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Magnet-less gyrotropy using time-periodic modulation of permittivity

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Abstract: Time-varying (TV) media in electromagnetics have unlocked new paths to important electromagnetic effects such as non-reciprocity, frequency conversion, and parametric amplification. In light of such advances over the past decade, a curious question arises as to whether chirality or/and gyrotropy, may also be achieved using TV dielectrics. In this paper, we propose a suitable time-modulation of the permittivity tensor in a static achiral and non-gyrotropic crystal and thereby *emulate* gyrotropy without the need of magnetic materials and external magnetic bias field. The *emulated* gyrotropy is owed to the temporal rotation of the principal axes of the permittivity tensor, which sustains only circularly/elliptically polarized eigenmodes. A possible realization using modulated electro-optic effects in a nonlinear crystal is proposed, showing a feasible approach to realize gyrotropy by time variation in non-magnetic achiral media.

1. Introduction

Time-varying (TV) electromagnetic media enable novel and alternative ways to reach new and existing electromagnetic (EM) phenomena and effects, attracting considerable research attention in the past decade or so in different applications [1,2]. Non-reciprocity using linear TV media was one major finding [3–5] enabling magnet-less TV isolators [6–15] and TV circulators [11,16]. The key significance was that non-reciprocity was achieved by simple time-modulation of the permittivity of an ordinary dielectric, circumventing the need for magnetic materials and magnetic bias fields, thus paving the way for much simpler, more compact, and amenable devices for high frequency and optical integrated circuits. Generating frequency harmonics with time-periodic modulations of a medium, has also led to novel approaches to applications such as frequency conversion [17–20], frequency combs [21,22], and parametric amplification [23,24]. These effects can also be seen in TV circuits and transmission lines realizing different applications [25–27].

Chirality and gyrotropy are two important electromagnetic effects that both bring about polarization rotation, and circular dichroism [28], with numerous applications in various fields such as biology, chemistry, and pharmaceutical industries to name a few. Naturally chiral materials such as sugar and DNA [29,30], or artificially structured chiral metamaterials such as [31–35] lack mirror symmetry in the geometry of their molecules/constituents [36–38], which manifests itself as magneto-electric coupling terms in the constitutive relations of the medium [37,39,40]. Chiral media interact differently with right-hand circularly polarized (RCP) and left-hand circularly polarized (LCP) light which has led to extensive research and development in sensing and separation of chiral enantiomers [34,41–45]. Gyrotropy manifests itself as an anti-symmetric contribution in the off-diagonal elements of the permittivity/permeability tensor which stems from the presence of a gyration vector [46,47]. Faraday rotation is a prime application of such media used in optical sensors. The challenge with this type of materials is that we need to apply an external magnetic bias to induce a gyration vector.

It should be noted that chiral and gyrotropic media are not equivalent despite their similarity in enabling circularly polarized eigenmodes. Regardless of their similarities and differences,

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chiral and gyrotropic effects have so far been mostly attained using stationary (time-invariant) media. The rapid growth of TV media [3] begs the question as to whether time-variations of a non-gyrotropic/achiral material can lead to either gyrotropic or chiral effects. In [48], an array of spatio-temporal ring resonators forming a metasurface was explored to show Faraday rotation. In [49,50] third-order instantaneous nonlinear phenomena and circularly polarized pumps were used for Faraday rotation. In [51], time modulation in a structurally chiral medium (Archimedes screw) is used to drag and amplify light.

In this paper, we utilize simple time variations of the permittivity of an ordinary achiral and non-gyrotropic anisotropic dielectric resulting in the temporal rotation of the principal axes of the medium to mimic gyrotropy. Thus, the use of naturally or artificially engineered gyrotropic media, which typically require applying external magnetic bias field to the particular magnetic materials, is avoided and a new approach is unlocked to *mimic* gyrotropic effects with the aid of TV media. Such time-variation of the permittivity tensor, breaks temporal translational symmetry and results in a non-reciprocal behavior, which combined with CP eigenmodes allows emulation of gyrotropy.

The paper is structured as follows. First, we introduce a way to characterize chirality and gyrotropy in time-varying media. Then, we propose a special type of time-periodic anisotropic media that mimics gyrotropic behavior. These are given in Sec. 2. To investigate chiral/gyrotropic effects, scattering from a time-periodic anisotropic slab structure is given in 3 and transmission from the media introduced in Sec. 2 is calculated as an example. A possible realization of the time-periodic gyrotropic media using the electro-optic Pockels effect and without any magnetic bias field is given in Sec. 4. Finally, the conclusions are given in Sec. 5.

2. Proposal for time-periodic gyrotropic media

Let us first address an important question on how to generally identify and recognize chiral/gyrotropic effects in propagation, be it in either stationary or TV medium, as no all-encompassing theory seems to exist for this purpose. EM propagation in linear media, be it waveguides or radiation in bulk, can be quantified in terms of the allowable eigenmodes and eigenvalues of the propagation problem, such that the electromagnetic response of two linear media with identical eigenmodes can be identical. The common feature of propagation in both chiral and gyrotropic media is that they allow only non-degenerate circularly-polarized (CP)/elliptical eigenmodes (CP eigenmodes with unequal propagation constants for the RCP and LCP modes) [52–54]. Therefore, we utilize the ellipticity of the eigenmodes as a means to identify chiral/gyrotropic behavior regardless of whether the medium is time-periodic or time-invariant. In time-periodic media, an eigenmode will be comprised of time-harmonics, which their polarization states may be explored individually. We then find the appropriate form of time modulation for permittivity tensor which leads to propagation with CP eigenmodes. If the polarization of the harmonics of eigenmodes is only CP, the medium essentially *emulates* either chiral or gyrotropic behaviour. The distinction between chiral and gyrotropic propagation is further found by investigating the non-reciprocity of the medium, which can be achieved by direct observation of the eigenmodes or using Lorentz reciprocity theorem extended for time-periodic media [55]. In the reciprocal case, the medium *would emulate* chirality while in the non-reciprocal case, the medium *emulates* gyrotropy. Such perspective does not establish true equivalence between our proposed TV medium and a chiral/gyrotropic medium, thus the term *emulate* is used to imply that the gyrotropic effect in propagation is mimicked by using the appropriate time variations.

As an instructive example, consider Fig. 1(a,b) where the eigenmodes of TEM (Transverse Electro-Magnetic) propagation in bulk time-invariant [bi]-isotropic chiral medium as well as in an anisotropic gyrotropic medium are shown, which both lead to two forward ($+z$) traveling CP, and two backward ($-z$) traveling CP modes with different phase velocities for the RCP and

LCP waves. The forward and backward RCP (or LCP) eigenmodes propagate with the same propagation constant in the purely chiral medium (Fig. 1(a)) resulting in reciprocal behavior. However, in the gyrotropic medium (Fig. 1(b)), the propagation constants of the forward and backward RCP (or LCP) modes are different thus resulting in non-reciprocity. Also, chiral anisotropic media such as uniaxial omega media have elliptically polarized eigenmodes with different propagation constants. This is while other types of media such as the static achiral isotropic and non-gyrotropic anisotropic media have linearly polarized eigenmodes. Therefore, to realize gyrotropy/chirality, we will seek TV media that yield CP eigenmodes of propagation, be it in a reciprocal or non-reciprocal fashion.

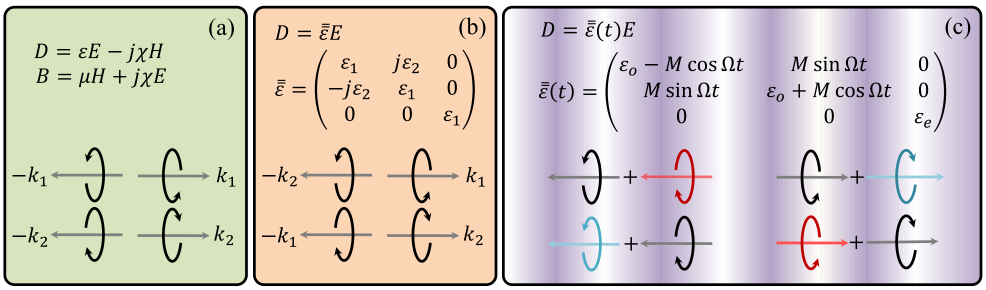

Fig. 1. (a) Static chiral bi-isotropic medium and its eigenmodes while assuming $\chi^2 < \mu\varepsilon$, (b) Static gyrotropic medium and its eigenmodes, (c) Sinusoidal time-modulated anisotropic medium and its eigenmodes. Eigenmodes of the TV medium have two non-zero harmonics where the main harmonic(ω_0), and two harmonics ω_1 and ω_{-1} are shown in black, blue, and red, respectively.

Without loss of generality, consider the propagation along the z-axis throughout a non-magnetic ($\bar{\bar{\mu}} = \mu_0\mathbf{I}$) TV biaxial anisotropic medium (without any magneto-electric coupling term) with the governing constitutive relations

$$\mathbf{D}(\mathbf{r},t) = \varepsilon_0\bar{\bar{\varepsilon}}_{st} + \varepsilon_0\Delta\bar{\bar{\varepsilon}}(t)\mathbf{E}(\mathbf{r},t), \quad \mathbf{B}(\mathbf{r},t) = \mu_0\mathbf{H}(\mathbf{r},t), \tag{1}$$

where an instantaneous response for polarizability is assumed and the time variation is added as an extra term to a static achiral biaxial permittivity tensor. By doing so we neglect dispersion and assume slowly-varying modulations. A time-periodic sinusoidal time variation $\Delta\bar{\bar{\varepsilon}}(t)$, where tensor elements $\Delta\varepsilon_{ij}, i,j = 1,2$ are sinusoidal with the period $2\pi/\Omega$ (Ω being the modulation frequency) are applied such that

$$\Delta\bar{\bar{\varepsilon}}(t) = \varepsilon_0\begin{pmatrix} \delta\varepsilon_{xx}\cos(\Omega t + \phi_{xx}) & \delta\varepsilon_{xy}\cos(\Omega t + \phi_{xy}) & 0 \\ \delta\varepsilon_{yx}\cos(\Omega t + \phi_{yx}) & \delta\varepsilon_{yy}\cos(\Omega t + \phi_{yy}) & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{2}$$

where $\delta\varepsilon_{ij}, i = 1,2$ shows the modulation strengths and $\phi_{ij}, i = 1,2$ is the phase of each cosine term.

In order to investigate the polarization states of the eigenmodes for propagation in z in this time-modulated medium, it is necessary to solve the associated eigenvalue problem. Using the Fourier series of the permittivity tensor as

$$\varepsilon_{ij}(t) = \varepsilon_0\sum_m \varepsilon_{m,ij}e^{jm\Omega t}, \quad i,j = 1,2. \tag{3}$$

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and invoking the Floquet-Bloch theorem, the electromagnetic field in the medium can be written as the Fourier series of waves at frequencies $\omega_n = \omega_0 + n\Omega$ as

$$\begin{aligned}\mathbf{E}(z,t) &= \sum_n \mathbf{E}_n(z,\omega)e^{j\omega_n t} + c.c., \\ \mathbf{H}(z,t) &= \sum_n \mathbf{H}_n(z,\omega)e^{j\omega_n t} + c.c..\end{aligned} \tag{4}$$

By Taking a finite number of $2N+1$ harmonics, Eqs. (4) and (1) can be substituted in Maxwell's equations, and by assuming the orthogonality between $2N+1$ harmonics, the harmonic balance technique is used [56]. After applying the Fourier transform in the z-direction, we arrive at a set of $4(2N+1)$ equations which can be written as the eigenvalue equation in matrix form as

$$\begin{aligned}\frac{\partial}{\partial\xi}\begin{pmatrix}\mathbf{E}_{x,N}\\ \mathbf{E}_{y,N}\\ \mathbf{H}_{x,N}\\ \mathbf{H}_{y,N}\end{pmatrix}_{(8N+4)\times 1} &= \mathbf{M}_{(8N+4)\times(8N+4)}\begin{pmatrix}\mathbf{E}_{x,N}\\ \mathbf{E}_{y,N}\\ \mathbf{H}_{x,N}\\ \mathbf{H}_{y,N}\end{pmatrix}_{(8N+4)\times 1} \\ &= \begin{pmatrix}0 & 0 & \phi & \mathbf{M}^{EH}_{xy}\\ 0 & 0 & \mathbf{M}^{EH}_{yx} & 0\\ \mathbf{M}^{HE}_{xx} & \mathbf{M}^{HE}_{xy} & 0 & 0\\ \mathbf{M}^{HE}_{yx} & \mathbf{M}^{HE}_{yy} & 0 & 0\end{pmatrix}\begin{pmatrix}\mathbf{E}_{x,N}\\ \mathbf{E}_{y,N}\\ \mathbf{H}_{x,N}\\ \mathbf{H}_{y,N}\end{pmatrix},\end{aligned} \tag{5}$$

where $\xi = \Omega/c_0 z$ and c_0 is the light velocity in free space, and

$$\begin{aligned}\mathbf{E}_{x/y,N} &= [E_{x/y,-N},\ldots,E_{x/y,0},\ldots,E_{x/y,N}]^T\\ \mathbf{H}_{x/y,N} &= [H_{x/y,-N},\ldots,H_{x/y,0},\ldots,H_{x/y,N}]^T\end{aligned} \tag{6}$$

are the field vectors with a truncated number of harmonics at frequencies of $\omega_0 + n\Omega$ $(-N<n<N)$, and also the elements of matrix $\mathbf{M}$ are

$$\begin{aligned}\mathbf{M}^{HE}_{xx,(n,m)} &= +\frac{j}{\eta_0}\left(\frac{\omega}{\Omega}+n\right)\varepsilon_{yx,n-m},\\ \mathbf{M}^{HE}_{yy,(n,m)} &= -\frac{j}{\eta_0}\left(\frac{\omega}{\Omega}+n\right)\varepsilon_{xy,n-m},\\ \mathbf{M}^{HE}_{xy,(n,m)} &= +\frac{j}{\eta_0}\left(\frac{\omega}{\Omega}+n\right)\varepsilon_{yy,n-m},\\ \mathbf{M}^{HE}_{yx,(n,m)} &= -\frac{j}{\eta_0}\left(\frac{\omega}{\Omega}+n\right)\varepsilon_{xx,n-m},\\ \mathbf{M}^{EH}_{xy,(n,m)} &= -j\eta_0\left(\frac{\omega}{\Omega}+n\right)\delta_{mn},\\ \mathbf{M}^{EH}_{yx,(n,m)} &= +j\eta_0\left(\frac{\omega}{\Omega}+n\right)\delta_{mn}.\end{aligned} \tag{7}$$

Given that the tensor $\mathbf{M}$ is invariant with respect to z, we can apply the Fourier transform in the z direction and then solve the eigenvalue problem. The eigenvalues of the matrix $\mathbf{M}$ can be written as $\lambda_p = k_p c_0/\Omega$, where k_p is the propagation constant of the mode. After solving the eigenvalue problem, we will have $8N+4$ eigenvalues $(k_p, p = 1,\ldots,8N+4)$ and $8N+4$ eigenvector $(\mathbf{V}_p)$, which half of them are forward propagating modes and the other half are backward propagating modes. The eigenvectors will have $2N + 1$ harmonics for each of components $\mathbf{E}_x$, $\mathbf{E}_y$, $\mathbf{H}_x$, and $\mathbf{H}_y$. In order to find the eigenvectors containing just the electric fields, the eigenvalue problem (5) can be written as

$$\begin{aligned}\left(\frac{-jkc_0}{\Omega}\right)^2\begin{pmatrix}\mathbf{E}_{x,N}(k,\omega)\\ \mathbf{E}_{y,N}(k,\omega)\end{pmatrix} &= \mathbf{W}\begin{pmatrix}\mathbf{E}_{x,N}(k,\omega)\\ \mathbf{E}_{y,N}(k,\omega)\end{pmatrix}\\ &= \begin{pmatrix}\mathbf{W}_{xx} & \mathbf{W}_{xy}\\ \mathbf{W}_{yx} & \mathbf{W}_{yy}\end{pmatrix}\begin{pmatrix}\mathbf{E}_{x,N}(k,\omega)\\ \mathbf{E}_{y,N}(k,\omega)\end{pmatrix},\end{aligned} \tag{8}$$

where k is the propagation constant in the z direction.

In this equation, $\mathbf{W}$ is a $(4N + 2) \times (4N + 2)$ matrix which is dependent on the Fourier components of $\bar{\bar{\varepsilon}}$ and frequency ω_n, that is $W_{ij,(n,m)} = -\varepsilon_{ij,(n-m)}\,(\omega/\Omega + n)^2$ where $i, j = 1, 2$.

Therefore, the eigenvalue problem can be written as

$$\begin{aligned}&\sum_m\left[\left(\frac{\omega_n}{\Omega}\right)^2\varepsilon_{n-m,xx}-\left(\frac{kc_0}{\Omega}\right)^2\delta_{nm}\right]E_{x,m}\left(k,\omega\right)+\left[\left(\frac{\omega_n}{\Omega}\right)^2\varepsilon_{n-m,xy}\right]E_{y,m}\left(k,\omega\right)=0,\\ &\sum_m\left[\left(\frac{\omega_n}{\Omega}\right)^2\varepsilon_{n-m,yy}-\left(\frac{kc_0}{\Omega}\right)^2\delta_{nm}\right]E_{y,m}\left(k,\omega\right)+\left[\left(\frac{\omega_n}{\Omega}\right)^2\varepsilon_{n-m,yx}\right]E_{x,m}\left(k,\omega\right)=0,\end{aligned} \tag{9}$$

and the dispersion equation of the modes can then be obtained by calculating $det(\mathbf{W}-(kc_0/\Omega)^2\mathbf{I}) = 0$. Solution of the eigenvalue problem for each wave vector k leads $8N + 4$ eigenvalues ω_p and $8N + 4$ eigenvectors $\mathbf{E}_p$. It should be noted that these eigenvalues are essentially only 4 distinct eigenmodes ($p = 1, 2, 3, 4$) within one Brillouin zone ($-0.5<\omega/\Omega<0.5$). In other words, for any given k, $\mathbf{W}_{(n,m)}(\omega + q\Omega) = \mathbf{W}_{((n-q),m)}(\omega)$, $k(\omega + q\Omega) = k(\omega)$ and the photonic band structure is periodic in ω with the period of Ω. The harmonic components, $\mathbf{e}_{np}$, of an eigenvector $\mathbf{E}_p$ ($\mathbf{E}_p = \sum_n \mathbf{e}_{np}$), also follow the relations, $e_{x,(n-q)p}(\omega+q\Omega) = e_{x,np}(\omega)$, $e_{y,(n-q)p}(\omega+q\Omega) = e_{y,np}(\omega)$. Therefore, by knowing the solution of the medium in ω, one can obtain the solution of the medium in $\omega + q\Omega$. For a given Bloch frequency, ω, the dispersion equation has $8N + 4$ eigenvalues k_p, and $8N + 4$ eigenvectors $\mathbf{E}_p$ ($p = 1, \ldots, 8N + 4$). These are different eigenmodes of the problem also shown with different colors in Fig. 2(a).

Equation (8) shows that in order to have an elliptically polarized eigenmode, W_{xy} and W_{yx} must be non-zero which as explained earlier are related to $\delta\varepsilon_{ij}$, resulting in the coupled equation for $\mathbf{E}_{x,N}$, and $\mathbf{E}_{y,N}$. The simplest form of sinusoidal TV medium is the isotropic medium which has just one characterizing parameter to be time-variant, its refractive index, leading to $\delta\varepsilon_{xy} = \delta\varepsilon_{yx} = 0$, thus $W_{xy} = W_{yx} = 0$. As the isotropic medium has just one refractive index along all axes, isotropic time variation in all directions does not result in a 90 degrees' phase difference between two components of the electric field. Therefore, there isn't any isotropic time-variations which could lead to CP eigenmodes, thus a time-periodic isotropic dielectric can by no means *emulate* gyrotropy nor chirality.

Let us now turn our attention to the interesting case of Eq. (10) where a sinusoidal time-variation is applied to an anisotropic medium in the following form

$$\bar{\bar{\varepsilon}}(t) = \varepsilon_0\begin{pmatrix}\varepsilon_o - M\cos(\Omega t) & M\sin(\Omega t) & 0\\ M\sin(\Omega t) & \varepsilon_o + M\cos(\Omega t) & 0\\ 0 & 0 & \varepsilon_e\end{pmatrix}. \tag{10}$$

This particular configuration is chosen by carefully examining all the various combinations for $\delta\varepsilon_{ij}$ and Φ_{ij} in Eq. (2), which are not shown here for brevity. The form given in Eq. (10), not only

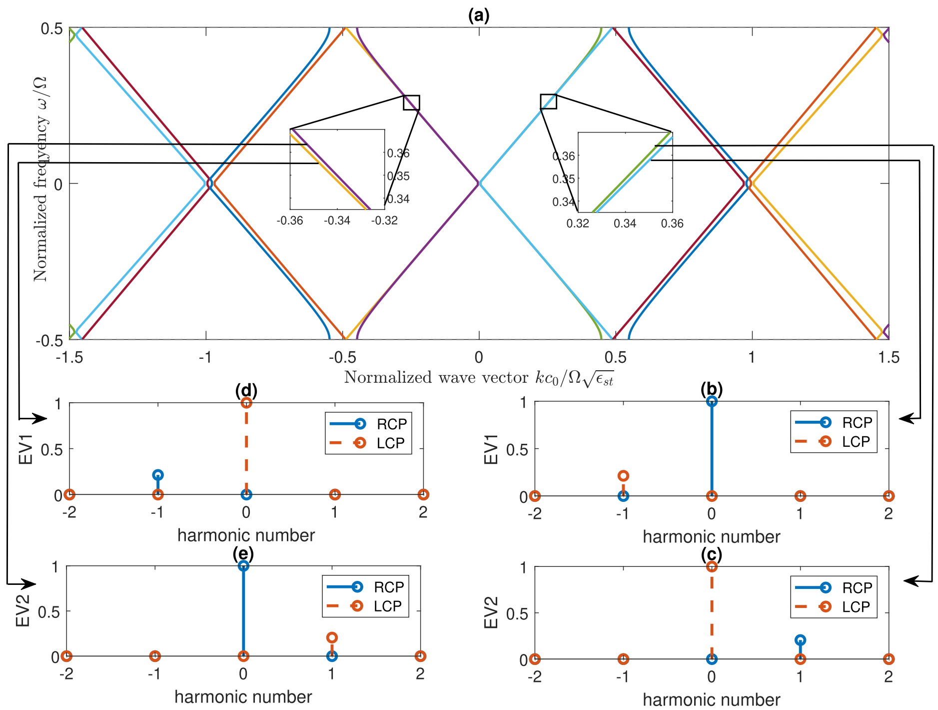

Fig. 2. (a) Dispersion diagram of a sinusoidal time-modulated anisotropic medium. (b-c) The RCP and LCP components of two first forward propagation eigenmodes and (d-e) RCP and LCP components of two first backward propagation eigenmodes. The medium has a sinusoidal time variation with parameters $\varepsilon_{st} = 1$, and $M = 0.2$.

yields our desired response, it also provides a physically viable solution that can be implemented using crystals with electro-optic effect which will be explained later in the paper. The opposite signs of the diagonal terms stem from the physical feasibility which will also become clear while discussing the implementation.

As an example, assuming a medium with the permittivity, Eq. (10), where $\varepsilon_{o,e} = 1$ and $M = 0.2$, the dispersion diagram is depicted in Fig. 2(a). Calculating the eigenmodes of the medium using the eigenvalue problem, Eq. (9), in MATLAB leads to eigenmodes with only two non-zero CP harmonics as depicted graphically in Fig. 1(c). The RCP and LCP components of two main forward propagating eigenmodes, shown on the dispersion diagram, are depicted precisely in Fig. 2(b,c), and the RCP and LCP components of two main backward propagating eigenmodes are depicted precisely in Fig. 2(d,e). This figure shows that the eigenmodes have a pair of non-zero frequency components, either $(\omega_0, \omega_0 + \Omega)$ or $(\omega_0, \omega_0 - \Omega)$ that are all circularly polarized, and the handedness of the main harmonic (ω_0) is opposite of the handedness side harmonics $(\omega_0 \pm \Omega)$. To verify the obtained result, we further investigate the problem using coupled mode theory (CMT) provided in the Supplement 1.

In order to check the reciprocity condition from eigenmodes of the media, as explained in the Supplement 1, it is needed to check whether the backward propagating modes in the frequency domain are the complex conjugate of forward propagating modes or not. The first forward propagating mode ($EV1$) of Fig. 1(b) has a right-hand circularly polarized 0 harmonic and a left-hand circularly polarized -1 harmonic and its complex conjugate will be a backward propagating wave with a left-hand circularly polarized 0 harmonic and a right-hand circularly polarized $+1$ harmonic which is different from the first backward propagating mode, ($EV1$) of

Fig. 1(d). These figures show that the backward propagating modes are not complex conjugates of the forward propagating modes which implies non-reciprocal propagation. The Lorentz reciprocity theorem extended for time-periodic media [55] confirms this non-reciprocity as $\bar{\bar{\varepsilon}}_{+1} \neq \bar{\bar{\varepsilon}}^T_{-1}$($\bar{\bar{\varepsilon}}_{\pm1}$ being the $+1$ and -1 Fourier components of the permittivity tensor, Eq. (10)). This non-reciprocity stems from the fact that the permittivity tensor, Eq. (10) creates a sense of rotation in time for the principal axes of the crystal, and thereby prevents time-reversal symmetry. It is worth noting that this medium satisfies z-reversal symmetry which is evident from Fig. 1(b) and (d) or (c) and (e).

Therefore, our proposed medium in Eq. (10) allows only non-degenerate non-reciprocal CP eigenmodes, and thus *emulates* a gyrotropic medium. It is worth emphasizing that this gyrotropy is achieved by merely time modulating the permittivity tensor of a non-gyrotropic medium and without the use of any magnetic materials or magnetic bias as needed in conventional gyrotropic media. This is also a fundamentally different physics compared to the rotation of the principal axes of the permittivity tensor along the direction of propagation (changing the variable t to z in Eq. (10)). Such rotation of anisotropy in space is well-known as used in twisted fibers [57], and helical or other metamaterials [38] yielding to chirality as it does not have mirror symmetry but maintains time-reversal symmetry. Again investigating the eigenmodes of propagation in the medium confirms that the forward and backward RCP(LCP) modes verify the reciprocal yet chiral behavior of this medium. It is worth noting that temporal rotation in an anisotropic surface is investigated previously in [58] to achieve non-reciprocal scattering. Similar to the approach presented in [58], by applying a rotating frame with a rotating angle $\Omega/2t$, the permittivity tensor, Eq. (10) becomes time-invariant in the rotated coordinate, allowing the fields to be expressed as simple phasors showing CP eigenmodes.

By considering an excitation at $z = 0$ in the bulk medium that excites only two first forward eigenmodes shown in Fig. 2(b,c), the electric field at $z = \ell$ inside the medium can be written as

$$\begin{aligned}\mathbf{E}(\ell, t) = &\left(\mathbf{e}_{0,1}e^{j\omega_0 t} + \mathbf{e}_{-1,1}e^{j\omega_{-1} t}\right) e^{-jk_1\ell} \\ &+ \left(\mathbf{e}_{0,2}e^{j\omega_0 t} + \mathbf{e}_{1,2}e^{j\omega_1 t}\right) e^{-jk_2\ell} + c.c.,\end{aligned} \tag{11}$$

where $\mathbf{e}_{0,1} = [1, j]^T$, $\mathbf{e}_{-1,1} = a[1, -j]^T$, $\mathbf{e}_{0,2} = [1, -j]^T$, and $\mathbf{e}_{1,2} = a[1, j]^T$ where a is the ratio of the amplitude of ±1 harmonics to the amplitude of the main harmonic. Carrying out some mathematical calculations, it can be seen that the main harmonic has a linear polarization that rotates while propagating in the medium and the angle of rotation is calculated as $\theta = (k_2 - k_1)\ell/2$. However, this excitation is not monochromatic and some fractions of power are in the ±1 harmonics. We usually excite a medium using a monochromatic wave which is essentially an iso-energy excitation. The monochromatic excitation needs to consider an infinite number of modes and causes reflection, resulting in polarization conversion.

3. Scattering from a time-periodic anisotropic slab structure

In this section, we solve the transmission and reflection from a 1D structure consisting of spatial slabs (piecewise constant homogeneity in space) whose permittivity of each slab is periodic in time in the form of Eq. (2) with the permeability tensor $\bar{\bar{\mu}} = \mu_0\mathbf{I}$, shown in Fig. 3. In order to solve the transmission and reflection from such a structure, we need to use Maxwell's equations and the Floquet-Bloch theorem to obtain the eigenvalue problem in each layer. Using the solution of the eigenmode problem given in the previous section, the electric field inside the TV medium

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can be written as the summation of eigenmodes at frequency ω_0 as follows

$$\begin{aligned}\mathbf{E}(z,t) &= \sum_{p=1}^{4N+2} c_p^{\pm}\mathbf{E}_p^{\pm}(k_p,t)e^{-jk_p^{\pm}z} + c.c. \\ &= \sum_{p=1}^{4N+2}\sum_{n=-N}^{N} c_p^{\pm}\mathbf{e}_{np}^{\pm}e^{j\omega_n t}e^{-jk_p^{\pm}z} + c.c.,\end{aligned} \tag{12}$$

where $\mathbf{E}_p^{\pm}(k_p,t)$ shows the eigenmodes of the medium corresponding with $k = k_p$ so that $4N+2$ of them are forward propagating and $4N+2$ are backward propagating. Also, $\mathbf{e}_{np}^{\pm}$ shows the amplitudes of the harmonics $\omega_0 + n\Omega$ of the eigenvector $\mathbf{E}_p^{\pm}$. This can be written similarly for the magnetic field. Therefore, in a slab structure, the electric and magnetic fields in the i^{th} layer can be written as

$$\begin{pmatrix}\mathbf{E}_{x,N}\\ \mathbf{E}_{y,N}\\ \mathbf{H}_{x,N}\\ \mathbf{H}_{y,N}\end{pmatrix}_i = \sum_{p=1}^{p=4N+2} c_{p,i}^{\pm}\mathbf{V}_{p,i}^{\pm}e^{\mp k_{p,i}(z-z_{i-1})}, \tag{13}$$

where $\mathbf{V}_{p,i}$ and $\lambda_{p,i}$ are the eigenvector and eigenvalues of the layer with $z_{i-1}<z<z_i$, respectively, and $c_{p,i}$ are the unknown coefficients that can be determined using the boundary conditions. Note that, because the eigenvalues of the matrix $\mathbf{W}$ are real, the propagation constants, k_p, are complex conjugate, and the eigenmodes can be forward or backward propagating which is identified using $+$ and $-$ signs.

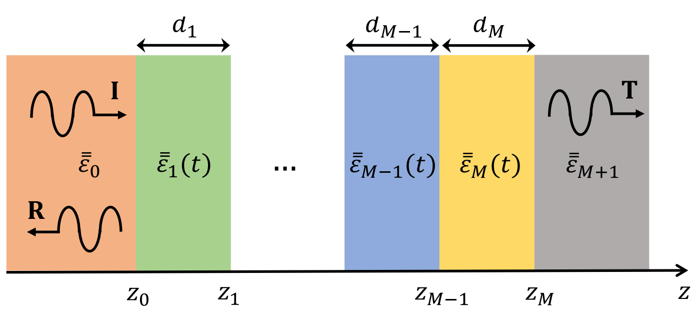

Fig. 3. Transmission and reflection from a spatially piecewise constant homogeneous multi-layer structure where each of layers is a non-magnetic time-periodic modulated anisotropic medium.

The boundary conditions at every spatial boundary, $(z = z_i)$, is the continuity of transverse electric and magnetic fields, which meas that $\mathbf{E}(z = z_i, t) = \mathbf{E}(z = z_{i+1}, t)$, and $\mathbf{H}(z = z_i, t) = \mathbf{H}(z = z_{i+1}, t)$ [15] resulting in

$$\begin{pmatrix}\mathbf{E}_{x,N}\\ \mathbf{E}_{y,N}\\ \mathbf{H}_{x,N}\\ \mathbf{H}_{y,N}\end{pmatrix}_i = \begin{pmatrix}\mathbf{E}_{x,N}\\ \mathbf{E}_{y,N}\\ \mathbf{H}_{x,N}\\ \mathbf{H}_{y,N}\end{pmatrix}_{i+1}. \tag{14}$$

Substituting Eq. (13) in the boundary condition Eq. (14), will reach

$$\left[\mathbf{V}_i^+|\mathbf{V}_i^-\right]\begin{bmatrix}\mathbf{X}_i & 0\\ 0 & \mathbf{X}_i^{-1}\end{bmatrix}\begin{pmatrix}\mathbf{C}_i^+\\ \mathbf{C}_i^-\end{pmatrix} = \left[\mathbf{V}_{i+1}^+|\mathbf{V}_{i+1}^-\right]\begin{pmatrix}\mathbf{C}_{i+1}^+\\ \mathbf{C}_{i+1}^-\end{pmatrix}, \tag{15}$$

and therefore, the relation between the amplitudes of waves can be written as

$$\begin{pmatrix}\mathbf{C}_{i+1}^+\\ \mathbf{C}_{i+1}^-\end{pmatrix} = \mathbf{Q}_{i\to i+1}\begin{pmatrix}\mathbf{C}_i^+\\ \mathbf{C}_i^-\end{pmatrix}, \tag{16}$$

where

$$\mathbf{Q}_{i\to i+1} = \left[\mathbf{V}_{i+1}^+|\mathbf{V}_{i+1}^-\right]^{-1}\left[\mathbf{V}_i^+|\mathbf{V}_i^-\right]\begin{bmatrix}\mathbf{X}_i & 0\\ 0 & \mathbf{X}_i^{-1}\end{bmatrix}, \tag{17}$$

and

$$\begin{aligned}\mathbf{X}_{i,(m,n)} &= e^{k_{m,i}d(i)\delta_{m,n}} = e^{\lambda_{m,i}d(i)\Omega/c_0\delta_{m,n}},\\ \mathbf{V}_i^\pm &= \left[\mathbf{V}_{-N,i}^\pm,\ldots,\mathbf{V}_{0,i}^\pm,\ldots,\mathbf{V}_{N,i}^\pm\right].\end{aligned} \tag{18}$$

Therefore, if an electromagnetic wave incident normally on the structure from the left layer, transmission and reflection can be calculated using

$$\begin{pmatrix}\mathbf{T}_x\\ \mathbf{T}_y\\ 0\\ 0\end{pmatrix} = \Pi_{i=0}^{M}\mathbf{Q}_{(M-i)\to(M+1-i)}\begin{pmatrix}\mathbf{I}_x\\ \mathbf{I}_y\\ \mathbf{R}_x\\ \mathbf{R}_y\end{pmatrix}, \tag{19}$$

where $\mathbf{I}_x, \mathbf{I}_y$ are two vectors whose non-zero components determine the incident electromagnetic field and $(\mathbf{T}_x, \mathbf{T}_y)$ and $(\mathbf{R}_x, \mathbf{R}_y)$ show the harmonic vectors of transmission and reflection shown in Fig. 3. The left-most and right-most layers are stationary, where fields enter/exit the overall time-modulated slab. Thus scattered wave from the TV structure radiate away into time-invariant space. In writing Eq. (19), it is assumed that the incident wave comes merely from left and not from right resulting in zero field values in the corresponding vector in Eq. (19).

By defining $\mathbf{Q} = \Pi_{i=0}^{M}\mathbf{Q}_{(M-i)\to(M+1-i)}$, the transmission $(\mathbf{T}_x, \mathbf{T}_y)$ and reflection $(\mathbf{R}_x, \mathbf{R}_y)$ coefficients can be calculated as

$$\begin{aligned}\mathbf{R} &= -\mathbf{Q}_{22}^{-1}\mathbf{Q}_{21}\mathbf{I},\\ \mathbf{T} &= \left(\mathbf{Q}_{11} - \mathbf{Q}_{22}^{-1}\mathbf{Q}_{21}\right)\mathbf{I}.\end{aligned} \tag{20}$$

Let us now investigate a slab of the proposed sinusoidally time-modulated anisotropic media under normal (along z) plane-wave incidence and observe if any of the effects such as polarization rotation can indeed be seen from an actual excited structure similar to what is achieved with static chiral/gyrotropic materials. We assume a 1D slab structure with the TV permittivity tensor as Eq. (10), with the length of L in free space and excite the slab with a plane wave with the angular frequency ω_0 that is incident normal to the slab. The transmission and reflection from the slab are calculated using the transfer matrix method presented here.

In order to investigate the possibility of realizing polarization rotation, transmission from such a time-modulated anisotopic slab with the permittivity tensor Eq. (10) is considered when a linearly polarized incident field normally excites the slab. The transmission from the TV slab with

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the length of $L_{norm} = L/\lambda_g$ ($\lambda_g = 2\pi c/(\omega_0\sqrt{\varepsilon_{st}})$) and with the permittivity tensor Eq. (10) with parameters $M = 0.2$ and $\Omega = \omega_0/400$ which is matched to free space ($\varepsilon_{st} = 1$), is demonstrated in Fig. 4 where the linearly polarized incident plane wave with the angular frequency of ω_0 has a polarization of $\mathrm{E}_{in} = E_0/\sqrt{2}(\hat{x}+\hat{y})$. It is worth noting that the slab has the same permittivity in the absence of time modulation, which matches to the permittivity of the outside space. Under such circumstance partial reflections caused by applying time modulation become almost negligible and transmission plays the vital role, and therefore only the transmission is reported herein. The transmitted power of three harmonics $\omega_0, \omega_0 \pm \Omega$ and the axial ratio and tilt angle of the main harmonic versus L_{norm} are depicted in Fig. 4. This figure shows that the polarization of the main harmonic of the transmitted wave is dependent on the length of the slab, and with the choice of the proper length, the polarization of the main harmonic can become linear, circular, or elliptical. It should be noted that in Fig. 4, only a limited range of slab lengths is shown for clarity, and thus a seemingly constant tilt angle may appear for the LP transmitted wave. However, by extending the length of the slab to a wider range, full range of [−90°, +90°] is covered. Therefore, this structure can be used for polarization conversion. The length of the slab can be engineered to convert linear polarization to an arbitrary polarization.

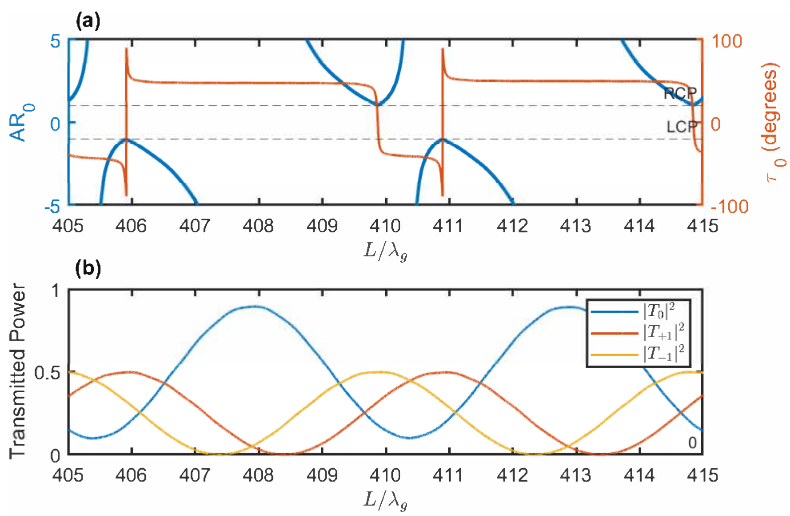

Fig. 4. (a) Axial ratio and tilt angle of the main harmonic and (b) the transmitted power of the main and ±1 harmonics in transmission form a slab with modulation strength of 0.2 and modulation frequency of $\Omega = \omega/400$. Here, the length of the slab is normalized to $\lambda_g = 2\pi c/(\omega_0\sqrt{\varepsilon_{st}})$.

As Fig. 4 shows, one can engineer the length of the slab to have polarization conversion from linear to LCP/RCP. As the transmitted wave may have three non-zero harmonics ($\omega_0, \omega_0 \pm \Omega$), one can choose a length where the transmitted power is maximum at 0^{th} harmonic. The transmission of the slab with the length of $L_{norm} = 996$, at which the zeroth order trnasmission reaches its maximum is shown in Fig. 5. It can be seen that the main harmonic is RCP with the maximum power of 0.5. Also, another harmonic is present which is avoidable in this structure under linearly polarized excitation. It is worth noting that unlike the static anisotropic medium that only converts the linear to CP polarization using a specified linearly incident plane-wave with 45 degrees tilt angle, our proposed TV slab can be used for linear to CP polarization conversion for any linearly polarized plane wave with any tilt angle.

In order to observe the polarization rotation of the slab, transmission from the slab with the length of $L_{norm} = 59.7$ which is excited by a horizontally polarized electric field at the frequency

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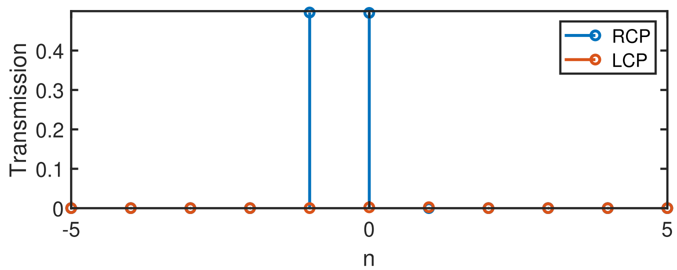

Fig. 5. The LCP and RCP components of different harmonics of the transmitted wave from a slab with the length of $L_{norm} = 996$ while excitation has linear polarization.

ω_0 is demonstrated in Fig. 6(a). It is shown that the plane of polarization of the main harmonic is rotated. However, the plane of polarization does not rotate continuously during the slab.

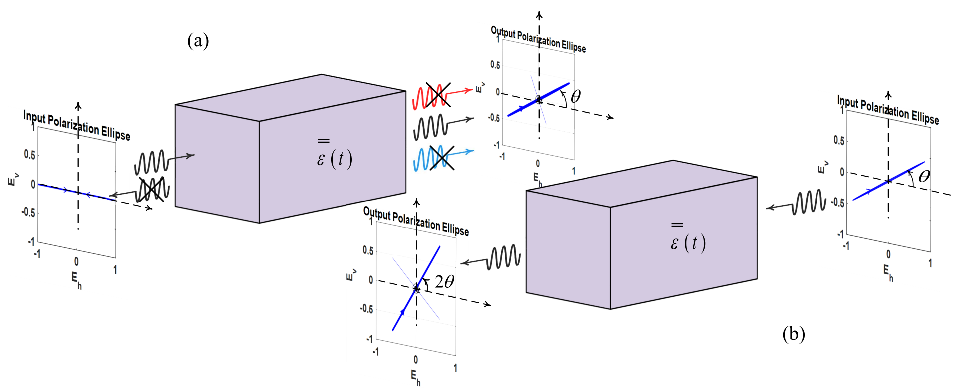

Fig. 6. (a) Transmission in the +z direction and (b) -z direction from a sinusoidal time-modulated slab with $\varepsilon_{st} = 1$, $L_{norm} = 59.7$, $M = 0.2$, and $\omega/\Omega = 400$ while exciting by a linearly polarized horizontal electric field. Here, the length of the slab is normalized to $\lambda_g = 2\pi c/(\omega_0\sqrt{\varepsilon_{st}})$.

To investigate the reciprocity of such a slab, we can use the propagation of a linear polarized horizontal electric field and then reflect it in the backward propagation which is depicted in Fig. 6(b). It can be seen that if the direction of propagation is reversed, the plane of polarization rotates in the same direction showing the slab has a non-reciprocal behavior. Therefore, our proposed time-modulated anisotropic media represents a Faraday rotator behaviour without any magnetic bias field. This behaviour is also similar to a medium in which the electric field density is dependent on the curl of the electric field which is introduced in [59] as a non-reciprocal chiral medium.

It should be noted that this structure can act as a frequency converter if it is excited by RCP/LCP waves and if it has the proper length at which the polarization conversion from liner to CP occurs. For example while exciting a slab of length $L_{norm} = 996$ by an LCP plane wave, the total power can be converted to the blue-shifted harmonic. The frequency conversion is also reported in the reflection from a rotating anisotropic surface with the rotating frequency of $\Omega/2$ that constructs a similar permittivity tensor [58]. However, our proposed structure is a bulk material and it is fundamentally different from the rotating anisotropic surface.

4. Possible realization using electro-optic effects

Our proposed time-periodic permittivity tensor may indeed be physically realized simply with time-variations of achiral crystal. Note that by applying an electric field to a nonlinear crystal with Pockels electro-optic effect, the ion lattice will slightly deform and the impermeability tensor ($\bar{\bar{\varepsilon}}^{-1}$) will change as $\eta_{ij}(\mathbf{E}) = \eta_{ij}(0) + r_{ijk}E_k$, where $\mathbf{E}$ is the applied electric field and the coefficients r_{ijk} are the linear (Pockels) electro-optic coefficients [47]. The impermeability tensor is also dependent on the quadratic electro-optic effect. However, in most practical applications, the quadratic effect is small compared to the linear effect and can be neglected when the linear effect is present. Given that the medium is lossless and optically inactive, the permittivity tensor is symmetric, and $r_{ijk} = r_{jik}$ and the abbreviated notation can be used which uses 2 indices instead of 3 indices in the form that $r_{11k} = r_{1k}, r_{22k} = r_{2k}, r_{33k} = r_{3k}, r_{23k} = r_{32k} = r_{4k}, r_{13k} = r_{31k} = r_{5k}, r_{12k} = r_{21k} = r_{6k}$ [47]. Our aim is to see whether the permittivity tensor Eq. (10) can be implemented by applying an electric field to one of these crystal classes or not.

Among the 32 point groups, the $\bar{6}m2$ symmetry group is our candidate of choice. The electro-optic tensor r_{ijk} and the permittivity tensor ε_{ij} in this symmetry group are in the form

$$r_{ijk} = \begin{pmatrix} r_{11} & 0 & 0 \\ -r_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -r_{11} & 0 \end{pmatrix}, \quad \varepsilon = \varepsilon_0 \begin{pmatrix} n_o^2 & 0 & 0 \\ 0 & n_o^2 & 0 \\ 0 & 0 & n_e^2 \end{pmatrix}. \tag{21}$$

By applying an electric field in the form of $\mathbf{E} = E_x\hat{x} + E_y\hat{y}$ to this crystal, the impermeability tensor can be calculated. After some mathematical calculations and by assuming $r_{11}^2(E_x^2 + E_y^2) \ll 1/n_o^4$, the permittivity tensor can be written as

$$\varepsilon(\mathbf{E}) \simeq \varepsilon_0 \begin{pmatrix} n_o^2 - r_{11}n_o^4E_x & r_{11}n_o^4E_y & 0 \\ r_{11}n_o^4E_y & n_o^2 + r_{11}n_o^4E_x & 0 \\ 0 & 0 & n_e^2 \end{pmatrix}, \tag{22}$$

in which the higher order terms $O(r_{11}^2)$ and $O(r_{11}^3)$ are neglected. The details are presented in the Supplement 1. Now, by considering an external bias electric field in the form of $E_x = E_0 \cos\Omega t$ and $E_y = E_0 \sin\Omega t$, the $xx(yy)$ component of the permittivity tensor will be $\varepsilon_{xx(yy)} = \varepsilon_0(\varepsilon_{xx}(0) \mp r_{11}\varepsilon_{xx}^2(0)E_0 \cos\Omega t)$ where the $-$ sign is related to xx and $+$ is related to the yy component and the xy and yx components will be $\varepsilon_{xy} = \varepsilon_{yx} = \varepsilon_0(r_{11}\varepsilon_{xx}^2(0)E_0 \sin\Omega t)$. By comparing these relations with the tensor Eq. (10), it becomes clear that $\Delta\varepsilon_{xx} = -r_{11}\varepsilon_{xx}^2(0)E_0$, $\Delta\varepsilon_{yy} = r_{11}\varepsilon_{xx}^2(0)E_0$, and $\Delta\varepsilon_{xy} = r_{11}\varepsilon_{xx}^2(0)E_0$. A pictorial implementation of such a structure is depicted in Fig. 7. The bias electric field may be applied across two pairs of plates which are placed parallel to the x and y directions. The voltages that are applied in the x and y directions must have a phase difference of 90 degrees.

A potential candidate with the point group symmetry of $\bar{6}m2$, is GaSe (Gallium Selenide) with the ordinary refractive index of $n_o = 2.91$ and an electro-optic coefficient of $n_o^3 r_{22} = 27.5 \times 10^{-12} m/V$ at an operating wavelength of 1.06 um. To achieve a giant polarization rotation of -90 degrees, a modulation strength of $\Delta\varepsilon/\varepsilon_o \approx 10^{-4}$ and modulation frequency of $\Omega = \omega_0/3000$ would suffice. Such modulation may be realized by applying bias voltage of about

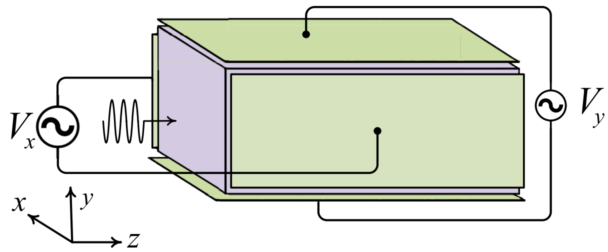

Fig. 7. Implementation of the sinusoidal time-modulated anisotropic slab using crystals with an electro-optic effect.

10 volts across a waveguide with half wavelength thickness. An interesting research venue is to explore various materials and methods for experimentation of the central ideas put forth in this paper.

5. Conclusion

In conclusion, we first showed that time modulations of isotropic achiral media can not result in CP eigenmodes and thus remain achiral. Therefore, we need to start from a static achiral anisotropic medium and introduce a suitable time modulation emulating a temporal rotation of the principal axes of the permittivity tensor which results in non-degenerate CP eigenmodes. The form of our proposed permittivity tensor does not have time-reversal symmetry and is non-reciprocal. Thus we *emulate* a gyrotropic medium without the need for any magnetic bias. While there isn't any true equivalence between our proposed medium and gyrotropic media, our proposed medium *mimics* the behavior of gyrotropic media. Transmission from a slab of our proposed time-modulated anisotropic media shows polarization conversion and the length of the slab can be chosen to convert any linear polarization to CP waves. This phenomenon can be realized by using the linear electro-optic effect in $\bar{6}m2$ crystals.

Disclosures. The authors declare no conflicts of interest.

Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

Supplemental document. See Supplement 1 for supporting content.

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