an infinite elastic solid (cf. Cambridge and Dublin Mathematical Journal, 1848).

It may be noted that the density of an isotropic solid, which does not vary with the coordinates (x, y, z), is expressed by the ratio,

$$\varrho = [(m+n)\nabla^2\delta]/(\mathrm{d}X/\mathrm{d}x+\mathrm{d}Y/\mathrm{d}y+\mathrm{d}Z/\mathrm{d}z)\,. \tag{43}$$

But by *Poisson*'s equation we have

$$\nabla^2 V+4\pi\varrho = 0 \qquad \varrho = -\nabla^2 V/4\pi \tag{44}$$

or

$$\varrho = -(1/4\pi)(\partial^2 V/\partial x^2+\partial^2 V/\partial y^2+\partial^2 V/\partial z^2)\,. \tag{45}$$

By comparing (43) and (45), we find that if a mass of density,

$$\varrho = 1/[4\pi(m+n)]\cdot\left(\frac{\mathrm{d}X}{\mathrm{d}x}+\frac{\mathrm{d}Y}{\mathrm{d}y}+\frac{\mathrm{d}Z}{\mathrm{d}z}\right) \tag{46}$$

be distributed throughout space, we may conclude that its potential at (x, y, z) will be identical with the dilatation of the elastic solid substance:

$$\delta = \partial\alpha/\partial x+\partial\beta/\partial y+\partial\gamma/\partial z\,. \tag{47}$$

For if we divide (42) by $(m+n)$, and subtract from it the first of (44), we get:

$$\nabla^2\delta+(\mathrm{d}X/\mathrm{d}x+\mathrm{d}Y/\mathrm{d}y+\mathrm{d}Z/\mathrm{d}z)/(m+n)+ \\ -\nabla^2 V-4\pi\varrho = 0 \tag{48}$$

which gives

$$\nabla^2(\delta-V) = 0 \tag{49}$$

if

$$\mathrm{d}X/\mathrm{d}x+\mathrm{d}Y/\mathrm{d}y+\mathrm{d}Z/\mathrm{d}z-4\pi\varrho(m+n) = 0 \tag{50}$$

or the density is defined by the expression:

$$\varrho = 1/[4\pi(m+n)]\cdot(\mathrm{d}X/\mathrm{d}x+\mathrm{d}Y/\mathrm{d}y+\mathrm{d}Z/\mathrm{d}z)\,. \tag{51}$$

This specifies the density throughout space of the infinite isotropic solid, that of the finite solid body in (41) being unity per unit of volume.

To reach Lord *Kelvin*'s result most directly, we let R denote the resultant of the forces, X, Y, Z, at any point (x, y, z), at the distance $r = \sqrt{(x^2+y^2+z^2)}$ from the origin, whether discontinuous and vanishing in all points outside some finite closed surface, or continuous and vanishing at all infinitely distant points with sufficient convergency to make Rr converge to 0 as r increases to ∞. Then the convergency of Xr, Yr, Zr to zero, when r is infinite, clearly makes $V = 0$ for all infinitely distant points. Accordingly, if S be any closed surface round the origin of coordinates, everywhere infinitely distant from it, the function $(\delta-V)$ is zero for all points of it, and satisfies the equation $\nabla^2(\delta-V) = 0$ for all points within it. Therefore $\delta = V$ throughout the infinite isotropic solid.

Now let X', Y', Z' denote the values of X, Y, Z at any point (x, y, z), and by a triple integration throughout all space, we shall have for the potential V or dilatation δ:

$$\delta = 1/[4\pi(m+n)]\cdot\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(\mathrm{d}X'/\mathrm{d}x'+\mathrm{d}Y'/\mathrm{d}y'+\mathrm{d}Z'/\mathrm{d}z')/\sqrt{[(x-x')^2+(y-y')^2+(z-z')^2]}\cdot\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z'. \tag{52}$$

For the element of the mass is

$$\varrho = 1/[4\pi(m+n)]\cdot(\mathrm{d}X'/\mathrm{d}x'+\mathrm{d}Y'/\mathrm{d}y'+\mathrm{d}Z'/\mathrm{d}z') \tag{53}$$

and the mutual distances of the elements of mass filling the element of space $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$ is

$$r = \sqrt{[(x-x')^2+(y-y')^2+(z-z')^2]}\,. \tag{54}$$

These expressions may be rendered more convenient by integrating by parts, and noticing the prescribed condition of convergence, according to which when x' is infinite,

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}X'/\sqrt{[(x-x')^2+(y-y')^2+(z-z')^2]}\cdot\mathrm{d}y'\,\mathrm{d}z' = 0\,. \tag{55}$$

And, therefore, for the three components of finite value, resolved along the coordinate axes, and integrated throughout all space, we have:

$$\delta = 1/[4\pi(m+n)]\cdot\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[X'(x-x')+Y'(y-y')+Z'(z-z')]/\sqrt{[(x-x')^2+(y-y')^2+(z-z')^2]}\cdot\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z'. \tag{56}$$

We may integrate each of the equations (38) in the same way, for α, β, γ respectively. The result for these displacements is:

$$\alpha = u+U \qquad \beta = v+V \qquad \gamma = w+W \tag{57}$$

where u, v, w, U, V, W denote the potentials at (x, y, z) of distributions of matter through all space of densities respectively

$$(m/4\pi n)\,\partial\delta/\partial x \qquad (m/4\pi n)\,\partial\delta/\partial y \qquad (m/4\pi n)\,\partial\delta/\partial z \qquad X/4\pi n \qquad Y/4\pi n \qquad Z/4\pi n\,. \tag{58}$$

In other words the functions are such that throughout all space

$$\nabla^2 u+(m/n)\,\partial\delta/\partial x = 0 \quad \nabla^2 U+X/n = 0 \qquad \nabla^2 v+(m/n)\,\partial\delta/\partial y = 0 \quad \nabla^2 V+Y/n = 0 \\ \nabla^2 w+(m/n)\,\partial\delta/\partial z = 0 \quad \nabla^2 W+Z/n = 0\,. \tag{59}$$

Accordingly, if X'', Y'', Z'' denote the values of X, Y, Z for a point (x'', y'', z''), we find

$$\alpha = (1/4\pi n)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(m\cdot\partial\delta''/\partial x''+X'')/\sqrt{[(x-x'')^2+(y-y'')^2+(z-z'')^2]}\cdot\mathrm{d}x''\,\mathrm{d}y''\,\mathrm{d}z''$$

$$\beta = (1/4\pi n)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(m\cdot\partial\delta''/\partial y''+Y'')/\sqrt{[(x-x'')^2+(y-y'')^2+(z-z'')^2]}\cdot\mathrm{d}x''\,\mathrm{d}y''\,\mathrm{d}z'' \tag{60}$$

$$\gamma = (1/4\pi n)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(m\cdot\partial\delta''/\partial z''+Z'')/\sqrt{[(x-x'')^2+(y-y'')^2+(z-z'')^2]}\cdot\mathrm{d}x''\,\mathrm{d}y''\,\mathrm{d}z''\,.$$