodiode) P_{B1} behind it. The "reflected" light pulse returns back to point A where it illuminates another photo-receiver P_{A2}; the produced electric pulse of the same duration T_0 is applied to an oscilloscope Osc_A and to the shutter Sh_A which shoots a new light pulse that performs exactly the same trip and action. The "transmitted" light pulse illuminates the photo-receiver P_{B1} behind the semi-transparent mirror at B; the produced electric pulse is applied to an oscilloscope Osc_B at point B. As the situation at point B is exactly the same as at point A, on the screen of the oscilloscope at A (as well as on that at B) one will see two pulses: one produced by the "to-and-fro" going light pulse, originating from the A-laser and one produced by the "to" going light pulse originating from the B-laser. By shifting the semi-transparent mirror one changes the period $T = 2d/c$ of the light clock. By changing for a while the path of the electric pulse from the photo-receiver to the shutter one changes the time t_ψ between the openings of the shutters which we shall call "phase difference". Let us suppose that the periods of the clocks are exactly equal and that $t_\psi = T/2$. If the axis of the apparatus is perpendicular to its absolute

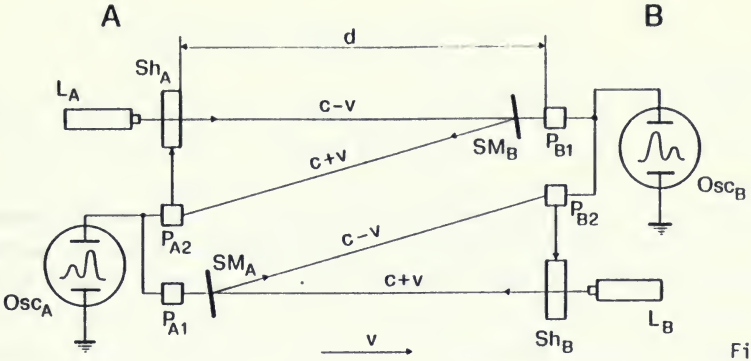

Fig. 56.1

velocity, the two pulses on the screens of both oscilloscopes will coincide. When the axis becomes parallel to the absolute velocity v, the "to" pulse on the A-oscilloscope will become shifted to a "distance" $\Delta t_A = dv/c^2$ in advance with respect to the "to-and-fro" pulse, while the "to" pulse on the B-oscilloscope will become shifted to the same distance $\Delta t_B = dv/c^2$ in retard with respect to the "to-and-fro" pulse. These shifts appear because of the Marinov effect (see formula (52.1)). When the axis AB becomes anti-parallel to v everything will be vice versa.

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Let us compare the "synchronous light clocks" experiment with our "rotating axle" experiments.

Advantages. 1) There is no mechanical part. 2) The axis of the apparatus can be made long enough; so, at d = 1.5 km, one has $T = 10^{-5}$ sec and for v = 300 km/sec the Marinov effect is Δt = 10 ns. Such a shift can reliably be registered on an oscilloscope.

Disadvantages. Since both light clocks are independent, it is a difficult problem to make their periods equal and maintain them equal during a whole day in which the measurement is to be made. Let us note that a difference Δd in the "arm" of the light clock causes a change in its period $\Delta T = 2\Delta d/c$ and a "phase shift" in a second $\Delta t_\psi = \Delta T/T = \Delta d/d$. Thus to have $\Delta t_\psi \leq \Delta t$, the "arms" of the light clocks must be equal to within $\Delta d = vd^2/c^2$ (= 3×10^{-3} cm, for v = 300 km/sec). To have $\Delta t_\psi \leq \Delta t$ in a day, Δd must be with 5 orders lower. It is clear that the maintenance of equal or nearly equal periods is a very difficult technical problem. The period will also be influenced by thermal and other disturbances in the electrical tracts of the pulses. This is the most important disadvantage of the "synchronous light clocks" experiment.

§57. THE "DRAG-OF-LIGHT" EXPERIMENTS

A. The first-order in v/c effects

The drag of light by a moving medium (see §40) was observed for the first time by Fizeau (1951) in the so-called "WATER TUBE" EXPERIMENT. In this experiment water is allowed to flow with a velocity v down a tube of length L. A light beam is split so that one beam travels through the tube in the direction of the water flow (the "direct" beam), while the other beam travels counter the water flow (the "opposite" beam). The two beams are then allowed to interfere and the inteference pattern is noted as a function of the velocity of water.

According to the theory presented in §40, a pulse of light traveling in the "direct" beam will arrive before a pulse traveling in the "opposite" beam by a time interval

$$\Delta t = \frac{L}{c_m^-} - \frac{L}{c_m^+} = \frac{2Lv}{c^2}(n^2 - 1), \tag{57.1}$$

where formula (40.16) has been used, for $\theta = 0, \pi$, to within first order in v/c.

Here n is the refractive index of water for the frequency ν of the monochromatic light used. However, since the molecules of the liquid move with respect to the light source, a Doppler effect occurs, and the water molecules receive photons with a frequency

$$\nu_o = \nu(1 \pm v/c), \tag{57.2}$$

where the sign "+" is for the "opposite" photons and the sign "-" is for the "direct" photons.

A Taylor expansion of the refractive index as a function of ν yields

$$n(\nu_o) = n(\nu) \pm \frac{dn}{d\nu}d\nu = n \pm \frac{v}{c}\nu\frac{dn}{d\nu}. \tag{57.3}$$

The change of the refractive index because of a change in the frequency of light is called DISPERSION. Thus putting (3) into (40.16), for $\theta = 0, \pi$, we obtain for the velocity of the photons in flowing water, taking into account the dispersion,

$$c_m = \frac{c}{n} \pm v(1 + \frac{\nu}{n^2}\frac{dn}{d\nu} - \frac{1}{n^2}). \tag{57.4}$$

Including the dispersion, the time delay (1) becomes

$$\Delta t = \frac{2Lv}{c^2}(n^2 - \lambda\frac{dn}{d\lambda} - 1), \tag{57.5}$$

where λ is the wavelength of the emitted light.

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Lorentz (1909) and Einstein (1914) give for the time delay in the "water tube" experiment the formula

$$\Delta t = \frac{2Lv}{c^2}(n^2 - n\lambda\frac{dn}{d\lambda} - 1). \qquad (57.6)$$

These authors derived formula (6) by proceeding not from relation (2), but from the relation

$$\nu_o = \nu(1 \pm \frac{v}{c/n}). \qquad (57.7)$$

They argued as follows: When the photons enter the water tube they first meet water molecules at rest and the frequency received by these shield molecules will be equal to the incident frequency ν. Thus in the "water tube" experiment there is a Doppler effect for light "emitted" by water molecules at rest and then "received" by the moving water molecules. However, according to these authors, the Doppler effect in water then involves the velocity of light in the medium c/n and not c.

According to our "hitch-hiker" model (see §40), the photons move only in vacuum and always with velocity c. One measures velocity c/n in a medium only because for a certain time the photons are hitched to the molecules and c/n is their average velocity.

Formula (5) was derived by Lorentz and Einstein for the case of moving solid block. Thus they made a substantial distinction between the propagation of light in a liquid flowing in tubes and in a moving block of matter. For our theory there only molecules, vacuum, and photons (free or hitched); and a Doppler effect always appear when the "emitting" and "receiving" molecules move with different velocities.

B. The Marinov "water tube" experiment

To establish which "drag-of-light" formula, (5) or (6), corresponds to physical reality, Marinov (1977) repeated the "water tube" experiment, using the sensitive bridge method for measurement of interference shifts (see §52B).

Our experiment was as follows (see 56.1): Light emitted by the source S was split by the semi-transparent mirror SM into two beams. The beam reflected by SM was split additionally by the semi-transparent mirror SM_1 into two daughter beams: the beam reflected by SM_1, after reflecting on mirror M_1, proceeded through the tube T_1, while the beam transmitted by SM_1 proceeded through the tube T_2 and, after reflecting on mirror M_2, met again the beam which had crossed T_1 at the semi-transparent mirror SM_2. These two beams interfered and illuminated uniformly the photoresistor P_2 which was in one arm of a Wheatstone bridge.

The situation was similar for the beam transmitted by SM. It was also split into two daughter beams at SM_2 which met at SM_1 to interfere and illuminated also uniformly the photoresistor P_1 put in the other arm of the Wheatstone bridge.

The procedure of measurement was as follows:

When the water is at rest the illumination of P_1 and P_2 must be equal, since the phase difference between the daughter beams in any pair are the same. If the water is set in

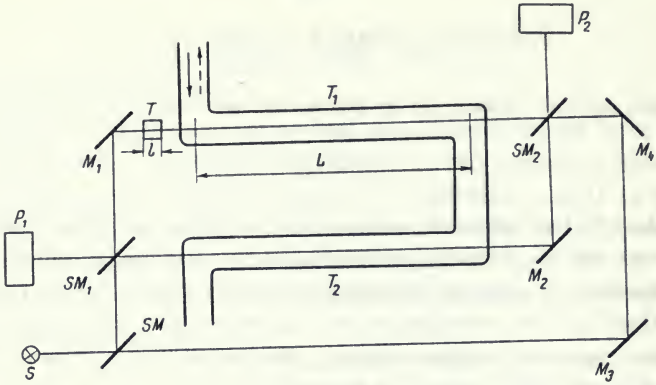

Fig. 57.1

motion, so that it flows in tube T_1 from left to right and in tube T_2 from right to left, the illuminations of P_1 and P_2 will begin to change oppositely. With the increase in the water's velocity, the bridge comes into greater and greater disequilibrium, passing through a maximum disequilibrium. As the path difference $\Delta = \Delta_1 + \Delta_2$ (where Δ_1 is the change in the difference between the light paths of the daughter beams propagating from left to right and Δ_2 is the change in the difference between the light paths of the daughter beams propagating from right to left) becomes equal to λ, the bridge comes again into equilibrium. Thus we can verify formulas (1), (5), and (6), where we must put $\Delta = c\Delta t = \lambda$.

The sensitivity of the method depends on the sum of the differences of the light paths of the beams going through T_1 and T_2 when the water is at rest. If this sum is exactly equal to $n\lambda$ (here n is an integer), no disequilibration of the bridge occurs when the velocity of the water is increased. If, however, this sum equals $(2n+1)(\lambda/2)$, the sensitivity of the bridge is the greatest.

We could adjust the apparatus to attain a maximum sensitivity by changing the temperature of a "tuner" T, a small piece of glass introduced into the light path, as shown in fig. 57.1. Its length was $l = 1$ cm, and the temperature coefficient of its refractive index was about $dn/dT = 5\times10^{-6}$ degree^{-1}. By changing its temperature by 6^{o}C, we could change the light paths of the beams proceeding through T and T_1 by 300 nm. The temperature of T could be thermostabilized to within $\delta T = \pm 0^{o}.04$ C, so that we could maintain the light path through T and T_1 constant in the limits of $\delta\Delta = \pm 2$ nm. The water was circulated from a reservoir where it was thermostabilized at a temperature $20^{o} \pm 0^{o}.3$ C.

Maximum sensitivity was established and maintained in the following way: By changing the temperature of the tuner T we changed the level of illumination over P_1 and P_2 from minimum (when the current in the arms of P_1 and P_2 is J_{min}) to maximum (when the current in the arms of P_1 and P_2 is J_{max}). If an additional resistance R, put in either of the arms of P_1 and P_2, reduced the current in the last case to J_{min}, then we put a resistance R/2 in those arms. By again changing the level of illumination of P_1 and P_2, using the tuner T, we adjusted the current to be J_{min}. Under these conditions the sensitivity was the greatest, and the temperature of T was maintained so that the current J_{min} always flowed through the arms of P_1 and P_2. It was expedient to always maintain the current in the diagonal galvanometer equal to zero, transferring resistance from the arm of P_1 into the arm of P_2 (or vice versa) when changing the velocity of water. The maximum sensitivity of the bridge was $\delta\Delta = \pm 2.5\times10^{-4}\lambda$, since the fluctuations of the zero galvanometer were about 4000 times smaller than the current $J_{max} - J_{min}$.

The lengths of the tubes T_1 and T_2 were L = 260.0 ± 0.5 cm, when measured as shown in the figure. The manometer was calibrated with a precision $\delta v/v = \pm 4\times10^{-3}$. The values for n and $dn/d\lambda$, corresponding to the different wavelengths, were taken from a graph which we plotted on the basis of the data given in Landolt-Börnstein (1962). The inaccuracies estimated by us were, respectively, $\delta n/n = \pm 2\times10^{-4}$ and $\delta(dn/d\lambda)/(dn/d\lambda) = \pm 5\times10^{-3}$.

The light source was a tuned dye laser with neodymium glass oscillator. The inaccuracy in the chosen wavelength was $\delta\lambda/\lambda = \pm 10^{-3}$.

Table 56.1

λ nm	n	dn/dλ mm^{-1}	v m/s	k
236	1,3850	- 662	6.30	0.99
250	1.3770	- 516	6.98	1.00
280	1.3644	- 310	8.44	1.01
300	1.3586	- 242	9.36	0.99
360	1.3438	- 130	11.96	0.98
390	1.3438	- 108	13.16	1.01

In table 56.1 we give the values of n and dn/dλ for the used wavelengths λ, the observed velocities of water v, and the calculated values of the factor

$$k = \frac{1}{\lambda(dn/d\lambda)}\left(n^2 - 1 - \frac{c\Delta}{4Lv}\right), \qquad (57.8)$$

where we put $\Delta = c\Delta t = \lambda$.

Taking into account all possible errors, we obtain $\delta k/k = \pm 0.10$, for λ = 300 nm.

The factor k should be zero according to Fresnel's formula (1), where dispersion is not considered. The factor should be n, according to Lorentz-Einstein's formula (6), and it should be unity, according to Marinov's formula (5). As it is seen from table 56.1, the experiments confirms our formula.

C. The second-order in v/c effects

An experimental confirmation of our drag-of-light formula (40.16) to within an accuracy of second order in v/c is still not made, and at the present state of technique such an experiment is to be considered as a challenge to the experimenters.

This experiment, proposed by Marinov (1974), will appear as follows:

A liquid with refractive index n in a tube with length L is to be put in one arm of a Michelson interferometer (see §49A). One should observe a certain interference pattern. Let then set the liquid in motion with velocity v. According to formula (40.16), when the liquid is moving the light beam, proceeding to and fro along the arm with the tube, will return to the semi-transparent mirror with a time delay

$$\Delta t = \frac{L}{c_m^+} + \frac{L}{c_m^-} - \frac{2L}{c/n} = \frac{2Lv^2}{c^3} n(n^2 - 1). \tag{57.9}$$

This effect is to be measured from the change in the interference pattern.

§58. THE "DRAG ABERRATION" EXPERIMENTS

A. The Jones experiment

The drag aberration formula (42.3) can easily be verified by silvering the parallel planes of a transparent slab with a great refraction index n, making them light reflecting, so that the beam, after having entered once into the medium, has to undergo a high number of reflection before leaving it (cf. fig. 52.8). In this way the "effective" length of the light beam will be considerably increased. The lateral displacement (QR in fig. 42.1) can then be reliably registered when rotating the slab with a high velocity.

Recently a similar "DRAG ABERRATION" EXPERIMENT was carried out by Jones (1975). However, the experiment was performed for the case $\phi = 0$ only (see fig. 42.1), where our formula (42.3) and the formula (42.11) obtained from the Lorentz transformation lead to the same result. Jones' measurements have verified both formulas for this special case. The task remains for performing the experiment under the condition $\phi \neq 0$, when, we are convinced, our formula will be confirmed, as formula (42.11) is unsound.

B. The Airy experiment

In the XIXth century one has supposed that an effect of "drag aberration" can be registered when the observer moves together with the medium. So it was supposed that if the aberration formula for an astronomical tube filled with vacuum is given by formula (39.4), for a tube filled with a medium having refraction index n, the aberration formula must be

$$\alpha = \frac{v}{c/n} \sin\theta_m = \frac{vn}{c} \sin\theta_m, \tag{58.1}$$

as the velocity of light in a medium is c/n and not c.

With the aim to establish which of the two formulas (39.4) and (58.1) is the true one, Airy (1871) repeated Bradley's observations by the help of an astronomical tube filled with water. He established that also in this case the right formula is Bradley's formula (39.4).

This result is obvious if one proceeds from our "hitch-hiker" model of light propagation. Indeed, during the time when the photons are hitched to the water molecules in the astronomical tube, they proceed with the same velocity as the tube (and the Earth); only when the photons move in the gaps between the molecules, their motion leads to an aberrational effect. For a tube filled with water the free motion of the photons is the same as for a tube filled with vacuum.

§59. THE "ROTATING DISK" EXPERIMENT

A. General remarks

The "ROTATING DISK" EXPERIMENT, performed by Harress (1912) and Sagnac (1913) in a laboratory and repeated on the rotating Earth by Michelson, Gale, and Pearson (1925), was the first, and until the performance of our light kinematic experiments, the unique experiment which has revealed the direction dependence of light velocity. Nevertheless, sixty years after its first performance, hundreds of pens have tried to reconcile the decisive positive "aether wind" effect in this experiment with the dogmatic one-way light velocity constancy, aiming to convince the scientific community that black is white. The history of this experiment and of its mistreatment by official physics is very instructive, having shown that when the experiments speak the gods keep silence but the theoreticians do not.

We have briefly analysed this experiment in §44E. Now we shall consider it in detail.

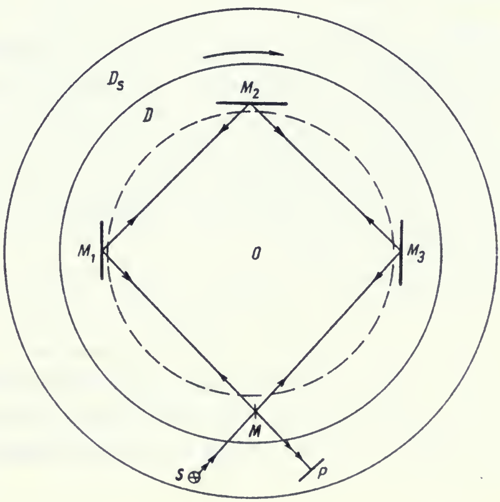

Fig. 59.1

The scheme of the "rotating disk" experiment is presented in fig. 59.1:

Monochromatic parallel light emitted by the source S is partially reflected and partially transmitted by the semi-transparent mirror M. The "reflected" beam reflects successively on mirrors M_1, M_2, M_3, M, and illuminates the screen P. The "transmitted" beam reflects successively on mirrors M_3, M_2, M_1 and, passing through the semi-transparent mirror M, also illuminates screen P, over which an interference pattern can be observed.

The mirrors M, M_1, M_2, and M_3 are mounted on the disk D; the source S and the screen P are mounted on the supplementary disk D_s. The disk D can rotate alone or together with disk D_s about an axis at the center O. It is of no importance for the result of the experiment whether the disks D and D_s are rigidly connected or the disk D_s is at rest. Thus only the rotation of the disk D produces the anisotropy effect.

The optical paths of both beams are not equal because the beam proceeding in a "direct" (clockwise) direction, in which the disk rotates, is reflected two times by the semi-transparent mirror M, while the beam proceeding in the "opposite" (counter-clockwise) direction is two times transmitted by M. If the disk D is set in rotation, the interference pattern will change, as the "direct" beam has to cover a longer distance with respect to absolute space than the "opposite" beam, and thus the "direct" photons will return to M with a certain time delay Δt after the "opposite" photons.

Suppose that a medium with index of refraction n is put between the mirrors (marked with the dotted line in the figure). Let all mirrors rotate with angular velocity Ω together with the medium or without it, or let the mirrors be at rest and let rotate only the medium. So four combinations are possible which we name as follows:

1. The HARRESS-SAGNAC EXPERIMENT, performed first by Sagnac (1913), in which the mirrors rotate and as a medium a vacuum is taken.

2. The HARRESS-MARINOV EXPERIMENT, performed first by Harress (1912) and repeated very carefully by Pogany (1928), in which the mirrors rotate together with the medium. We give to this variation the name "Harress-Marinov experiment" from a methodological point of view (see below), as here one measures the velocity of light when both observer and medium move with respect to absolute space. This effect, for an inertial motion of source and observer, was observed first by Marinov (see §51 and §52).

3. The HARRESS-DUFOUR EXPERIMENT, performed first by Dufour and Prunier (1942) and repeated by Marinov (1977) in a slightly different arrangement (see §59B), in which the mirrors rotate and the medium is at rest.

4. The HARRESS-FIZEAU EXPERIMENT, performed by Fizeau (1851) in a substantially different arrangement (called the "water tube" experiment -see §57), in which the medium rotates and the mirrors are at rest. Marinov's (1977) performance of the Harress-Fizeau experiment can be considered as the first one, however, for the sake of methodological purposes (see below), we give to this variation the name "Harress-Fizeau experiment".

Our nomenclature of the different variations of the "rotating disk" experiment is chosen from a historical and a methodological point of view and will be preserved also for the denomination of the different variations of the "moving platform" experiment (see §64). The first name "Harress" designates that this is a "rotating disk" experiment which was performed for the first time by Harress. The second name designates the kind of the effect when observer and medium move with respect to one another or together with respect to absolute space, namely: When the observer is at rest and the medium moving, the effect has the name of Fizeau (see §40B); when the medium is at rest and the observer moving, the effect has the name of Dufour (see §40C); when observer and medium move together, the effect has the name of Marinov (see §40D); if in the last case one takes vacuum as a medium, the effect has the name of Sagnac (see §40D).

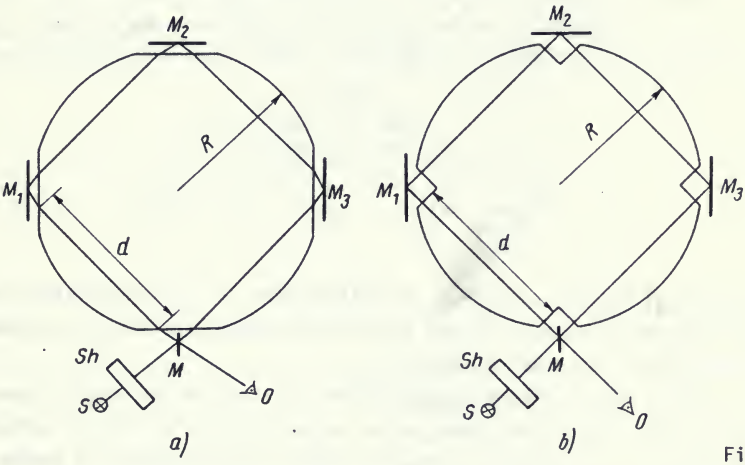

Fig. 59.2

In fig. 59.2 are presented two possibilities for inserting the medium between the mirrors. The arrangement in fig. 59.2a was used by Marinov (1977), and one can call this the "Marinov variation". The arrangement in fig. 59.2b was used by Dufour and Prunier (1942), and one can call this the "Dufour variation". Obviously, the difference between these two variations is not substantial (see §59B). The designation of the different elements is the same as in fig. 59.1. The element Sh is a shutter governed by the rotating turnabout which lets light pass only when the rotating part has a strictly defined position with respect to the laboratory, i.e., when the light beam observed by O comes from the same point.

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In fig. 59.3 is presented the actual scheme of our set-up for the performance of the "rotating disk" experiment (reported in §59F):

A medium can rotate with the semi-transparent mirrors SM, SM_A, SM_B and the mirrors M_1,

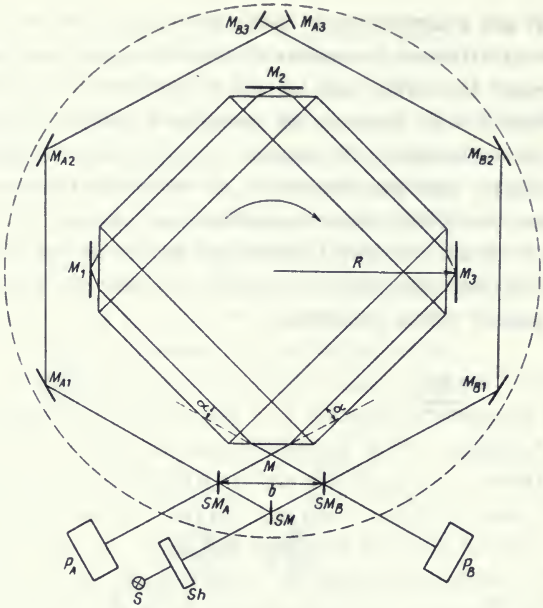

Fig. 59.3

M_2, M_3, M_{A1}, M_{A2}, M_{A3}, M_{B1}, M_{B2}, M_{B3}, or without them, or only the mirrors can rotate and the medium remain at rest. In the last case a medium with refractive index $n = 1$, i.e., vacuum (air), can also be taken.

S is a light source emitting coherent light. Sh is a shutter which is governed by the rotating disk. It allows short light pulses to pass only at a strictly defined position of the disk when the diametrically opposite facets of the transparent medium are exactly parallel to the mirrors M_1, M_2, M_3. The areas of the facets are small and the mirrors are placed near to the medium. Thus we can assume that the photons travel between the single mirrors along the corresponding chords of a circle with radius R. P_A and P_B are two photoresistors put in the arms of a Wheatstone bridge. When the shutter Sh allows light to pass, the photoresistors are illuminated uniformly by interfered light. With the aim of explaining the character of the interference, let us consider four photons which are emitted by S at the same moment and cover the following paths:

First photon: SM - SM_A - M_{A1} - M_{A2} - M_{A3} - M_{A2} - M_{A1} - SM_A - P_A.

Second photon: SM - SM_B - M_1 - M_2 - M_3 - SM_A - P_A.

Third photon: SM - SM_B - M_{B1} - M_{N2} - M_{B3} - M_{B2} - M_{B1} - SM_B - P_B.

Fourth photon: SM - SM_A - M_3 - M_2 - M_1 - SM_B - P_B.

The first and third photons cover the same paths when the apparatus is at rest, as well as when it is in motion. Actually, differences of second order in v/c occur which are analysed in §63. In §59 we shall consider only the effects of first order in v/c.

The second photon (which we shall call "direct") travels in the direction of rotation, and the fourth photon (which we shall call "opposite") travels against the direction of rotation. The differences in the optical paths of the first and second photons, on one hand, and of the third and fourth photons, on the other hand, will change oppositely with the rotational velocity. At rest the illumination over both photoresistors is the same, and the bridge is in equilibrium. When increasing the rotational velocity, the bridge comes into greater and greater disequilibrium, passes through a state of maximum disequilibrium, and at a certain rotational velocity Ω comes again into equilibrium. If the time spent by the second (or fourth) photon for covering its path at the angular velocity Ω differs by Δt_A (Δt_B) from the time spent at rest, and we introduce the notation $\Delta t = \Delta t_A + \Delta t_B$, then $\Delta = c\Delta t$ equals the wavelength λ of the light used.

The results obtained by the help of our "rotating disk" experiment are reported in §59F.

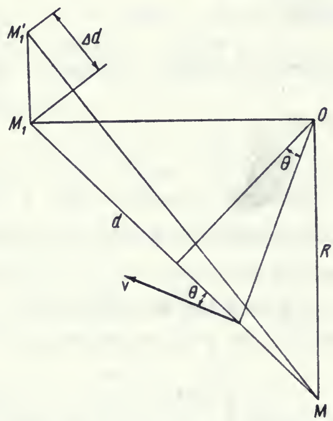

Fig. 59.4

B. The Harress-Dufour experiment

First we shall consider the Harress-Dufour experiment. Its schemes, in the Marinov and Dufour variations, respectively, are given in fig. 59.2a and 59.2b.

Suppose that the mirrors rotate in a "direct" direction and the medium remains at rest in absolute space. According to fig. 59.4 (see also fig. 59.2 and fig. 59.3), a "direct" photon which separates from an "opposite" photon at the semi-transparent mirror M will reflect not at point M_1, where it reflects when the mirrors are at rest, but at a point M_1', and thus in the case of rotation its path will be longer by

$$\Delta d = \Omega R \frac{d}{c/n} \cos\frac{\pi}{4} = \frac{\Omega R^2}{c} n, \tag{59.1}$$

where $d = R/\cos(\pi/4)$ is its path when the mirrors are at rest and R is the distance of the mirrors from the center of rotation. In the case of rotation the path of an "opposite" photon between M and M_3 will be with Δd less than in the case at rest.

Since mirror M (or mirrors SM_A and SM_B in fig. 59.3) moves, the Doppler effect causes the frequencies of the "direct" photons received by the molecules of the medium to be

$$\nu_o = \nu(1 + 2\frac{v}{c}\cos\frac{\pi}{4}) = \nu(1 + \sqrt{2}\,\frac{\Omega R}{c}), \qquad (59.2)$$

while the frequencies of the "opposite" photons received by the molecules of the medium will remain the same.

If we take into account dispersion, the refractive index of the medium for the "direct" photons becomes

$$n^+ = n(\nu_o) = n + \sqrt{2}\,\frac{\Omega R}{c}\nu\frac{dn}{d\nu}. \qquad (59.3)$$

Thus, if the mirrors rotate, a "direct" photon will return to mirror M after an "opposite" photon with the following time delay:

a) for the Marinov variation

$$\Delta t_{H-D} = 4\,\frac{d + \Delta d}{c/n^+} - 4\,\frac{d - \Delta d}{c/n} = 8\,\frac{\Omega R^2}{c^2}(n^2 - \lambda\frac{dn}{d\lambda}), \qquad (59.4)$$

b) for the Dufour variation

$$\Delta t_{H-D} = 4(\frac{d}{c/n^+} + \frac{\Delta d}{c}) - 4(\frac{d}{c/n} - \frac{\Delta d}{c}) = 8\,\frac{\Omega R^2}{c^2}(n - \lambda\frac{dn}{d\lambda}). \qquad (59.5)$$

Let us note that in our realization of the Harress-Dufour experiment (see fig. 59.3), we have to take into account the difference in the light paths which appears along the contour SM - SM_A - M - SM_B - SM when the mirrors rotate. Denoting by b the distance between SM_A and SM_B, we obtain for the area encircled by this contour

$$S = \frac{b^2}{2}\cot(\frac{\pi}{4} + \alpha) = b^2\,\frac{(2 - n^2)^{1/2}}{2n}, \qquad (59.6)$$

where α is the difference between the angles of incidence and refraction, and we have used Snell's law (41.4), which for our case is

$$\sin(\frac{\pi}{4} + \alpha) = \frac{\sqrt{2}}{2}n. \qquad (59.7)$$

Thus, when the mirrors rotate with an angular velocity Ω, the second photon will come for a rendezvous with the first photon on mirror SM_A with the following additional time advance

$$\Delta t_{add} = \frac{S\Omega}{c^2} = \frac{\Omega b^2}{c^2}\,\frac{(2 - n^2)^{1/2}}{2n}, \qquad (59.8)$$

while the fourth photon will come for a rendezvous with the third photon on mirror SM_B with the same time delay.

C. The Harress-Fizeau experiment

The Harress-Fizeau experiment can also be performed in the two different arrangements shown in fig. 59.2a and 59.2b. We shall consider only the first one, as only this variation has been carried out by Marinov (1977).

Suppose that the medium rotates in "direct" (clockwise) direction and the mirrors remain at rest in absolute space. Now, as can be seen from fig. 59.4, the velocity of a molecule of the medium that makes an angle θ with the direction of propagation of the "direct" photons will have a magnitide

$$v = \frac{\sqrt{2}}{2}\frac{\Omega R}{\cos\theta}. \tag{59.9}$$

Since the medium moves with respect to the mirrors, a Doppler effect occurs. The frequencies of the "direct" and "opposite" photons received by the molecules of the medium will be, respectively,

$$\nu_o = \nu(1 \mp \frac{v}{c}\cos\theta) = \nu(1 \mp \frac{\sqrt{2}}{2}\frac{\Omega R}{c}). \tag{59.10}$$

Hence, if we take into account dispersion, the refractive indices for the "direct" and "opposite" photons become, respectively,

$$n^{\pm} = n(\nu_o) = n \mp \frac{\sqrt{2}}{2}\frac{\Omega R}{c}\,\frac{dn}{d\nu}. \tag{59.11}$$

Thus, when the medium rotates, a "direct" photon returns to mirror M before an "opposite" photon with the time advance given by (using formula (40.16) and comparing with formula (57.5))

$$\Delta t_{H-F} = \frac{4d}{c_m^-} - \frac{4d}{c_m^+} = 8\frac{\Omega R^2}{c^2}(n^2 - \lambda\frac{dn}{d\lambda} - 1). \tag{59.12}$$

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For the Harress-Fizeau experiment performed with a medium representing a solid disk, the Lorentz-Einstein conceptions (see p. 219) must lead also to formula (12). According to the Lorentz-Einstein conceptions, the same formula (12) is to be written also if the medium is a liquid (imagine it closed in a disk with compartments, as shown in fig, 59.5). However, if the partitions are not connected with the rim (see fig. 59.5), and the partitions can rotate dragging the liquid between them as paddles, but the rim of the disk stays at rest, then we should realize an analogue to the Fizeau's "water tube" experiment (see §57). In such a case, since water molecules will stick on the rim, proceeding from the Lorentz-Einstein conceptions, we have to write instead of formula (12) the following one (cf. formulas (57.5) and (57.6))

$$\Delta t_{H-F} = \frac{8\Omega R^2}{c^2}(n^2 + n\lambda\frac{dn}{d\lambda} - 1). \tag{59.13}$$

The experiment (see §59F) has shown that our formula (12) corresponds to physical reality.

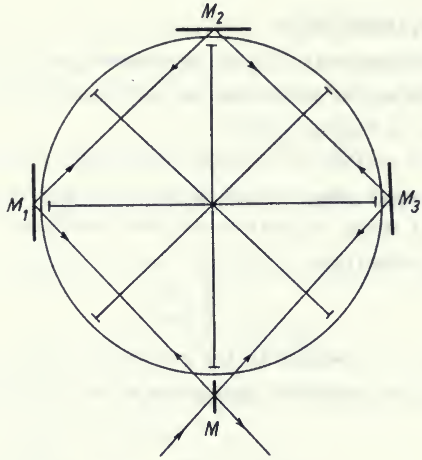

Fig. 59.5

D. The Harress-Marinov experiment

In the Harress-Marinov experiment the time delay with which a "direct" photon returns to mirror M after an "opposite" photon is equal to the difference in the time delays in the Harress-Dufour and Harress-Fizeau experiments. Thus from formulas (4) and (12) we obtain

$$\Delta t_{H-M} = \Delta t_{H-D} - \Delta t_{H-F} = 8\frac{\Omega R^2}{c^2}. \qquad (59.14)$$

We must emphasize once more (see p. 154) that in the Harress-Dufour and Harress-Fizeau experiments there is the same relative motion between mirrors and medium. However, the effects in these two experiments are substantially different because in the Harress-Dufour experiment the medium is at rest in absolute space, while in the Harress-Fizeau experiment the mirrors are at rest in absolute space. The theory of relativity cannot explain the appearance of this difference.

E. The Harress-Sagnac experiment

The formula for the effect in the Harress-Sagnac experiment may be obtained from (4) and (5) for n = 1, yielding

$$\Delta t_{H-S} = 8\frac{\Omega R^2}{c^2}. \qquad (59.15)$$

F. Marinov's performance of the Harress-Dufour and Harress-Fizeau experiments

One difference between our arrangement (see fig. 59.3) and the traditional arrangement (see fig. 59.1) is that in our arrangement the "direct" and "opposite" light pulses interfere with light pulses that always cover the same path. Thus the illuminations over the photoresistors P_A and P_B change oppositely, and we can use the sensitive bridge method described in §57B. A second difference consists in the following:While in the traditional method (see fig. 59.1) the mirrors M_1, M_2, M_3 are tangent to the circumference of the medium, the semi-transparent mirror M is not tangent and cannot be placed close enough to the medium. In our arrangement the separation of the photons that will later interfere proceeds first from the semi-transparent mirror SM and then from the semitransparent mirrors SM_A and SM_B, so that instead of mirror M (see fig. 59.2) there is an effective point of separation M (see fig. 59.3) which can lie close enough to the circumference of the medium. Proceeding from formulas (4) and (8), we obtain the following formula for the effect in the Harress-Dufour experiment when using our set-up (for the light path difference)

$$\Delta_{H-D} = 8\frac{\Omega R^2}{c}(n^2 - \lambda\frac{dn}{d\lambda}) - \frac{\Omega b^2}{c}\frac{(2 - n^2)^{1/2}}{n}. \tag{59.16}$$

It can easily be seen that the formula for the effect in the Harress-Fizeau experiment when using our set-up, as obtained directly from (12), is

$$\Delta_{H-F} = 8\frac{\Omega R^2}{c}(n^2 - \lambda\frac{dn}{d\lambda} - 1). \tag{59.17}$$

As a medium distilled water was used in a metallic vessel which had the form shown in fig. 59.3. Glass windows were placed at points where light beams must cross the walls of the vessel. Glass windows also were placed in the metallic interfaces which divide the ring into compartments. Taking into account the thikness of the glass plates and their refractive index, we have put the mirrors M_1, M_2, M_3 in such positions that the actual light path (distance multiplied by refractive index) along the contour M - M_1 - M_2 - M_3 - M was equal to the light path that would be covered if mirrors M_1, M_2, M_3 had been immersead in water.

The light source was a He-Ne laser. For light of wavelength λ = 632.8 nm the refractive characteristics of water are n = 1.3317 and $dn/d\lambda = - 2.7\times10^{-5}$ nm^{-1}. We assumed that the errors in n and $dn/d\lambda$ can be neglected, i.e., we accepted $\delta n = 0$, $\delta(dn/d\lambda) = 0$. The geometrical parameters were R = 30.6 ± 0.2 cm, b = 10.0 cm, assuming $\delta b = 0$; we considered a large error $\delta R = \pm 0.2$ cm, which also had to compensate for possible errors introduced in the measurement of the thikness of the glass plates and errors that could appear from the replacement of the actual light path by an idealized light path only in water.

We made the light paths of the first and second photons (as well of the third and fourth photons) equal. However, since laser light with good coherence was used, this was by no means necessary, and the light paths of the first and third photons could have been substantially reduced.

The sensitivity of our bridge method is annalysed in §52B and §57B. Let us repeat that a maximum sensitivity was obtained when the sum of the differences of the light paths of the first and second photons and of the light paths of the third and fourth photons at rest is $(2n+1)\lambda/2$, where n is an integer, being $\delta\Delta = \pm 2.5\times10^{-4}\lambda$. When this sum is equal to $n\lambda$, the sensitivity falls to zero. We did not search for the highest sensitivity using a "tuner", as described in §57B. We have taken an average sensitivity $\delta\Delta = \pm 10^{-2}\lambda$. The "tuner" described in §57B could also have been used for calibration during the run, however, in our method, where we change the rotational velocity until $\Delta = c\Delta t$ becomes equal to λ, no calibration needed to be made.

The number of revolutions per second of the disk, $N = \Omega/2\pi$, was measured using a light stroboscopic cyclometer and maintained automatically with precision $\delta N = \pm 0.02$ rev/sec. We rotated the disk first counter-clockwise with angular velocity Ω_1, and then clockwise with angular velocity Ω_2, taking $\Omega = (1/2)(\Omega_1 + \Omega_2)$.

When the disk is at rest, the Wheatstone bridge can be set into equilibrium by a micrometrical movement of mirror M_{A3} or M_{B3}. If we do not care to do this and if at the beginning the bridge is in disequilibrium witn a certain "positive" current, then at the angular velocity Ω (counter-clockwise or clockwise) the bridge will be in disequilibrium with the same "negative" current. However, here the shutter has to operate with the same chopping frequency at rest and at the rotational velocity Ω. In the case where the apparatus is thermostabilized, a maximum sensitivity can be achieved by a micrometrical movement of the mirrors M_{A3} and M_{B3}.

We obtained $N = 22.68 \pm 0.04$ rev/sec for the Harress-Dufour experiment and $N = 50.60 \pm 0.04$ rev/sec for the Harress-Fizeau experiment. Substituting the numerical values into formulas (16) and (17), we obtain, supposing that the velocity of light is an unknown quantity,

$$c_{H-D} = (3.01 \pm 0.07)\times 10^8 \text{ m/sec},$$
$$c_{H-F} = (2.97 \pm 0.07)\times 10^8 \text{ m/sec}, \qquad (59.18)$$

where $\delta c = \pm 7\times 10^6$ m/sec was the maximum error.

G. The inertial "rotating disk" experiment

The explanation of the "rotating disk" experiment given by the theory of relativity is full of contradictions. Certain relativists assert that this experiment can be explained only by the apparatus of general relativity because the motion is not inertial. However, other relativists assert that this can be done also in the frame of special relativity, making use of the Lorentz transformation.

The mirrors M, M_1, M_2, and M_3 in fig. 59.1 move with a normal acceleration. Nevertheless this normal acceleration is not decisive for the result of the acceleration. Indeed, we propose the following modificabion of the Harress-Sagnac experiment (called by us the INERTIAL "ROTATING DISK" EXPERIMENT), which will give the same result and where any non-inertial motion is excluded.

Let (see fig. 59.6) mirrors M_1, M_2, and M_3 be at rest and let the semi-transparent mirror S rotate with angular velocity Ω about some center C. We assume that when the semi-transparent mirror S is vertical, then, over some small angle α, a "finger" reduces the rotational motion to a translational one with velocity v = R, where R is the radius of the rotational motion of S.

Let a light pulse fall over S and split into "direct" and "opposite" portions. If the semitransparent mirror S is at rest, a certain interference pattern will be observed produced by the "direct" and "opposite" photons after their unification. If now S is set in motion, the interference pattern will change because of the time delay with which the "direct" photons will return to S after the "opposite" photons, and this time delay will be given by formula (15).

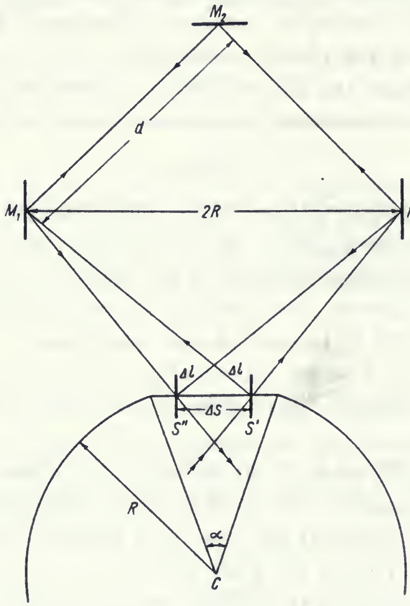

Fig. 59.6

Indeed, the time t in which the "direct" and "opposite" photons cover path 4d is equal to

$$t = 4d/c = 4\sqrt{2}\,R/c. \qquad (59.19)$$

In this time the semi-transparent mirror will cover a distance

$$\Delta s = vt = 4\sqrt{2}\,\Omega R^2/c \qquad (59.20)$$

between the positions S' and S".

If Δl is the difference between the paths of the "direct" and "opposite" photons when S is in motion and at rest, then the time with which the "direct" photons will return to the semi-transparent mirror S after the "opposite" photons will be

$$\Delta t = 2\,\Delta l/c = \sqrt{2}\Delta s/c = 8\,\Omega R^2/c^2. \qquad (59.21)$$

§60. THE "ROTATING DISK" EXPERIMENT WITH NEUTRONS

The "ROTATING DISK" EXPERIMENT WITH NEUTRONS was carried out for the first time by Werner, Staudemann, and Collela (1979) and we considered it briefly in §44E. Werner's experiment represents an analogue to the Michelson-Gale-Pearson (1925) experiment where for the first time a Sagnac effect with photons was measured on the spinning Earth as a rotating disk. We should like to emphasize (see p. 151) that with these two experiments one measures the net angular velocity of the laboratory which represents the geometrical sum of all angular velocities in which the laboratory takes part. Michelson et al. and Werner et al. have registered only the diurnal angular velocity as that about the Sun is 365 times smaller and that around the galactic center further 200,000,000 times smaller and thus both latter are to be disregarded with respect to the former.

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Werner et al. used a perfect-silicon-crystal interferometer of the type first developed for x rays by Bonseand Hart (1965). The first demonstration that such a device can be used for neutrons was achieved by Rauch et al. (1974).

The interferometer represents three parallel Si-crystal slabs. A monoenergetic neutron beam of wavelength λ = 1.262 Å incident on the interferometer is coherently split in the first Si slab by reflection from the lattice planes. The two resulting beams, which make an angle θ ($< \pi/2$) are coherently split again at the second Si slab. The two of the produced beams, which are parallel to the beams split at the first slab, meet under the same angle θ at the third Si slab where, after being split once more, overlap and interfere. The outgoing interfering beams are detected by two ^{3}He proportional detectors. Thus the coherent neutrons which separate at the first slab and meet again at the third slab confine a parallelogram with an area S = 8.864 cm^2.

The time difference with which the "direct" neutrons (those proceeding along the direction of rotation) will come at the rendezvous point after the "opposite" neutrons (those proceeding against the direction of rotation) is given by the half of formula (44.19), as the "direct" and "opposite" neutrons do not meet at the point of separation and cover only the half perimeter of the interferometer parallelogram. Thus, taking also the half of formula (44.24), we obtain that the phase difference between the "opposite" and "direct" neutrons will be (see the text following formula (44.24))

$$\Delta\psi = \frac{4\pi}{h} m \Omega S, \qquad (60.1)$$

where m is the neutron mass and h is the Planck constant.

The interferometer's plane has been maintained vertically. Rotating the interferometer about the vertical axis of the incident beam, one changes the angular rotational velocity of the ineterferometer's area S from zero (when the plane of the interferometer is parallel to the geographical meridian of the laboratory) to a maximum value $\Omega = \Omega_E \sin\phi$, where

Ω_E is the Earth's diurnal angular velocity and ϕ is the geographical latitude (when the plane of the interferometer is perpendicular to the meridian). The appearing phase shift between the "direct" and "opposite" photons verified formula (1) with a high precision.

§61. THE INTERRUPTED "ROTATING DISK" EXPERIMENT

The proper time delay in the "rotating disk" experiment performed with light can be written in the form (see formula (44.19) and use formula (4.28))

$$\Delta t_o = \int_o^d \frac{dr}{(c_o')_{dir}} - \int_o^d \frac{dr}{(c_o')_{opp}} = \frac{2}{c^2}\int_o^d v\cos\theta' dr, \qquad (61.1)$$

where d is the closed path covered by the "direct" and "opposite" photons on the rotating disk with the velocity $(c_o')_{dir}$ and $(c_o')_{opp}$, respectively, and θ' is the angle between the "direct" photons' velocity and the velocity v of the respective points of the disk.

When considering effects of first order in v/c (as we shall do in this section) a difference between universal and proper time is not to be made, and we can write $\Delta t_o = \Delta t$.

Easily can be seen that formula (1) can be presented in the form

$$\Delta t = \frac{2}{c^2}\int_o^d \vec{v}.d\vec{r}, \qquad (61.2)$$

where $d\vec{r}$ is the vector element of the light path of the "direct" photons and $\vec{v}$ is their absolute velocity.

In the "rotating disk" experiment (see §59) the point of separation of the "direct" and "opposite" photons is the same as the point of their meeting, so that the light paths of the interfering photons are closed curves. If we should interrupt these closed paths and make the points of separation and meeting different, the light paths of the "direct" and "opposite" photons which are different at rest and motion of the disk can be made straight lines. Such an INTERRUPTED "ROTATING DISK" EXPERIMENT was performed by Marinov (1977). This experiment showed patently that the velocity of light is direction dependent along a straight line on a rotating disk. Its scheme is the following (see fig. 61.1):

The light sourse S was a He-Ne laser. Sh was a shutter which was governed by the rotating disk and let light pass only at a strictly defined position of the disk when both photoresistors P_A, P_B were illuminated. Later we realized that since the areas of the photoresistors are small, the shutter is even not necessary. If S, P_A, and P_B were also to be mounted on the rotating disk, the shutter Sh would be entirely unnecessary. SM was a semi-transparent mirror, M a mirror, and SM_C a corrective semi-transparent mirror which reduced the number of photons along the path to SM_A to the number of photons along the path to SM_B.

Let four photons be emitted by S at the same moment and suppose that they cover the following paths:

First photon: S - SM - SM_C - SM_A - SM'_A - P_A.

Second photon: S - SM - SM_C - SM_A - SM_B - SM'_B - P'_B.

Third photon: S - SM - M - SM - SM_B - SM'_B - SM'_A - P_A.

Fourth photon: S - SM - M - SM - SM_B - SM'_B - P_B.

Using formula (2) and fig.61.1, we find that in the case of rotation (with respect to the case at rest) the time it takes for the third (fourth) photon to reach P_A (P_B) is shorter than the time it takes for the first (second) photon to reach P_A (P_B) by the amount

$$\Delta t_A = \frac{2\Omega R^2}{c^2} \tan\frac{\theta}{2} \qquad \left(\Delta t_B = \frac{\Omega R^2}{c^2} \sin\theta\right). \tag{61.3}$$

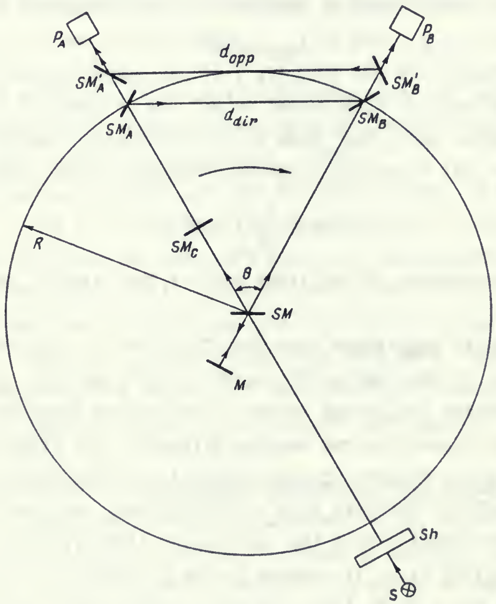

Fig. 61.1

The photoresistors P_A, P_B were put in the arms of a Wheatstone bridge. They were illuminated <u>uniformly</u> by interfered light. When the disk was at rest, the bridge was put into equilibrium, so that both photoresistors were illuminated by equal light intensities. This was achieved by adjusting micrometrically SM'_A and SM'_B and changing in such a way the path difference between the first and third photons until the bridge comes into equilibrium. Then we set the disk in rotation. With increasing rotational velocity, the bridge came into greater and greater disequilibrium, passing through a state of maximum disequilibrium. At a certain angular velocity Ω, when the sum of the differences in the optical paths $\Delta = (\Delta t_A + \Delta t_B)c$ became equal to the wavelength λ of the light used, the bridge was again in equilibrium. In this case

$$\lambda = \Delta = \frac{\Omega R^2}{c} \left(2\tan\frac{\theta}{2} + \sin\theta\right). \qquad (61.4)$$

We experimentally checked this formula, taking $\delta\Delta = \pm\ 10^{-2}\lambda$ (concerning the sensitivity of our bridge method see §57B and §59F), $\lambda = 632.8$ nm, $\theta = 60\overset{\circ}{.}0 \pm 0\overset{\circ}{.}5$, $R = 40.0 \pm 0.2$ cm. The number of revolutions per second $N = \Omega/2\pi$ was measured by a light stroboscopic cyclometer and maintained automatically with a precision $\delta N/N = \pm\ 2\times10^{-4}$. We registered $N = 92.90 \pm 0.02$ rev/sec. Putting the figures into formula (4), we obtained, supposing the velocity of light is unknown,

$$c = (2.98 \pm 0.07)\times10^{8} \text{ m/sec}, \qquad (61.5)$$

where $\delta c = \pm\ 7\times10^{6}$ m/sec was the maximum error.

§62. THE "COUPLED SHUTTERS ON A ROTATING DISK" EXPERIMENT

The "COUPLED SHUTTERS ON A ROTATING DISK" EXPERIMENT, proposed by Marinov (1975), represents a variant of the interrupted "rotating disk" experiment for the case where the rotation of the disk cannot be changed at will, as it is if our Earth is taken as a rotating disk. Its essence is as follows:

The mirrors $M_1,\ldots..M_k$ are placed along the rim of a disk as shown in fig. 62.1. Light emitted by the source S_A (or S_B) passes through the semi-transparent mirror M_A (M_B) and through the high-frequency operating shutter Sh_A (Sh_B). The choped light, reflecting from the mirrors $M_1,\ldots..M_k$, passes through the shutter Sh_B (Sh_A). After being reflected by the semi-transparent mirror M_B (M_A), it is observed by the observer O_A (O_B). The shutters operate with the same chopping frequency f, being driven by the common resonator Res put at the center of the disk. Thus the shutters operate synchronously both at rest and at motion of the disk. Indeed, since the distances between the common resonator and the shutters which the electromagnetic signals have to cover are equal (with respect to the disk but also with respect to absolute space), the shutters will open and close together.

Let us suppose that the disk is at rest and let us denote by d the light path between both shutters. If $n = (d/c)f$ is an integer (or an integer plus 1/2), both observers will register maximum (minimum) photon flux. If the disk is set in rotation in a clockwise direction, then the observer O_A will register a maximum photon flux when $n_A = (d/c)f(1+v/c)$ is an integer, while the observer O_B will register a maximum photon flux when $n_B = (d/c)f(1-v/c)$ is an integer.

If d and v are given and f changes, then both observers consequently should register "equal" or "opposite" pictures, e.g., "O_A and O_B together see maximum light", or "O_A sees maximum light when O_B sees minimum light". Indeed, we have

$$n_A = n_B \frac{c+v}{c-v} \cong n_B + 2\frac{v}{c}n = n_B + \Delta n. \qquad (62.1)$$

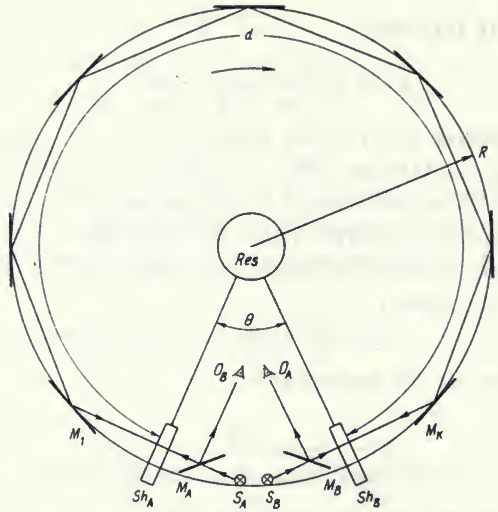

Fig. 62.1

Thus if $\Delta n = 2dvf/c^2$ is an integer, the observers 0_A and 0_B should register "equal" pictures and if Δn is equal to an integer plus 1/2, the observers 0_A and 0_B should register "opposite" pictures.

If the angle θ is almost equal to 2π and the radius of the disk R is very large, then we can consider the motion of the coupled shutters as essentially uniform liner motion, i.e., as inertial. This can be practically realized if one takes as a rotating disk our Earth and puts the common resonator at the pole. As shutters two Kerr cells can be used, separated by a distance d (say, 100 km) along the equator (the shutters may be placed on the peaks of two mountains). As light sources lasers can be used. The commanding signals can be sent from the pole to the shutters by the help of several relay stations. For v = 0.45 km/sec (the approximate linear rotational velocity of the Earth's equator) and d = 100 km, one should have $\Delta n \cong 0$ for f low, $\Delta n = 1/2$ for $f = 5\times10^8$ Hz, $\Delta n = 1$ for $f = 10^9$ Hz, and so on. Thus, changing the commanding frequency in this range, one should change the pictures registered by both observers from "equal" to "opposite", again to "equal", and so on. In this manner the linear rotational velocity of the Earth's equator can be measured using the direction dependence of light velocity along a straight line.

It can be easily seen that if the resonator is put on a parallel with latitude ϕ and the coupled shutters along a parallel with latitude ϕ_0, then

$$\Delta n = 2\,\frac{\Omega R}{c^2}\,d\,f(\cos\phi_0 - \cos\phi), \qquad (62.2)$$

where Ω is the Earth's angular velocity.

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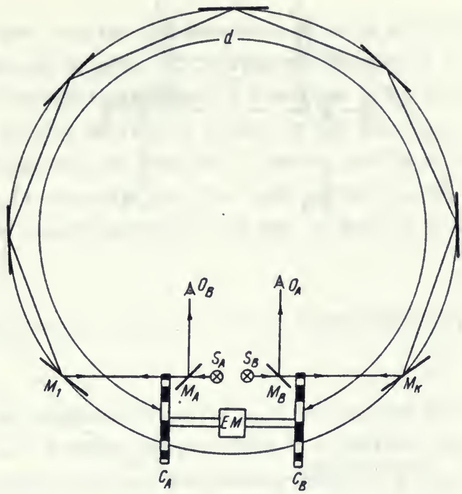

Fig. 62.2

Let us now suppose that our coupled shutters represent two cog-wheels fixed on a common shaft which is driven by an electromotor EM and let this shaft be mounted on a rotating disk (see fig. 62.2). Suppose that the distance between the wheels is much less than the circumference of the disk. The direct light beam emitted by the source S_A passes through the semitransparent mirror M_A and, after its chopping by the cog-wheel C_A and reflection by the mirrors $M_1,\ldots\ldots M_k$, arrives at the cog-wheel C_B. If $n = (d/c)f$, where n is an integer, d the path of the light beam between the cog-wheels C_A and C_B, and f the chopping frequency, then this chopped beam will pass through the cog-wheel C_B and, being reflected by the semi-transparent mirror M_B, will be observed by the observer O_A. The same happens with the opposite light beam.

Let the disk be at rest. If we change the frequency of chopping, the observers O_A and O_B will register together maximum and minimum photon fluxes. However, if we set the disk in rotation, the observer O_A will register a maximum photon flux for the integer values of the quantity

$$n_A = f\left(\frac{d}{c} + \int_0^d \vec{v}.d\vec{r}/c^2\right) = f\,\frac{d}{c}\left(1 + \frac{\Omega R}{c}\right), \tag{62.3}$$

where R is the radius of the disk, Ω is the angular velocity of rotation, $\vec{v}$ is the linear velocity of rotation, $d\vec{r}$ is the differential element of the light path, and we have assumed that the mirrors $M_1,\ldots\ldots M_k$ are close to each other, so that we can write $\vec{v}.d\vec{r} = v\,dr$.

The observer O_B will register a maximum photon flux for the integer values of the quantity

$$n_B = f\,\frac{d}{c}\left(1 - \frac{\Omega R}{c}\right). \tag{62.4}$$

Suppose now that the distance d decreases gradually. Formulas (3) and (4), which des-

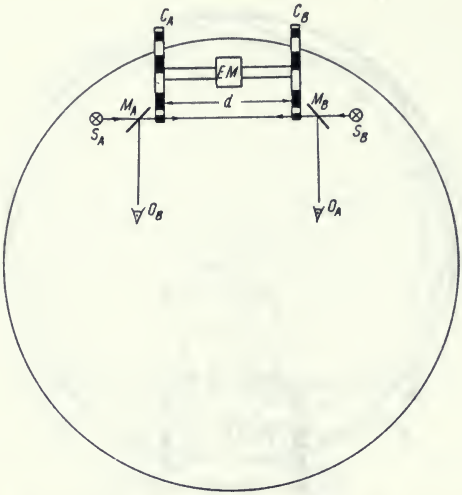

Fig. 62.3

cribe the effect in this interrupted rotating disk experiment, will remain valid. Hence with such a gradually greater and greater interruption we shall come to the situation presented in fig. 62.3, where the paths of both light beams are much smaller than the circomference of the disk. Now, for time intervals $t \ll 2\pi R/v = 2\pi/\Omega$, the motion of the coupled shutters can be considered as inertial, and thus we come to the "coupled shutters" experiment analysed in §50. Since any component of the Earth's absolute motion - the daily rotation about its axis, the yearly rotation around the Sun, the revolution around the galactic center of mass, etc. - represents some rotational motion, it follows that the "coupled shutters" experiment must establish the resultant velocity, which we have measured with our "coupled mirrors" experiment.

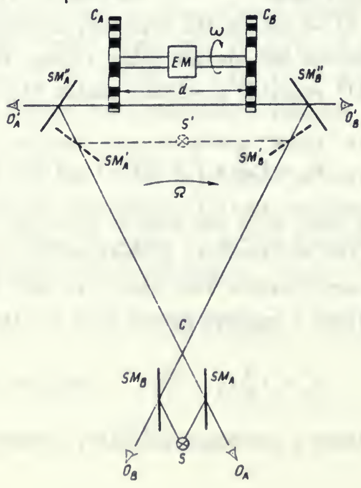

Fig. 62.4

The transition from the "coupled shutters on a rotating disk" experiment to the "coupled shutters" experiment can be made easily also by the help of the scheme shown in fig. 62.4, where all mirrors are semi-transparent. First the light source S is considered at the center of the disk and the anisotropy effect in the velocity of light measured by the observers O_A and O_B, placed also at the disk's center, can be explained thoroughly as a Sagnac effect. However, if we should use the light source S', the observers O_A' and O_B' will measure exactly the same anisotropy effect but now it must be treated as a Marinov effect.

§63. THE SECOND-ORDER EFFECTS IN THE "ROTATING DISK" EXPERIMENT

A. General remarks

The measurement of the second-order effects in the Harress-Dufour and Harress-Fizeau experiments is a difficult problem because there is a relative motion between mirrors and medium. Thus only the Harress-Marinov and Harress-Sagnac experiments are convenient for the measurement of the second-order in v/c effects. Nevertheless, since these effects are very small, such an experiment is difficult for execution. For this reason Marinov (1976) proposed the "ROTATING DISK" EXPERIMENT FOR MEASUREMENT OF THE SECOND-ORDER EFFECTS, without entering into practical details. When calculating the effects we shall not take into account the dispersion of the medium, i.e., we shall suppose $dn/d\lambda = 0$.

The arrangement for measuring second-order effects in the "rotating disk" experiment is shown in fig. 63.1. All elements are rigidly connected with the rotating disk. The light source S, the semi-transparent mirror SM, the mirror M, and the photoresistor P are put in the hollow part of the medium which has the form of a ring with outer radius R. The photoresistor illuminated by interfering light beams is put in one arm of a Wheatsone bridge and in the other arm there is a variable resistor. We assume that the mirrors M_1, M_2,.....M_k are placed near to the rim of the medium's ring and close to each other. Thus we can assume that the photons travel essentially around the circumference of a circle and cover a path $d = 2\pi R$.

Light emitted by the source S is split by the semi-transparent mirror SM into reflected and transmitted beam. The "reflected" beam reflects from mirror M and, after transmission through SM, illuminates P. The "transmitted" beam reflects successively from M_1,.....M_k clockwise, from M_k,.....M_1 counter-clockwise and, after reflection from SM, illuminates P.

When the disk is set into rotation, the "reflected" beam will not change the time it takes to cover its path, because it moves only along the radius of the rotating disk. However, the "transmitted" beam (which has the same radial motion if the distances from SM to M and M_1 are equal!) changes the time it takes to cover its path by the amount Δt. This time for the four different variations of the "rotating disk" experiment is derived below.

B. The Harress-Dufour experiment

Using formula (40.23) for the case $\theta_o = 0$ and writing, according to formula (4.29),

$$c'_m = c'_{om}(1 - v^2/c^2)^{1/2}, \tag{63.1}$$

we find the difference in the absolute times which the "transmitted" beam has to spend covering its path in the cases of rest and rotation

$$\Delta t_{H-D} = \frac{d}{c'^+_m} + \frac{d}{c'^-_m} - \frac{2d}{c_m} = \frac{dv^2}{c^3} n\,(2n^2 - 1). \tag{63.2}$$

C. The Harress-Fizeau experiment

Using formula (40.12) for the case $\theta_o = 0$, we find the difference in the absolute times which the "transmitted" beam has to spend covering its path in the cases of rest and rotation

$$\Delta t_{H-F} = \frac{d}{c'^+_m} + \frac{d}{c'^-_m} - \frac{2d}{c_m} = \frac{2dv^2}{c^3} n\,(n^2 - 1). \tag{63.3}$$

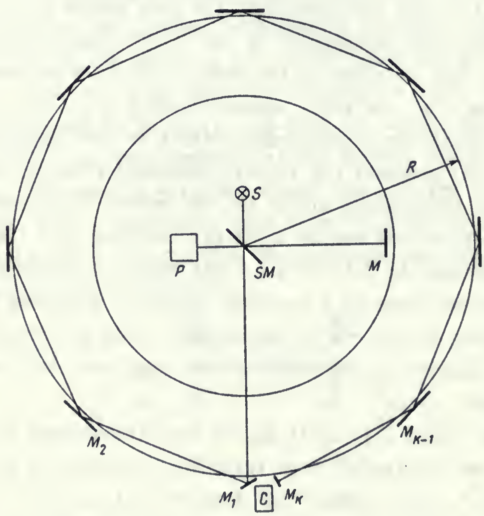

Fig. 63.1

D. The Harress-Marinov experiment

Using formula (40.28) for the case $\theta' = \theta = 0$ and formula (1), we find the difference in the absolute times which the "transmitted" beam has to spend covering its path in the cases of rest and rotation

$$\Delta t_{H-M} = \frac{d}{c'^+_m} + \frac{d}{c'^-_m} - \frac{2d}{c_m} = \frac{dv^2}{c^3} n. \tag{63.4}$$

E. The Harress-Sagnac experiment

For n = 1, i.e., for the second-order effect in the Harress-Sagnac experiment, we obtain from formulas (2) or (4)

$$\Delta t_{H-S} = \frac{dv^2}{c^3}. \tag{63.5}$$

F. Connection with kinematic time dilation

As introduced in §2, a light clock may be constructed using a light source and a mirror placed in front of it, between which a light pulse goes to and fro. Instead of one mirror we can have an arbitrary number. It is of importance only that a light pulse which leaves a given point returns to it and repeats this cycle uninterruptedly. Thus our mirrors $M_1, M_2, \ldots\ldots M_k \ldots\ldots M_2, M_1$ also represent a light clock.

Let the time which a light pulse spends covering path d to and fro be T when the mirrors are at rest. Thus

$$T = 2d/c \tag{63.6}$$

is the period of our clock when at rest. When the disk is set in motion with a rotational velocity of the rim $v = \Omega R$, where Ω is the angular velocity, the period of the light clock in motion measured in absolute time (i.e., by the help of a clock at rest in absolute space) will be, using formula (4.19) for $\theta = \theta' = 0$,

$$T_o = \frac{d}{c'^+} + \frac{d}{c'^-} = \frac{2d}{c(1 - v^2/c^2)^{1/2}} = \frac{T}{(1 - v^2/c^2)^{1/2}}. \tag{63.7}$$

The same period measured in proper time (i.e., by the help of a clock which is attached to the rim of the moving disk) will be, using formula (4.28) for $\theta = \theta' = 0$,

$$T_{oo} = \frac{d}{c_o'^+} + \frac{d}{c_o'^-} = \frac{2d}{c} = T. \tag{63.8}$$

Thus, the period of our light clock rotating with velocity v in absolute space, as well as the period of any light clock proceeding as a whole with velocity v, becomes longer according to formula (7). We have called this effect the absolute kinematic time dilation (see §7).

According to our tenth axiom, the time unit for any observer is determined by the period of a light clock which has the same "arm" for all observers. When the "arm" is d = 150,000 km, this time unit is called a second. If the observer is at rest in absolute space, his second is called universal. If the observer moves with a certain velocity in absolute space, his second is called proper. Any proper second is larger than the universal second and the relation is given by formula (7), where the durations of T and T_o are given in the same time. Thus the change in the duration of the period of a light clock when in motion can be established by comparing its period with a periodical process of a system which is at rest in absolute space (in general, one that does not change its velocity when the light clock under investigation changes its velocity). If we shoudl compare the period of the light clock considered with the periodical process of a system which constantly moves with the same velocity as the light clock, then no change can be registered, as follows from formula (8), where the period of the moving clock is given in proper time and the period of the same clock at rest in universal time.

All these assertions of our absolute space-time theory can be verified experimentally if one measures the second order effects in the Harress-Sagnac experiment.

The second-order effects in the Harress-Sagnac experiment have been treated by Burcev (1974) who also proposed an experiment for their measurement. Burcev's proposal consists in the following: Let us have a number (≥ 3) of artificail satellites moving along the same circular trajectory round the Earth with a certain velocity v. If a radar pulse is emitted from one of the satellites, then, by means of reflections in the other satellites, this radar pulse can be again received after having covered a closed path round the Earth and the time of delay can be measured with a high precision. If we suppose that the satellites are placed close enough to each other, the trajectory of the radar wave can be assumed as circular and the gravitational potential at all points crossed by the wave as equal. We can treat Burcev's proposal by considering our figure 63.1, assuming that clock C (an atomic clock) is attached to mirrors M_1 and M_k, so that the time in which a light pulse covers the path from M_1 to M_k and from M_k to M_1 can be measured.

According to Einstein's theory of general relativity, this time, for the "direct" (+) and "opposite" (-) pulses, respectively, is

$$t_E^{\pm} = t \frac{1 \pm v/c}{(1 - v^2/c^2)^{1/2}}, \tag{63.9}$$

where $t = d/c = 2\pi R/c$ is the time registered on the same clock if the disk is at rest.

According to the traditional Newtonian aether theory, this time is

$$t_N^{\pm} = \frac{t}{1 \pm v/c}. \tag{63.10}$$

According to our absolute space-time theory, this time is (use formula (4.28))

$$t_M^{\pm} = t_o^{\pm} = d/c_o'^{\pm} = t(1 \pm v/c). \tag{63.11}$$

If the time will be measured on a clock which rests in absolute space, this time is (use formula (4.18))

$$t^{\pm} = d/c'^{\pm} = t \frac{1 \pm v/c}{(1 - v^2/c^2)^{1/2}}. \tag{63.12}$$

When we try to measure the universal time interval $t^{\pm}$ by the help of a clock which rests in absolute space, the problem arises about the time synchronization of spatially separated clocks. This problem is solved by us (theoretically and experimentally) with the help of a rotating shaft (see §50, §51, §52) and with the help of light signals (see

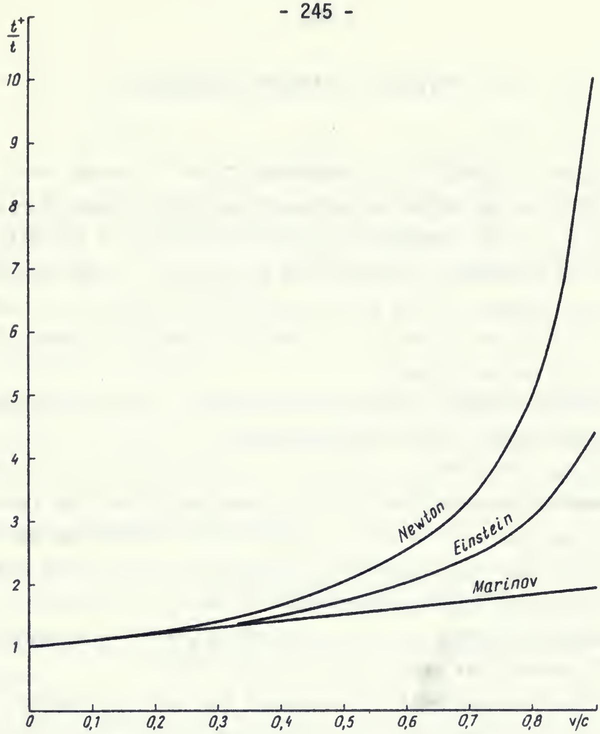

Fig. 63.2

§56). In the "rotating disk" experiment the problem about the synchronization of spatially separated separated clocks can be eliminated if we choose an appropriate velocity v, so that the light pulse emitted by M_1 when it passes near to the clock C which is at rest will arrive at M_k when M_k passes near to C. Thus, if the short distance between M_1 and M_k is $2\pi R - d$, one must have

$$\frac{d}{c} = \frac{2\pi R - d}{v}. \qquad (63.13)$$

For the meàsrement of the time spent by light to cover the distance d from M_k to M_1, one has to put M_1 to the right from M_k over the same distance $2\pi R - d$.

In fig. 63.2 we give the graphs of the relation t^+/t versus v/c drawn according to formulas (9), (10), and (11). Thus an experiment as that proposed by Burcev can choose between these three rival theories. We think, however, that there is no need to perform this costly experiment, since, in the light of the present book, it is obvious that our formula will correspond to physical reality.

§64. THE "MOVING PLATFORM" EXPERIMENT

A. General remarks

The "MOVING PLATFORM" EXPERIMENT is an analogue of the "rotating disk" experiment, where the motion of the medium or/and the mirrors (the interferometer) is not rotational but inertial. Now, again, four combinations can be realized which we call:

1. The ZEEMAN-FIZEAU EXPERIMENT, in which the mirrors are at rest and the medium moves. This experiment was performed first by Fizeau (1851) with water and by Michelson and Morley (1886) with a solid medium. It was very carefully repeated by Zeeman (1914, 1915, 1920, 1922) with liquid and solid media.

2. The ZEEMAN-DUFOUR EXPERIMENT, in which the medium is at rest and the mirrors move. This experiment was performed first by Marinov (1977).

3. The ZEEMAN-MARINOV EXPERIMENT, in which mirrors and medium move together. This experiment was performed by Marinov (1977) and, as a matter of fact, it can be carried out by anyone who would take the care to observe whether the interference picture in a Zeeman-type implement, in which mirrors and medium are at rest, should change during a day when the absolute velocity of the implement changes as a result of the Earth's rotation.

4. The ZEEMAN-SAGNAC EXPERIMENT, in which the mirrors move and a vacuum (air) is taken as a medium. This experiment was performed by Marinov (1977).

The nomenclature for the different variations of the "moving platform" experiment is similar to that of the "rotating disk" experiment (see p. 224). The first name "Zeeman" designates that this is a "moving platform" experiment, as Zeeman has performed (after Fizeau and Michelson-Morley) during a period of many years the most accurate measurements of the convection of light by liquid and bulk materials. The second name designates the kind of the effect when observer and medium move with respect to one another or together with respect to absolute space.

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Let us reduce the "moving platform" experiment, stripped of all fundamentally irrelevant details, to the following ideal arrangement (see fig. 64.1): The box S contains a monochromatic light source together with a device producing two parallel coherent beams - B_v which propagates in vacuum and B_m which propagates in a medium with refractive index n. After travelling a distance L, they enter a second box O in which they are united and their interference observed. Both boxes are halfway immersed in the medium. First the boxes (also called mirrors) and the medium are at rest and a specific interference picture is observed. Then one realizes the four different combinations mentioned above, and from the difference in the observed interference patterns conclusions can be drawn about the character of light propagation. We suppose that the motion of the medium, or of the boxes, or both, proceeds from left to right.

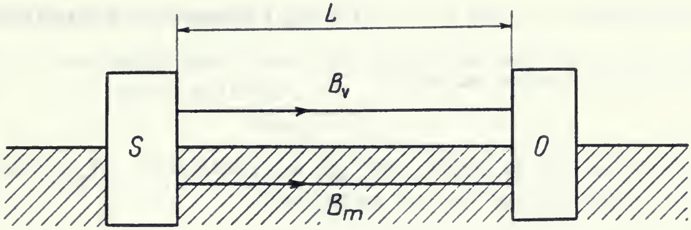

Fig. 64.1

B. The Zeeman-Fizeau experiment

In §57A we have obtained the formula for the Zeeman-Fizeau (i.e., for the "water tube") experiment (see formula (57.5)). For methodological reasons we shall again deduce this formula.

The velocity of light in a medium moving with velocity v, if measured by an obsever at rest, is (see formulas (40.16) and (57.4))

$$c_m = \frac{c}{n} + v\left(1 + \frac{\nu}{n^2}\frac{dn}{d\nu} - \frac{1}{n^2}\right)\cos\theta, \tag{64.1}$$

where ν is the frequency of the light used and θ is the angle between v and the direction of light propagation.

When mirrors and medium in fig. 64.1 are at rest, a photon proceeding along the path B_m (a B_m-photon) will arrive at box O with the following time delay after a photon proceeding along the path B_v (a B_v-photon)

$$t = \frac{L}{c/n} - \frac{L}{c} = \frac{L}{c}(n - 1). \tag{64.2}$$

When the medium is set in motion, the time delay becomes (in fig. 28.1 $\theta = 0$)

$$t_{Z-F} = \frac{L}{c_m} - \frac{L}{c} = \frac{L}{c}\{n - 1 - \frac{v}{c}(n^2 - \nu\frac{dn}{d\nu} - 1)\cos\theta\}. \tag{64.3}$$

Hence, for the Zeeman-Fizeau experiment, the effect to be observed in the interference picture will correspond to a time difference

$$\Delta t_{Z-F} = t - t_{Z-F} = \frac{Lv}{c^2}(n^2 + \nu\frac{dn}{d\nu} - 1)\cos\theta. \tag{64.4}$$

C. The Zeeman-Dufour experiment

The velocity of light measured by an observer who moves with the velocity v with respect to a medium at rest in absolute space is (use formula (40.26), taking into account formula (59.3))

$$c'_m = \frac{c}{n} - v\left(\frac{\nu}{n^2}\frac{dn}{d\nu} + 1\right)\cos\theta, \tag{64.5}$$

where ν is the frequency of light and θ is the angle between v and the direction of light propagation.

For n = 1, i.e., for vacuum, we obtain

$$c' = c - v\cos\theta. \qquad (64.6)$$

When the mirrors are set in motion, the time delay with which a B_m-photon arrives at box 0 after a B_v-photon becomes (in fig. 64.1 $\theta = 0$)

$$t_{Z-D} = \frac{L}{c'_m} - \frac{L}{c'} = \frac{L}{c}\{n - 1 + \frac{v}{c}(n^2 + \nu\frac{dn}{d\nu} - 1)\cos\theta\}. \qquad (64.7)$$

Hence, for the Zeeman-Dufour experiment, the effect to be observed in the interference pattern will correspond to a time difference

$$\Delta t_{Z-D} = t - t_{Z-D} = -\frac{Lv}{c^2}(n^2 + \nu\frac{dn}{d\nu} - 1)\cos\theta. \qquad (64.8)$$

Although the mathematical expressions for the effects produced by the Zeeman-Fizeau and Zeeman-Dufour experiments are exactly the same (cf. (4) and (8)), these two experiments are not equivalent physically, and the formulas used to obtain these identical results are different. According to the principle of relativity, no distinction can be made between the Zeeman-Fizeau and Zeeman-Dufour experiments, and special relativity considers the latter only as trivial tautology of the former.

With the aim of demonstrating that these two experiments are physically different, we performed them using the variation considered in §64F, where the identical results can be explained only if the formulas used for their calculation are different.

D. The Zeeman-Marinov experiment

The velocity of light in a medium moving with velocity v, if measured by an observer who moves with the same velocity is (see formula (40.28))

$$c'_m = \frac{c}{n} - \frac{v}{n^2}\cos\theta, \qquad (64.9)$$

where θ is the angle between v and the direction of light propagation.

When the mirrors are set in motion together with the medium, the time delay with which a B_m-photon arrives at box 0 after a B_v-photon is

$$t_{Z-M} = \frac{L}{c'_m} - \frac{L}{c'_m} = \frac{L}{c}(n - 1). \qquad (64.10)$$

Hence, for the Zeeman-Marinov experiment, the effect to be observed in the interference pattern will correspond to a zero time difference

$$\Delta t_{Z-M} = t - t_{Z-M} = 0, \qquad (64.11)$$

and thus no change can be registered.

E. The Zeeman-Sagnac experiment

The effect for the Zeeman-Sagnac experiment can be obtained immediately from formula (8), putting n = 1, or from formula (11), i.e.,

$$\Delta t_{Z-S} = 0. \qquad (64.12)$$

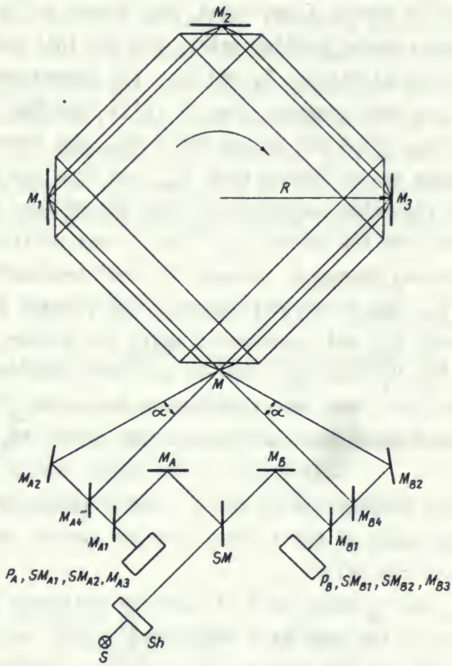

Fig. 64.2

F. The non-inertial "moving platform" experiment

The scheme of the "moving platform" experiment performed by Marinov (1977) is shown in fig. 64.2. We call this variation of the Zeeman experiment the NON-INERTIAL "MOVING PLATFORM" EXPERIMENT, while the variation shown in fig. 64.1 can be called the inertial "moving platform" experiment. The differences will become clear after giving the description of our set-up.

S was a light source. The shutter Sh, which is governed by the rotating turnabout, let light pulses (of a duration $\cong 10^{-6}$ sec) pass only when the mirrors M_1, M_2, M_3 were parallel to the diametrically opposite small sides of the medium. For a medium we used distilled water put in a metallic vessel of the form shown in fig. 64.2 (the same as that used in the "rotating disk" experiment - see §59). Glass windows were placed at points where the light beams must cross the walls. Glass windows were placed also in the metallic interfaces which divide the ring in compartments. Taking into account the thickness of the glass plates and their refractive index, we put the mirrors M_1, M_2, M_3 in such positions that the real light paths (distance multiplied by refractive index) along the contour $M - M_1 - M_2 - M_3 - M$ exactly equaled to the light path which would have been covered if mirrors M_1, M_2, M_3 had been immersed in water. The distance between mirrors M_1 and M_3, as well as between mirror M_2 and point M, was 2R.

The light beam emitted by source S was split into A-beam and B-beam at semi-transparent mirror SM. These two beams covered similar paths, and for this reason we shall follow only the A-beam. After reflection at mirrors M_A and M_{A1}, the A-beam was reflected by the semi-transparent mirror SM_{A1} and went upwards. Then it split into two daughter beams at the semi-transparent mirror SM_{A2} which was placed above SM_{A1} and their planes made a right angle. The A'-daughter beam was reflected from SM_{A2} and from the mirror M_{A2}, passed through the material medium, was reflected successively from the mirrors M_3, M_2, M_1 and, leaving the medium, was reflected from the mirror M_{B2}. Then it was reflected from the semi-transparent mirror SM_{B2} and, going downwards through the semi-transparent mirror SM_{B1}, illuminated the photoresistor P_B. The A"-daughter-beam passed through SM_{A2}, went further upwards, was reflected from M_{A3} and M_{A4} and, proceeding above the medium, was reflected successively from the mirrors M_3, M_2, M_1, and M_{B4}. Then it was reflected from the mirror M_{B3} and, going downwards through the semi-transparent mirror SM_{B2} (where it interfered with the A'-daughter-beam) and through the semi-transparent mirror SM_{B1}, illuminated the photoresistor P_B.

The angle α between the projections of the A'- and A"-daughter-beams in the plane of the figure, before the entrance of the A'-beam into the medium and after its exit, can be determined by the Snell law (41.4).

The photoresistors P_A and P_B which were illuminated uniformly by the interfering B- and A-daughter beams were put in the arms of a Wheatstone bridge, which is described in more detail in §52B and §57B. We did not search for the highest sensitivity with the help of a "tuner", as described in §57B; we assumed an average sensitivity $\delta\Delta = \pm 10^{-2}\lambda$.

The index of refraction for water taken from Landolt-Börnstein (1962) was $n = 1.3317 \pm 0.0003$ for $\lambda = 632.8$ nm of the He-Ne laser used. The error δn corresponded to a change in the refractive index with the temperature which was maintained at $T = 20^0 \pm 3^0$ C, since it is $dn/dT = 10^{-4}$ degree^{-1}. From the same source we have taken $dn/d\lambda = -2.7\times10^{-5}$ nm^{-1}, assuming $\delta(dn/d\lambda) = 0$. We had $R = 30.6 \pm 0.2$ cm. We measured $N_{Z-F} = 50.80 \pm 0.04$ rev/sec for the Zeeman-Fizeau experiment and $N_{Z-D} = 50.94 \pm 0.04$ for the Zeeman-Dufour experiment, having $N = \Omega/2\pi$. Putting these figures into formulas (4) and (8), and taking into account that it is

$$L = 4\sqrt{2}\,R, \qquad v = \frac{\sqrt{2}}{2}\frac{\Omega R}{\cos\theta}, \tag{64.13}$$

we obtained, supposing that the velocity of light is unknown

$$c_{Z-F} = (3.01 \pm 0.07)\times10^8 \text{ m/sec}, \qquad c_{Z-D} = (3.02 \pm 0.07)\times10^8 \text{ m/sec}, \tag{64.14}$$

where $\delta c = \pm 7\times10^{-6}$ m/sec was the maximum error.

In the Zeeman-Marinov and Zeeman-Sagnac experiments we registered no perceptible disequilibration of the bridge when rotating the disk.

The measuring procedure in the "rotating disk" experiment (see §57F) is very similar to that in the non-inertial "moving platform" experiment. We performed the "moving platform" experiment in its non-inertial variation, intending to use the set-up for our "rotating disk" experiment. On the other hand, we think that the results of the non-inertial variation are more important than the same results which could be obtained with the inertial variation. Indeed, since we already know that the velocity of light with respect to a rotating disk is direction dependent, the identical effects in the Zeeman-Fizeau and Zeeman-Dufour experiments can be explained only by our theory where these identical results are derived by proceeding from substantially different formulas.

§65. THE LIGHT DOPPLER-EFFECT EXPERIMENTS

A. The Ives-Stilwell longitudinal "canal ray" experiment

The second-order in v/c term in formula (38.5) was first verified experimentally by Ives and Stilwell (1938) who used light emitted by the moving ions in a canal ray tube; we called this the LONGITUDINAL "CANAL RAY" EXPERIMENT.

The arrangement of their experiment in a simplified version is indicated in fig. 65.1.

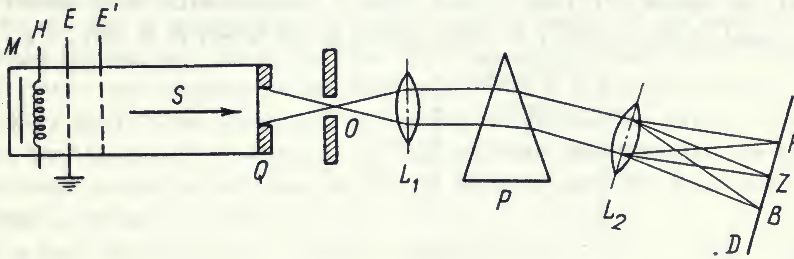

Fig. 65.1

Positive ions were produced in a hydrogen arc between the heater H and the perforated electrodes E and E'. Between E and E' the ions were acclerated by the electric field to form the beam S that represented the moving source. These ions proceeded with a constant velocity v which depended on the voltage applied between E and E'. The photons emitted by the excited ions, passing through the large slit Q, illuminated the narrow slit O which represented the observer at rest. To analyse the energies (i.e., the frequencies) of the photons, a spectroscope was used, of which we show the focusing lenses L_1, L_2, the refractive prism P and the screen D.

The mirror M reflected the light emitted by the ions which moved away from it with velocity v. Thus three groups of photons flowed bo the screen:

a) Photons with frequency ν emitted by the ions at rest, before being acclereated by the electrodes; they illuminated the zero point Z.

b) Photons with frequency ν^+ shifted to the "blue end" which were emitted by the moving ions; they illuminated point B.

c) Photons with frequency ν^- shifted to the "red end" which were emitted by the moving ions and then reflected by the mirror M; they illuminated point R.

According to formula (38.5), under the condition $\theta' = \theta = 0$,

$$\nu^{\pm} = \nu(1 \pm \frac{v}{c} + \frac{1}{2}\frac{v^2}{c^2}). \qquad (65.1)$$

Thus the middle of these two frequencies will be shifted from the frequency ν over a frequency interval

$$\Delta\nu = \frac{\nu^+ + \nu^-}{2} = \frac{1}{2}\nu\frac{v^2}{c^2}, \qquad (65.2)$$

that was experimentally verified by Ives and Stilwell.

B. The transverse "canal ray" experiment

The TRANSVERSE "CANAL RAY" EXPERIMENT has been proposed by Marinov (1970, 1977). Its scheme is shown in fig. 65.2. The system for production and acceleration of the ions is as in fig. 65.1. The photons emitted by the excited ions, passing through the large slit Q, illuminate the narrow slit O behind which there is a monochromator which detects the frequency ν equal to the frequency of light emitted by the molecules at rest, i.e., an in-

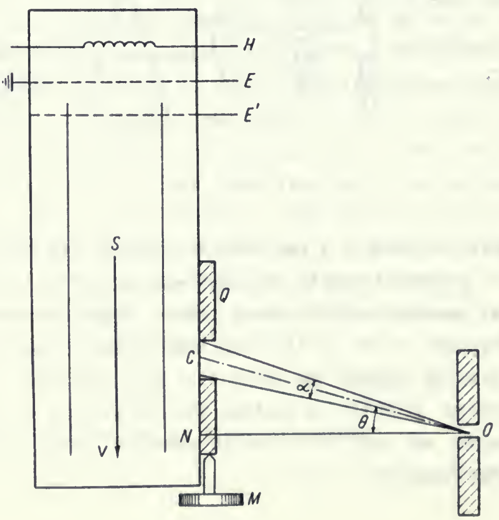

Fig. 65.2

dicator that gives indication only when photons with frequency ν are incident.

From fig. 65.2 and from the first formula (38.5) it may be seen that the moving ions will emit photons in the direction to illuminate O, the received frequency of which is

$$\nu_0 = \frac{\nu(1 - v^2/c^2)^{1/2}}{1 + \frac{v}{c}\cos(\frac{\pi}{2} + \theta \pm \frac{\alpha}{2})} = \nu\{1 + \frac{v}{c}(\theta \pm \frac{\alpha}{2}) - \frac{v^2}{2c^2}\}, \qquad (65.3)$$

where θ is the angle between the perpendicular ON to the ions' beam and the line OC connecting slit O with the center of slit Q; α is the angle under which slit Q is seen from point O. If we choose $\alpha \ll \theta$, then frequency ν will be observed only when

$$\theta = v/2c. \qquad (65.4)$$

Hence the experiment is to be performed as follows: For any voltage applied to the electrodes, i.e., for any velocity v of the ions, we search for a position of slit Q which will yield photons with the frequency ν. Then the theory can be verified plotting $2\theta c$ versus v; the locus must be a straight line dividing the quadrant.

C. The Hay "rotor" experiment

The "ROTOR" EXPERIMENT, performed first by Hay et al. (1960) using the Mössbauer effect, is diagrammed in fig. 65.3 (see also §36):

A radioactive ^{57}Co source was put on a rotating disk at a distance R from the center of rotation C. A thin ^{57}Fe absorber, representing the observer, was put around the circumference of the rotating disk at a distance R_0 from the center of rotation. A detector D at rest was used to measure the rate of the γ-photons emitted by the source which passed through the absorber. The transmission of the absorber was measured for various angular velocities. This transmission was found to increase as the angular velocity increased, indicating a shift in the characteristic frequency of the absorber relative to the source.

Since the line shape of the absorber at rest was known experimentally, the magnitude of the frequency could be estimated, and it was found to agree with the frequency shift calculated according to formula (38.17).

Let us make the calculation. From the triangles OPC and S'PC in fig. 65.3 we obtain the relation

$$R_0\cos\theta_0 = - R\cos\theta'. \qquad (65.5)$$

Now substituting

$$v = \Omega R, \qquad v_0 = \Omega R_0, \qquad (65.6)$$

where Ω is the angular velocity of rotation, into the first formula (38.17) and keeping in mind (5), we obtain the relation

$$\nu_0 = \nu\left(\frac{1 - v^2/c^2}{1 - v_0^2/c^2}\right)^{1/2}, \qquad (65.7)$$

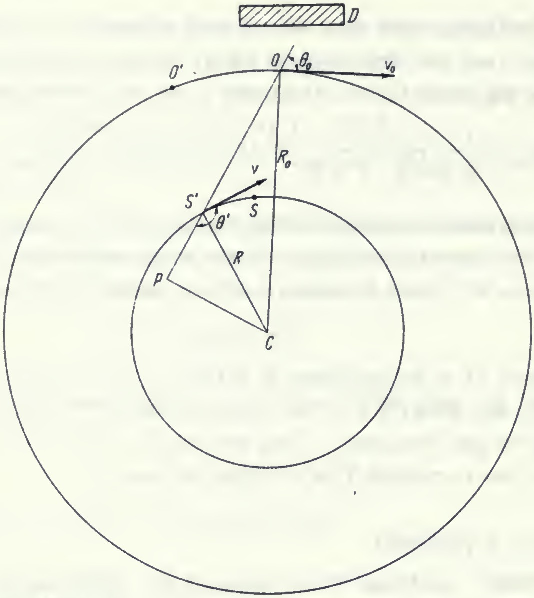

Fig. 65.3

wich was verified experimentally.

We emphasize that formula (7) is valid for any position of source and observer on the circumferences with radii R and R_o.

D. The "rotor-rotor" experiment

The "ROTOR-ROTOR" EXPERIMENT, proposed by Marinov (1977) and, to a certain extent, already carried out by Champeney et al. (1963), is diagrammed in fig. 65.4.

This experiment can be realized when the center of the rotor considered in §65C (which we shall call the "small rotor") rotates at angular velocity Ω and linear velocity v_o with respect to some center, thus making another "large rotor".The radii of the small and large rotors are denoted by r and R. The angular velocity of rotation of the small rotor about its own center is denoted by ω. It is assumed that a light source is placed at the rim of the small rotor and an observer at its center. If the linear velocity of rotation of the source on the small rotor is v_r, its absolute velocity will be

$$\vec{v} = \vec{v}_r + \vec{v}_o. \tag{65.8}$$

The angle between $\vec{v}_r$ and $\vec{v}$ is denoted by ψ. The angle between $\vec{R}$ and $\vec{r}$ is denoted by ϕ. The small angle between the observer's radii at the emission and reception moments is denoted by α, and the small angle under which the emission and reception positions of the observer are seen from the emission position of the source is denoted by β.

We have from fig. 65.4 (see also fig. 38.2)

$$\theta' = \frac{\pi}{2} - \psi + \beta, \qquad \theta_0 = \frac{\pi}{2} + \phi - \alpha - \beta, \tag{65.9}$$

where

$$\alpha = \frac{v_0 r}{cR}, \qquad \beta = \frac{v_0}{c}\cos\phi. \tag{65.10}$$

Putting (10) into (9) and taking into account that α and β are small quantities, we can write

$$\cos\theta' = \sin\psi\cos\beta - \cos\psi\sin\beta = \sin\psi - \frac{v_0}{c}\cos\phi\cos\psi, \tag{65.11}$$

$$\cos\theta_0 = -\sin\phi\cos(\alpha+\beta) + \cos\phi\sin(\alpha+\beta) = -\sin\phi + \frac{v_0}{c}\left(\frac{r}{R} + \cos\phi\right)\cos\phi. \tag{65.12}$$

From the figure we further obtain

$$\sin\psi = \frac{v_0\sin\phi}{v}, \qquad \cos\psi = \frac{v_r + v_0\cos\phi}{v}, \tag{65.13}$$

from which we get

$$v^2 = v_0^2 + v_r^2 + 2v_0 v_r\cos\phi. \tag{65.14}$$

Using the last four equations in the first of equations (38.17) and working to within an accuracy of second order in "velocity/c", we obtain the relation

$$\nu_0 = \nu\left(1 - \frac{v_r^2}{2c^2} - \frac{v_0^2}{c^2}\frac{r}{R}\cos\phi\right). \tag{65.15}$$

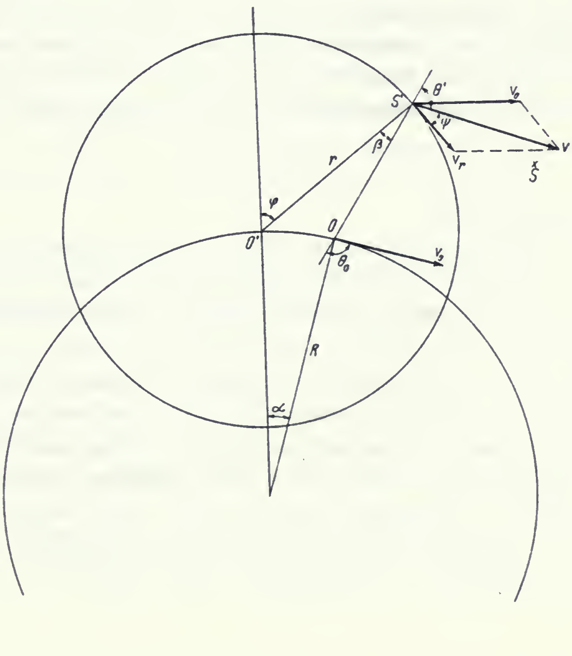

Fig. 65.4

Noting that

$$\frac{r\omega}{v_r} = \frac{R\Omega}{v_o}, \qquad (65.16)$$

we can write (15) in the form

$$\nu_o = \nu\left(1 - \frac{v_r^2}{2c^2} - \frac{v_r v_o}{c^2}\frac{\Omega}{\omega}\cos\phi\right). \qquad (65.17)$$

This result may then be confirmed with experiment.

Now we shall show that formula (17) is already checked to a certain degree by experiment. Indeed, if we suppose $\Omega \ll \omega$, then equation (17) shows that with the help of the "rotor" experiment one cannot register the absolute translational velocity v. Champeney et al. (1963) tried to register absolute effects using the "rotor" experiment, since, according to Newtonian theory, a rotor moving with the absolute translation velocity v_o should yield an effect given by formula (17) with $\Omega/\omega = 1$. The aim of Champeney was to measure the Earth's rotational velocity (which is 310 m/sec on the 45^0 parallel). He claimed to have shown that v_o must be less than 1.6 ± 2.8 m/sec, and this result was treated as a new and better (in comparison to the accuracy of the historical Michelson-Morley experiment) verification of the Einstein principle of relativity.

It is evident that this claim is not warranted. When we analyse Champeney's experiment using our formula (17), we see that, if $\Omega = 1.15\times10^{-5}$ rad/sec (the Earth's diurnal angular velocity), $\omega = 1.15\times10^{3}$ rad/sec (the rotor's angular velocity), $v_o = 310$ m/sec, then $(\Omega/\omega)v_o = 3.1\times10^{-6}$ m/sec. This effect is lower than the accuracy of Champeney's experiment by about six orders. Since in nature all motions of the celestial bodies are rotational, then (at least theoretically!) we can establish any such motion using the "rotor" experiment, i.e., the "rotor-rotor" experiment, where the large rotor represents the rotation of the celestial body (about its rotational axis, about the primary, or about the galactic center).

D. The Santos experiment

In the "rotor" experiment there is no relative motion between source and observer. With the aim of realizing a transverse Doppler-effect experiment where source and observer have to move with respect to one another, Santos (1976) proposed the following experiment:

Let us consider two disks rotating in opposite directions: (i) about two parallel axes, so that the disks lie in the same plane, or (ii) about the same axis, so that the disks lie in two parallel planes. A γ-ray emitter is placed at the rim of the one disk and a γ-ray absorber at the rim of the other, the linear rotational velocity of each being v. At the moment when their velocities are anti-parallel, their relative velocity will be 2v.

The result of this experiment can be found from formula (38.17), putting there $\theta' = \pi/2$, $\theta_o = \pi/2$. Within an accuracy of second order in v/c we obtain

$$\nu_o = \nu\{1 - (v^2 - v_o^2)/2c^2\}, \qquad (65.18)$$

and for $v = v_o$ we get $\nu_o = \nu$.

It is very instructive to note that in this transverse Doppler effect experiment, where source and observer are moving with equal velocities, the result is the same at anti-parallel and parallel directions of the velocities.

Because of the inevitable appearance of first-order in v/c effects, Santos' experiment can not be realized in fact. In order to perform this experiment, one would have to put a shielding of length d and aperture b between the rotating disks. Assuming, for simplicity's sake, that the trajectories of emitter and absorber are rectilinear and that the shielding is exactly perpendicular to them, we shall have for the different emitting and receiving atoms (see fig. 38.2)

$$\theta' = \pi/2 \pm b/d, \qquad \theta_o = \pi/2 \pm b/d, \qquad (65.19)$$

since the emitter and absorber are not point objects.

Substituting this result into equation (38.17) and assuming $v = v_o$, we obtain

$$\Delta\nu = \nu_o - \nu = \pm\, \nu v b/cd. \qquad (65.20)$$

Santos predicts, using the Einstein theory of relativity, that this experiment should give a positive result $|\Delta\nu| = 2\nu v^2/c^2$. Thus the requirement $b/d < v/c$ is to be satisfied. Supposing $v = 300$ m/sec, $d = 10$ cm, we obtain $b < 10^{-5}$ cm. Obviously, such an experiment cannot be practically realized.

It is worth noting that the shielding plays a very important role in Santos' experiment. If this shielding is at rest in the laboratory, being perpendicular to the trajectories of emitter and absorber, the experiment should yield a null (traverse) effect. If the shielding is attached to the absorber, there will be a post-traverse Doppler effect, and we shall have $\nu_o = \nu(1 - 2v^2/c^2)$. If the shielding is attached to the emitter, there will be an ante-traverse Doppler effect, and we shall have $\nu_o = \nu(1 + 2v^2/c^2)$.

§66. THE QUASI-WIENER "STANDING WAVES" EXPERIMENT

The "STANDING WAVES" EXPERIMENT, considered by Marinov (1977), represents a modification of the historical Wiener experiment with which Wiener (1890) measured the length of light waves in the most direct manner. Its essence is as follows:

Let a light source and an ideal mirror be placed on the x-axis of a frame K. If this frame is at rest in absolute space (or its absolute velocity is perpendicular to the x-axis), the electric intensities of the light waves incident and reflected by the mirrors will be

$$E^+ = E_{max}\sin(\omega t + kx), \qquad E^- = E_{max}\sin(\omega t - kx), \qquad (66.1)$$

where E_{max} is the amplitude of the electric intensity, $\omega = 2\pi\nu$ is the angular frequency, and $k = 2\pi\tilde{k}$ is the angular wave-number. The time t is registered on a clock attached to frame K, i.e., on an absolute clock if frame K is at rest or an a proper clock if K moves with velocity v, omitting in the last case, for brevity's sake, the subscript "o"; x is the distance from the frame's origin where at the zero initial moment the electric intensities of the incident (+) and reflected (-) waves are equal to zero.

The incident and reflected light waves will interfere. For the electric intensity of the produced standing waves we obtain

$$E = E^{+} + E^{-} = 2E_{max}\sin(\omega t)\cos(kx). \qquad (66.2)$$

Suppose that the frame K is now set into motion with the velocity v in the x-direction (or that we rotate the moving frame K so that its velocity v becomes parallel to the x-axis). Instead of ω and k in equation (1) we now have to write the quantities (see formulas (38.19) and (38.21))

$$\omega_o^{\pm} = \omega, \qquad k_o^{\pm} = 2\pi/\lambda_o^{\pm} = k(1 \pm v/c), \qquad (66.3)$$

and the electric intensity from (2) becomes

$$E = E^{+} + E^{-} = 2E_{max}\sin\{\omega(t + vx/c^2)\}\cos(kx). \qquad (66.4)$$

Hence the distances between the nodes of the standing waves when the Wiener experiment is performed at rest and in motion with respect to absolute space will be <u>exactly the same</u>, and <u>no</u> first- or second-order differences in the pattern can be registered. The unique difference is: When the laboratory is at rest in absolute space (or its velocity is perpendicular to the direction of light propagation), E obtains its maximum at all antinodes (i.e., for $x = n\pi/k$, where n is an integer) <u>at the same moment</u>, and when the velocity of the laboratory is parallel to the direction of light propagation, E obtains its maximum at the different antinodes <u>at different moments</u>. For a given moment t, the electric intensity in (4) obtains its maximum at the antinodes with coordinates near to

$$x = \left(\frac{2n + 1}{\omega}\frac{\pi}{2} - t\right)\frac{c^2}{v}, \qquad (66.5)$$

while for this moment t it is zero at the antinodes with coordinates near to

$$x = \left(\frac{n\pi}{\omega} - t\right)\frac{c^2}{v}. \qquad (66.6)$$

This is the unique effect which is offered by the quasi-Wiener experiment. We are sceptical about the possibilities for its experimental verification.

It may be pointed that the results of the historical Michelson-Morley experiment show that the quasi-Wiener experiment should not reveal any second-order effect in v/c. Indeed, if the standing waves were to have different lengths (within terms of second order in v/c) in the two cases when the pattern is parallel and when it is perpendicular to the absolute velocity, different numbers of wavelengths would be placed in the Michelson-Morley experiment between the semi-transparent mirror and the two mirrors placed at equal distances from it in parallel and perpendicular directions to the absolute motion. This would lead to a positive result in the Michelson-Morley experiment which has not been observed.

§67. THE "COHERENT LASERS" EXPERIMENT

A. General remarks

As is well known, the coherency of light emitted by lasers is much higher than the coherency of light emitted by other sources. The coherency of a laser beam can be maintained even over hours of time, while the coherency of ordinary light sources is only about 10^{-7} sec. For this reason, light emitted by two different lasers can be made to interfere with each other in the same manner as two beams interfere which were derived by splitting from a single ordinary light source. Before the invention of the laser, a single light source was always used in all optical experiments to search for an "aether wind". The inevitable result was that in all "inertial" experiments (i.e., those involving uniform linear motion), light beams had to cover the paths "out and back". The first-order effects in v/c were, thus, wiped out in the final result (as we have shown several times in this chapter, no second-order effects can appear either). The unique interferometric experiment in which first-order effects in v/c have been observed (excluding our own experimental activity!) represents the "rotating disk" experiment where the equipment does not move inertially.

However, if two different coherent light sources are used, then equipments involving first-order effects in v/c become possible when the equipments perform inertial motion. This was the intention of Carnahan (1962) who proposed the "COHERENT LASERS" EXPERIMENT and of Cialdea (1972) who carried it out.

B. The inertial "coherent lasers" experiment

The arrangement of the INERTIAL "COHERENT LASERS" EXPERIMENT is indicated in fig. 67.1: Light emitted from laser L_A (or L_B) is partlty reflected and partly transmitted by the semi-transparent mirror SM_A (SM'_B). The "transmitted" beam proceeding from L_A (L_B) intrerferes with the "reflected" beam from L_A (L_B), after the latter has covered the distance d in "opposite" ("direct") direction and has been further reflected by the semi-transparent mirror SM'_A (SM_B). The photodetector D_A (D_B) measures the result of the interference.

For simplicity it may be assumed that the semi-transparent mirrors SM_A and SM'_A (SM_B and SM'_B) lie at the same point (see fig. 67.2) which we shall call point A (B).

Let us suppose first that the apparatus is at rest in absolute space. Let the electric intensities of the light beams produced by L_A at point A and by L_B at point B be, respectively,

$$E_A = E_{max}\sin(\omega_A t + \alpha_A), \qquad E_B = E_{max}\sin(\omega_B t + \alpha_B), \qquad (67.1)$$

where

$$\omega_A = \omega + \frac{\Delta\omega}{2} = \frac{2\pi c}{\lambda_A} = \frac{2\pi c}{\lambda - \Delta\lambda/2}, \qquad \omega_B = \omega - \frac{\Delta\omega}{2} = \frac{2\pi c}{\lambda_B} = \frac{2\pi c}{\lambda + \Delta\lambda/2} \tag{67.2}$$

are the angular frequencies ($0 < \Delta\omega \ll \omega$), λ_A, λ_B are the wavelengths, α_A, α_B are the initial phases, and we have assumed that the amplitudes E_{max} of the two beams are equal.

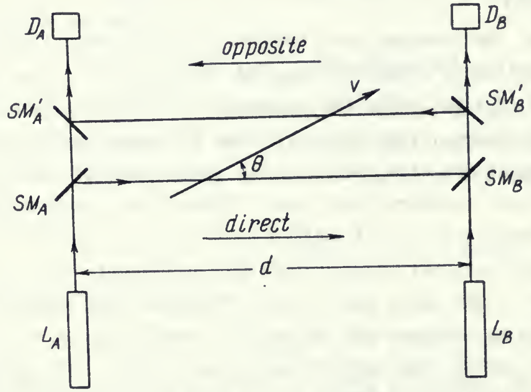

Fig. 67.1

If we wish to find the electric intensities at point A (or point B) after the addition, we have to add E_A and E_B, taking for the latter an additional phase shift $2\pi d/\lambda_B$ (taking for the former an additional phase shift $2\pi d/\lambda_A$).

Let then the apparatus be set in motion with a velocity v such that it makes an angle θ with the "direct" direction of its axis. To find in this case the electric intensity at point A (point B) after the addition, we have to add E_A and E_B, taking for the latter an additional phase shift $2\pi d/\lambda_{Bo}$ (taking for the former an additional phase shift $2\pi d/\lambda_{Ao}$), where λ_{Bo} (λ_{Ao}) is the observed wavelength of the "opposite" ("direct") beam. Thus, writing by t the proper time of a clock attached to the apparatus (for brevity we omit the subscript "o"), we shall obtain for the electric intensities at the points A and B, respectively, (see (38.19) and (38.21))

$$(E_A + E_B)_A = E_{max}\sin(\omega_A t + \alpha_A) + E_{max}\sin(\omega_B t + \alpha_B + \frac{2\pi d}{\lambda_{Bo}}) =$$

$$2E_{max}\sin\left[\frac{1}{2}\{2\omega t + \alpha_A + \alpha_B + \frac{d}{c}(\omega - \frac{\Delta\omega}{2})(1 - \frac{v}{c}\cos\theta)\}\right].$$

$$\cos\left[\frac{1}{2}\{\Delta\omega t + \alpha_A - \alpha_B - \frac{d}{c}(\omega - \frac{\Delta\omega}{2})(1 - \frac{v}{c}\cos\theta)\}\right] =$$

$$2E_{max}\cos(\frac{1}{2}\phi_A)\sin(\omega t + \beta_A) \tag{67.3}$$

and

$$(E_A + E_B)_B = E_{max}\sin(\omega_A + \alpha_A + \frac{2\pi d}{\lambda_{Ao}}) + E_{max}\sin(\omega_B t + \alpha_B) =$$

$$2E_{max}\sin\left[\frac{1}{2}\{2\omega t + \alpha_A + \alpha_B + \frac{d}{c}(\omega + \frac{\Delta\omega}{2})(1 + \frac{v}{c}\cos\theta)\}\right].$$

$$\cos\left[\frac{1}{2}\{\Delta\omega t + \alpha_A - \alpha_B + \frac{d}{c}(\omega + \frac{\Delta\omega}{2})(1 + \frac{v}{c}\cos\theta)\}\right] =$$

$$2E_{max}\cos(\frac{1}{2}\phi_B)\sin(\omega t + \beta_B). \qquad (67.4)$$

Let the photodetectors transform the incident light intensity into an electric potential which we lead to a point (call it point C) in the middle between points A and B. Designate by U_A, U_B the electric potential on the outputs of the detectors D_A, D_B. Since U_A, U_B are proportional to the squares of the variable amplitudes of $(E_A + E_B)_A$ and $(E_A + E_B)_B$, we can write

$$U_A = U_{max}\cos^2(\frac{1}{2}\phi_A) = \frac{1}{2}U_{max}(1 + \cos\phi_A),$$

$$U_B = U_{max}\cos^2(\frac{1}{2}\phi_B) = \frac{1}{2}U_{max}(1 + \cos\phi_B), \qquad (67.5)$$

where U_{max} is the amplitude of the electric potential whose angular frequency is $\Delta\omega$.

Leading the electric potentials U_A, U_B to the middle point C and taking into account the additional phase shifts for U_A and U_B because of the different velocities of propagation of the electromagnetic energy in "direct" and "opposite" directions, we obtain for their sum

$$(U_A + U_B)_C = \frac{1}{2}U_{max}\left[1 + \cos\{\phi_A + \frac{d}{2c}\Delta\omega(1 + \frac{v}{c}\cos\theta)\}\right] +$$

$$\frac{1}{2}U_{max}\left[1 + \cos\{\phi_B + \frac{d}{2c}\Delta\omega(1 - \frac{v}{c}\cos\theta)\}\right] =$$

$$U_{max}\{1 + \cos(\Delta\omega t + \alpha_A - \alpha_B + \frac{d}{c}\Delta\omega + \frac{d}{c^2}\omega v\cos\theta)\cos(\frac{d}{c}\omega)\}. \qquad (67.6)$$

Let us analyse this result. Obviously

$$\cos\frac{2\pi d}{\lambda} = \begin{cases} +1 & \text{for } d = n\lambda, \\ -1 & \text{for } d = (n \pm 1/2)\lambda, \\ 0 & \text{for } d = (n \pm 1/4)\lambda, \end{cases} \qquad (67.7)$$

n being an integer. Thus the "percentage modulation" of the resultant electric potential depends on the number of average wavelengths λ placed along distance d. Hence, to be able to measure a change $v\cos\theta$ in the component of the absolute velocity of the apparatus along its axis, the following two conditions must be met

$$\Delta\omega = 0, \qquad d \neq (n \pm 1/4)\lambda. \qquad (67.8)$$

In such a case, if during a definite time the component of the absolute velocity of the apparatus along its axis changes from 0 to $v\cos\theta$, this will lead to a phase shift in the argument of the resultant electric potential equal to $d\omega v\cos\theta/c^2$ radians, assuming that during this time the initial phases α_A, α_B remain constant.

However, it can easily be established, by taing into account the absolute time dilation (in a manner very similar to that used in §55D), that when the apparatus is rotated with respect to its absolute velocity, i.e., when "switching on an aether wind by rotation", the initial phases of the lasers do not remain constant. They change exactly in such a way that the absolute effect, which a traditional absolutist expects to be registered, will be annihilated.

Thus, with the "coherent lasers" experiment, we can measure only a real change in the velocity of the implement. Such an experiment may be performed as follows: Assuming that the conditions (8) are fulfilled, let us measure some phase α of the electric tension $(U_A + U_B)_C$. If α_A and α_B are known, $v\cos\theta$ can be calculated. However, since the initial phases of the leasers are unknown, we cannot do this. Let us then set the implement in motion with a certain velocity v along the "direct" direction. If the new phase which should be measured will be α', the phase difference

$$\alpha' - \alpha = \frac{d}{c^2}\omega v, \qquad (67.9)$$

will correspond to the following change in the sum of the phase shifts of the electric tensions U_A and U_B

$$(\phi'_A - \phi_A) + (\phi'_B - \phi_B) = \frac{2d}{c^2}\omega v. \qquad (67.10)$$

If $\omega/2\pi = 5\times10^{14}$ Hz, d = 1 m, v = 45 km/sec = 162 km/hr, a phase whift $\alpha' - \alpha = \pi/2$ will be realized.

C. The "coherent lasers on a rotating disk" experiment

To show more clearly why the inertial "coherent lasers" experiment is to be explained in the manner presented in §67B, we shall consider the "COHERENT LASERS ON A ROTATING DISK" EXPERIMENT, proposed by Marinov (1977). Its essence is as follows (see fig. 67.2):

The equipment in fig. 67.1 is mounted on a rotating disk. The electric potentials U_A, U_B are registered on the outputs of the detectors D_A, D_B. Let the first condition (8) be fulfilled. If the disk is first at rest and then set in rotation in a clockwise direction with a linear rotational velocity of its rim v, then the arguments of U_A and U_B will experience additional phase shifts (see formulas (3), (4), (5)) whose sum is given by formula (10). This "coherent lasers on a rotating disk" experiment, as can be seen immediately from fig. 67.2 if both lasers should be replaced by a unique light source S and the mirrors M_A, M_B, is analogous to the "rotating disk" experiment.

The substantial difference between the "rotating disk" experiment and the present "coherent lasers on a rotating disk" experiment consists in the fact that there are two sources emitting coherent light in the latter, while there is only a single light source in

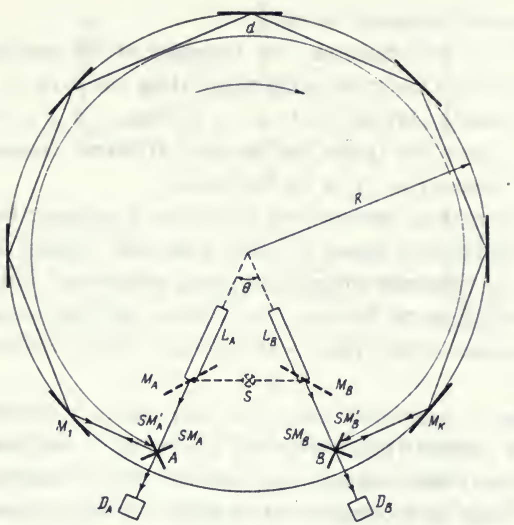

Fig. 67.2

the former. Thus if we should make angle θ in fig. 67.2 almost equal to 2π and should set the source S very near to the rim of the disk, then the "rotating disk" experiment could give no positive effect because the time lags which should appear along the path d would be cancelled out by the opposite time lags which would appear along the paths from S to M_A and M_B. However, the "coherent lasers on a rotating disk" experiment will always (at any angle in fig. 67.2) give the result (10) because the two coherent light sources are spatially separated and, when "switching on an aether wind", i.e., when setting the disk in rotation, the difference in the initial phases of both lasers cannot change because both lasers always move with the same absolute velocity.

Remember that for obtaining a positive effect in the "rotating disk" experiment for any angle θ, the source must be placed at the center of rotation (see §61).

D. The relationship between the "coherent lasers" and light Doppler-effect experiments

If the apparatus indicated in fig. 67.1 is accelerated with a constant acceleration u, the velocity at any time interval t will be v = ut. Putting this into (6) and assuming

$$\theta = 0, \qquad d = n\lambda, \qquad \alpha_A - \alpha_B + \frac{d}{c}\Delta\omega = 2\pi n, \tag{67.11}$$

yields

$$(U_A + U_B)_C = 2U_{max}\cos^2\{\tfrac{1}{2}(\Delta\omega + \Omega)t\}, \tag{67.12}$$

where

$$\Omega = \frac{d}{c^2}\omega u \tag{67.13}$$

represents some additional frequency increase.

Thus, when accelerating the apparatus, the frequency of the resultant electric potential should increase (the acceleration being taken along the axis of the apparatus). Using the data given after formula (10) and letting $t = 100$ sec, i.e., $u = 45$ cm/sec^2, we obtain $\Omega = (\pi/2)10^{-2}$ rad/sec. Hence the lasers can now have different frequencies ω_A, ω_B, and a change in the "beat" frequency $\Delta\omega$ is to be registered.

However, there is no need to perform this particular experiment because this would be essentially only a repetition of Bömmel's (1962) experiment. Bömmel observed the frequency change (13) by using the Mössbauer effect when gamma emitter and absorber were accelerated with $u = 10^6$ m/sec^2. Although the accuracy that can be obtained using lasers is no less than that for the Mössbauer effect ($\delta\omega/\omega = 10^{-12}$), such large accelerations cannot be realized with lasers.

The essence of Bömmel's experiment and of our ACCELERATED "COHERENT LASERS" EXPERIMENT is the same. Since the emitter (say, mirror SM_B' in fig. 67.1) and the receiver (mirror SM_A') are both accelerated, the frequency received will differ from the frequency emitted. Due to the fact that there is a certain time lapse during which light covers the distance d, the velocity of the reciver at the reception moment will be different (higher for u pointing along the emitter-receiver line) from the velocity of the emitter at the emission moment. Einstein (1911) considered this simple and clear phenomenon many years before its experimental confirmation.

The analysis of the "coherent lasers" experiment given here allows one to understand that when the emitter and receiver are accelerated, the shift in the received frequency leads to an additional phase shift which is equal to the product of the frequency shift and the time t of accelerated motion. Thus the number of light waves (wavelengths) placed along the distance between emitter and receiver changes (the number increases for $\vec{c}$ parallel to $\vec{u}$ and decreases for $\vec{c}$ anti-parallel to $\vec{u}$). Hence, as the velocity of light is the product of frequency and wavelength, the light velocity will be different for different velocities of the apparatus (with respect to the apparatus) because the frequency received remains the same.

§68. THE "WIRED PHOTOCELLS" EXPERIMENT

A. The inertial "wired photocells" experiment

The "WIRED PHOTOCELLS" EXPERIMENT was first performed by Godart (1974) who claimed to have observed a positive absolute effect. It was repeated by Marinov (1977) with a firm null result.

The essence of the INERTIAL "WIRED PHOTOCELLS" EXPERIMENT is as follows (see fig. 68.1):

Two photocells P_1, P_2 were put on the opposite sides of a light source S (an electric bulb). The cells and source were mounted on an optical bench and covered with a light-tight

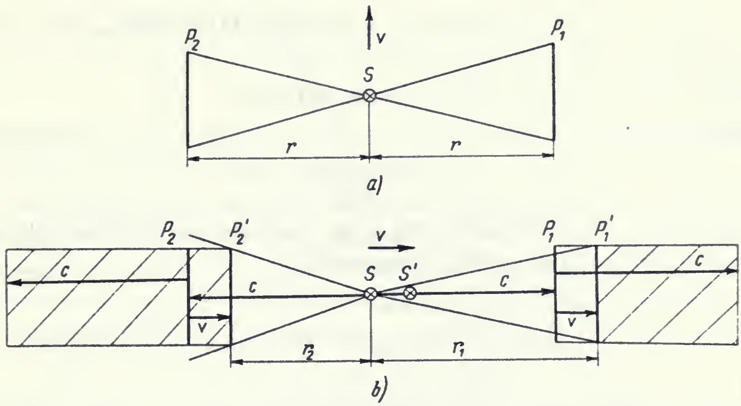

Fig. 68.1

cloth. The cells were wired in such a way that only the difference in the current from the two cells flowed through a galvanometer. In our realization, all elements (excluding the galvanometer) were mounted on a table which could be oriented at will. Godart (1974) claimed to have observed an increase in current when the apparatus was oriented in approximately the north-south direction. We were unable to obtain any effect what-so-ever during a rotation through 360^0.

The null result arises as a consequence of the mutual annihilation of two effects: the "aether wind" effect (analysed in §68B) and the effect of the anisotropy in the distribution of the radiation from a moving source (analysed in §68C).

We have been able to further substantiate our assertion about the mutual annihilation of these two high-velocity absolute effects in the "wired photocells" experiment by performing the "wired photocells on a rotating disk" experiment described below in §68D.

B. The "aether wind" effect

Fig. 68.1a indicates the case when the absolute velocity v of the laboratory is perpendicular to the axis of the apparatus; when the light source is at the middle position between the photocells P_1 and P_2 no current flow through the galvanometer.

When the apparatus is rotated so that the absolute velocity of the laboratory is directed from P_2 towards P_1 (see fig. 68.1b), it may be concluded from the "aether" concept of light propagation that more photons will strike P_2 per unit time than P_1. The currents produced by the photocells will be $J_1 = J - \Delta J/2$, $J_2 = J + \Delta J/2$, where J is the current produced in case (a), and the difference in current ΔJ will flow through the galvanimeter in case (b).

Indeed, the "effective" distances of the photocells from the source (i.e., the distances between the emission position of the source and the reception positions of the cells) will be

$$r_1 = r(1 + v/c), \qquad r_2 = r(1 - v/c), \tag{68.1}$$

where r is the actual distance.

Hence the energy flux density over P_1 and P_2 will be:

a) For the case in fig. 68.1a

$$I = \frac{P}{4\pi r^2}, \tag{68.2}$$

where P is the energy flux radiated by the whole light source (for simplicity we suppose the source as a point and the radiation isotropic).

b) For the case in fig. 68.1b

$$I_1' = \frac{P}{4\pi r_1^2} = \frac{P}{4\pi r^2(1 + v/c)^2}, \qquad I_2' = \frac{P}{4\pi r_2^2} = \frac{P}{4\pi r^2(1 - v/c)^2}. \tag{68.3}$$

However, we also have to take into account the fact that cell P_1 will collect in a unit of time all photons in a cylinder (suppose the photocells circular) whose axis is equal to c <u>minus</u> the photons in the cylinder P_1P_1' whose axis is equal to v, while the cell P_2 will collect in a unit of time all photons in a cylinder whose axis is equal to c <u>plus</u> the photons in the cylinder P_2P_2' whose axis is equal to v. Thus the actual energy flux densities will be

$$I_1 = \frac{I_1'}{1 + v/c} = \frac{P}{4\pi r^2(1 + v/c)^3}, \qquad I_2 = \frac{I_2'}{1 - v/c} = \frac{P}{4\pi r^2(1 - v/c)^3}, \tag{68.4}$$

Indeed, the photons which strike P_1 and P_2 have velocity (see formula (4.28))

$$c_o' = c \tag{68.5}$$

for the case (a) and, respectively, the velocities

$$c_{1o}' = \frac{c}{1 + v/c}, \qquad c_{2o}' = \frac{c}{1 - v/c} \tag{68.6}$$

for the case (b), if these velocities are measured on a proper clock. Thus we shall have

$$I_1/c_{1o}' = I_1'/c_o', \qquad I_2/c_{2o}' = I_{2o}'/c_o', \tag{68.7}$$

and from the last three formulas we obtain (4).

Since the electric currents produced by the photocells are proportional to the energy flux densities, we shall have:

a) For the case in fig. 68.1a

$$J = kI = \frac{kP}{4\pi r^2}, \tag{68.8}$$

where k is some constant.

b) For the case in fig. 68.1b

$$J_1 = kI_1 = \frac{kP}{4\pi r^2(1 + v/c)^3} = \frac{J}{(1 + v/c)^3}, \qquad J_2 = kI_2 = \frac{kP}{4\pi r^2(1 - v/c)^3} = \frac{J}{(1 - v/c)^3}. \tag{68.9}$$

For their difference we obtain to within an accuracy of first order in v/c

$$\Delta J = J_2 - J_1 = 6J\frac{v}{c}. \qquad (68.10)$$

In the experiment we performed $J = 5\times10^{-4}$ A, so that for v = 1 km/sec the "aether wind" effect by itself would have yielded a current difference, according to (10), of the amount $\Delta J = 10^{-8}$ A, an amount greater than the fluctuation error inherent to our equipment. No variation larger than 10^{-8} A was noted by us over the period of a day. Thus taking into account the results of our interferometric "coupled mirrors" experiment (see §52B), we have to conclude that the "wired photocells" experiment gives a null result.

It may be noted that if <u>only</u> the effect described by formula (3), i.e., the effect

$$\Delta J' = J_2' - J_1' = 4J\frac{v}{c}, \qquad (68.11)$$

is taken into account, it is equivalent to the effect obtained by shifting the light source to the photocell P_2 a (v/c)th part of the distance r. Assuming v = 300 km/sec and taking into account that we used r = 500 mm, the effect described by formula (11) is equivalent to shifting the light source a distance $\Delta r = (v/c)r = 0.5$ mm. Such a shift of the source produced an electric current ΔJ of about 2×10^{-6} A. This indicated that the effect of the Earth's motion could have been easily detected if only (11) were true.

C. The effect of the anisotropy of the radiation from a moving source

We explain the null result in the "wired photocells" experiment, taking into account the distribution in the radiation flux density of a rapidly moving light source. Such an effect has already been observed in betatrons and synchrotrons where the radiants are elementary particles. We assume that the same effect must exist also when the radiant is a macroscopic light source and we have confirmed this assumption with the help of the "wired photocells on a rotating disk" experiment (see §68D).

The theory of the angular distribution of the radiation from a moving light source is given in V, §53. Our theoretical analysis shows that the effect of the anisotropy in the radiation is exactly equal and opposite to the "aether wind" effect, i.e., the energy flux densities over P_1 and P_2 are given:

a) For the case shown in fig 68.1a by the formula (2).

b) For the case shown in fig. 68.1b by the following formulas (see formulas (V,53.52))

$$I_1 = \frac{P}{4\pi r^2(1 - v/c)^3}, \qquad I_2 = \frac{P}{4\pi r^2(1 + v/c)^3}. \qquad (68.12)$$

Thus, as a result of these two effects (compare formulas (9) and (12)), no positive effect should be expected in the "wired photocells" experiment, in agreement with our experimental findings.

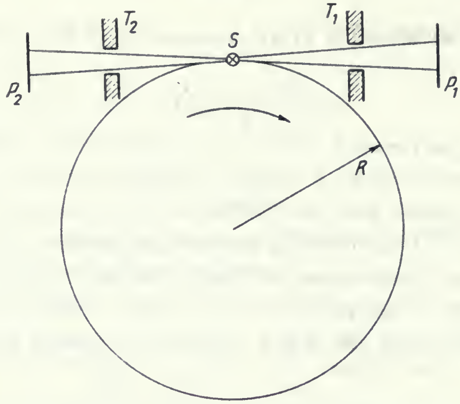

Fig. 68.2

D. The "wired photocells on a rotating disk" experiment

We further substantiated our theoretical prediction about the mutual annihilation of the absolute effects described in §68B and §68C by performing the "WIRED PHOTOCELLS ON A ROTATING DISK" EXPERIMENT indicated in fig. 68.2.

The source S and/or the photocells P_1, P_2 can be mounted on a turnabout which rotates clockwise. T_1 and T_2 are two slits which are always at rest. All other details are as in the "wired photocells" experiment (see §68A).

We performed this experiment with the following three variations:

a) Source moving, cells at rest. In this case one registers only the effect of the anisotropy in the radiation described by formulas (12). The fractional difference in current ΔJ (cf. with (9) and (10), taking into account that now $\Delta J = J_1 - J_2 > 0$) that we observed fitted the formula

$$\frac{\Delta J}{J} = 6\frac{v}{c}. \tag{68.13}$$

b) Cells moving, source at rest. In this case one registers only the "aether wind" effect described by formulas (7). The fractional difference in current ΔJ (= $J_2 - J_1 > 0$) that we observed fitted the formula

$$\frac{\Delta J}{J} = 2\frac{v}{c}, \tag{68.14}$$

since now the "aether wind" effect described by formulas (3) - see formula (11) - does not exist.

c) Source and cells moving. In this case the effects described by formulas (4) and (12) cancel each other out. In agreement with the theory, we registered no change in the difference in current when changing the velocity of rotation.

In all formulas relevant to the "wired photocells on a rotating disk" experiment, v is the component of the velocity of the source or the cells or both along the line of light propagation. Thus, supposing that the slits are narrow enough, we can assume that this is

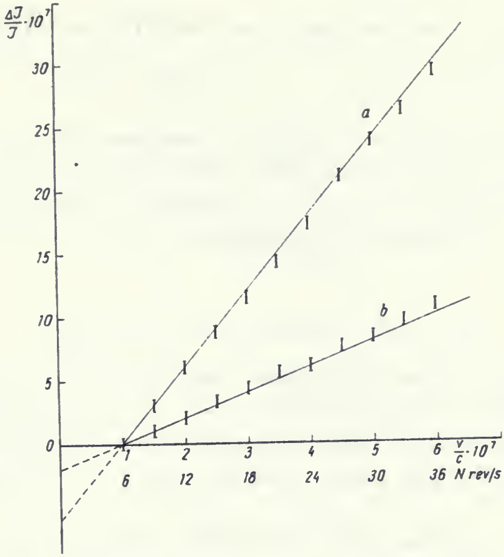

Fig. 68.3

the velocity of a point on the rotating disk whose radius R is equal to the distance from the center of rotation to the center of the light source.

In our experiment we used a stabilized gas discharge lamp as a light source. The conductors from the lamp and from the photocells were immersed in mercury at the center of the disk, so we did not register a substantional difference in the fluctuation of the galvanometer at rest and during rotation of the disk. The difference in current ΔJ was fed to a direct-current electronic amplifier with a low resistance input (i.e., which can be considered as a galvanometer) whose fluctuation corresponded to $\delta J = \pm 3.3\times 10^{-12}$ A (for maximum rotational velocity). At low rotational velocities the observed current J varied with the period of rotation. At a rate of rotation higher than 5 rev/sec the current became stable, equal to 6.5×10^{-5} A, and did not change with a further increase of the rate of rotation. For low rotational velocities, a slight "single-sinusoidal" variation in the difference in current ΔJ was also observed. The distance of the lamp from the center of rotation (measuring from the center of the lamp's window) was $R = 79.6 \pm 0.2$ cm, and the distance of the cells about 98 cm. The rate of rotation N was measured by a light stroboscopic cyclometer and maintained automatically with a precision $\delta N/N = \pm 2\times 10^{-4}$. We adjusted the difference in current to zero for $v/c = 10^{-7}$, corresponding to a rotational rate of N = 6.00 rev/sec, by a corresponding shift of the cells (case a) or of the lamp (case b). For case (c), ΔJ was made equal to zero when lamp and cell were at rest.

The results of our measurements for case (a) and (b) are shown in fig. 68.3. For case (c) no difference in current was registered which could be differentiated from the background fluctuations.

It is obvious that our working formulas were not (13) and (14) but

$$\frac{\Delta J}{J} = 6(\frac{v}{c} - 10^{-7}), \qquad \frac{\Delta J}{J} = 2(\frac{v}{c} - 10^{-7}). \qquad (68.15)$$

Only the fluctuation errors are indicated in the graph. The errors introduced by the inaccuracy δN are two small to be discussed. Also, we do not take into account the errors introduced by the inaccuracy $\delta R = \pm 0.2$ cm, because its relative error is ± 0.25%, while the minimum relative error due to the fluctuation (for $v/c = 6\times10^{-7}$ in case a) was ± 1.7%.

§69. THE "SYNCHROTRON" EXPERIMENT

Marinov (1977) proposed the "SYNCHROTRON" EXPERIMENT with the aim of showing that the ballistic (Ritz) model of light propagation (see p. 5) is not adequate to physical reality, i.e., that the velocity of light with respect to absolute space is always equal to c, and is not equal to the vector sum of $\vec{c}$ and the velocity $\vec{v}$ of the emitter, as assumed by Ritz.

The "synchrotron" experiment, which arose from an outline given by Karastoyanov (1972), is shown in fig. 69.1:

Electrons are accelerated in a circular accelerator A. Light is emitted at E in short regular intervals of time ΔT. These light pulses, after being reflected by the semi-transparent mirror M and passing through the slit S, reach the electrons revolving in the accelerator along the tangent to their trajectory. The photons, after being reflected by the electrons, turn back and, passing through the semi-transparent mirror M, are registered by the receiver R.

We can consider the revolving electrons (representing, as a matter of fact, a fast moving mirror - see § 35) as a new source of radiation. Changing the velocity of the electrons, we change the velocity of this light source. If the velocity of light depends on the velocity of the source of radiation, an increase in the velocity v of the revolving electrons will shorten the time it takes the photons to cover the distance from the accelerator to the mirror M. Hence if we obtain electric pulses from the emitted and received light pulses and if we lead them to the electrodes of an electronic oscillograph Osc, on its screen we should see the picture shown in the figure. The high peaks described by the electronic beam correspond to the light pulses emitted at E, and the low peaks correspond to the reflected light pulses received at R. If the velocity of light does not depend on the velocity of the source of radiation, the distance d between the high and low peaks will remain the same when the velocity of the electrons along the circular trajectory of the accelerator increases. If the velocity of light depends on the velocity of the source of radiation, distance d will change when the velocity of the electrons changes. Let us show this.

When the light pulses are emitted at intervals of ΔT, and D is the distance between

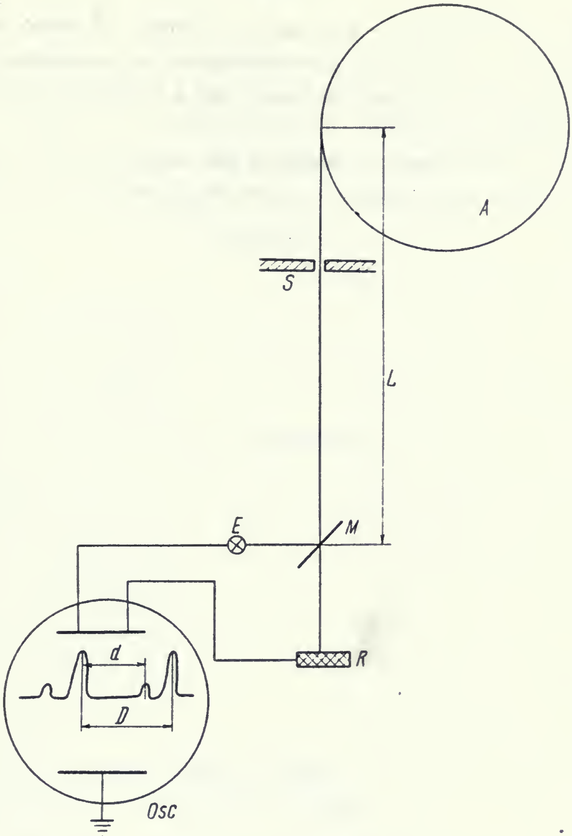

Fig. 69.1

them on the screen, then $D = k\,\Delta T$, where k is the scanning constant of the oscilloscope which equals the horizontal distance the electronic beam covers over the screen per unit of time.

Let the velocity of the electrons in the accelerator be first v and then $v + \Delta v$. If the velocity of the source must be added to the velocity of light, then the velocity of the photons on the track from the accelerator to mirror M will be $c_1 = c + v$ in the first case and $c_2 = c + v + \Delta v$ in the second case.

Hence the time Δt with which the light pulses will come earlier to the receiver in the second case will be (suppose $v, \Delta v \ll c$)

$$\Delta t = \frac{L}{c_1} - \frac{L}{c_2} = \frac{L\,\Delta v}{(c + v)(c + v + \Delta v)} = \frac{L\,\Delta v}{c^2}, \qquad (69.1)$$

where L is the distance between the accelerator and mirror M.

The difference between the distances d_1 and d_2 in the first and second cases will be $d = k\,\Delta t$.

If we choose $\Delta T = 10^{-10}$ sec, L = 9 m, and $\Delta v = c/300 = 10^6$ m/sec, we obtain $\Delta t = \Delta T$. Thus for such an increase of the electron velocity in the accelerator, the low peaks will be shifted with respect to the high peaks over a distance Δd equal to the distance D between the high peaks.

If the velocity of light does not depend on the velocity of the source, as our absolute space-time theory asserts, then $\Delta d = 0$ for any increase of the electrons' velocity.

CONTENTS:

Part III - HIGH-VELOCITY MECHANICS

VII. KINEMATICS OF LIGHT

VII. THE EXPERIMENTAL EVIDENCE FOR THE SPACE-TIME ABSOLUTENESS

HIGH-VELOCITY MECHANICS represents Part III of Marinov's enciclopaedic CLASSICAL PHYSICS which consists of the following five volumes: I. Mathematical apparatus. II. Axiomatics. Low-velocity mechanics. III. High-velocity mechanics. IV. Gravimagretism. V. Electromagnetism. CLASSICAL PHYSICS is constructed on the Newtonian notions of absolute space and time. The notion »aether« can be considered exclusively as a synonym to the notion »absolute space«, being defined as the space in which the center of mass of the universe is at rest, or the space in which the velocity of light is isotropic. The fundamental aspects of Marinov's absolute space-time theory are elaborated in Part III. Within effects of first order in v/c Marinov's theory is identical with Newton's theory. The differences are for effects of second (and higher) order in v/c. Marinov shows that all high-velocity light experiments can be explained if the laboratory velocity of light is taken not in the Newtonian form $c' = c - v\cos\Theta$, but in the form $c' = c^2/(c + v\cos\Theta)$, where Θ is the angle between the absolute velocity v of the laboratory and the direction of light propagation. Almost the half of Part III is dedicated to the analysis of experiments giving confirmation of Marinov's absolute space-time theory. A detailed description is given to Marinov's own numerous experiments, the most important of which is the »coupled-mirrors« experiment with whose help, as a first man in history, in 1973, Marinov succeeded in registering the absolute motion of the Earth in a laboratory; in 1975/76 he measured accurately enough the Sun's absolute velocity, obtaining a magnitude 303 km/sec with equatorial coordinates of the apex: declination -23°, right ascension 14h 17m. This experiment shows in the most direct way that Einstein's principle of relativity is invalid. Very important are also the various Marinov's experiments on the rotating disk.

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